1. $\int 2x\sqrt{3x-5}\,dx$

$u = 3x-5$ $\quad \int 2x \cdot u^{1/2} \cdot \frac{du}{3}$

$\frac{du}{dx} = 3$ $\quad \frac{2}{3}\int \left(\frac{u+5}{3}\right) \cdot u^{1/2} \cdot du$

$dx = \frac{du}{3}$ $\quad \frac{2}{9}\int u^{3/2} + 5u^{1/2}\,du$

$x = \frac{u+5}{3}$ $\quad \frac{2}{9}\left[\frac{2}{5}u^{5/2} + \frac{10}{3}u^{3/2} + C\right]$

$\frac{4}{45}(3x-5)^{5/2} + \frac{20}{27}(3x-5)^{5/2} + C$

2. $\int 3x\sin(2x^2)\,dx$

$u = 2x^2$ $\quad \int 3x \sin u \cdot \frac{du}{4x}$

$\frac{du}{dx} = 4x$ $\quad \frac{3}{4}\int \sin u\,du$

$dx = \frac{du}{4x}$ $\quad -\frac{3}{4}\cos u + C$

$-\frac{3}{4}\cos(2x^2) + C$

3. $\int \frac{-x\csc^2(2x^2)}{\cot(2x^2)}\,dx$

$u = \cot(2x^2)$ $\quad \int \frac{-x\csc^2(2x^2)}{u} \cdot \frac{du}{-4x\csc^2(2x^2)}$

$\frac{du}{dx} = -\csc^2(2x^2)\cdot 4x$ $\quad \frac{1}{4}\int \frac{1}{u}\,du$

$dx = \frac{du}{-4x\csc^2(2x^2)}$ $\quad \frac{1}{4}\ln|u| + C$

$\frac{1}{4}\ln|\cot(2x^2)| + C$

4. $\int \frac{3x^2 - 4\sqrt{x}}{\sqrt[3]{x}}\,dx$

$\int (3x^2 - 4x^{1/2})x^{-1/3}\,dx$

$\int 3x^{5/3} - 4x^{1/6}\,dx$

$\frac{9}{8}x^{8/3} - \frac{24}{7}x^{7/6} + C$

5. $\int \frac{4x^2}{\sqrt{2x-7}}\,dx$

$u = 2x-7$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$x = \frac{u+7}{2}$

$\int \frac{4x^2}{u^{1/2}} \cdot \frac{du}{2}$

$2\int \left(\frac{u+7}{2}\right)^2 u^{-1/2}\,du$

$2\int \left(\frac{u^2+14u+49}{4}\right) u^{-1/2}\,du$

$\frac{1}{2}\int u^{3/2} + 14u^{1/2} + 49u^{-1/2}\,du$

$\frac{1}{2}\left[\frac{2}{5}u^{5/2} + \frac{28}{3}u^{3/2} + 98u^{1/2} + C\right]$

$\frac{1}{5}(2x-7)^{5/2} + \frac{14}{3}(2x-7)^{3/2} + 49(2x-7)^{1/2} + C$

6. $\int \frac{e^{3x}}{e^{3x}-7}$

$u = e^{3x}-7$

$\frac{du}{dx} = e^{3x}\cdot 3$

$dx = \frac{du}{3e^{3x}}$

$\int \frac{e^{3x}}{u} \cdot \frac{du}{3e^{3x}}$

$\frac{1}{3}\int \frac{1}{u}\,du$

$\frac{1}{3}\ln|u| + C$

$\frac{1}{3}\ln|e^{3x}-7| + C$

7. $\int 3^{\cos(2x)} \sin(2x)\,dx$

$u = \cos(2x)$

$\frac{du}{dx} = -2\sin(2x)$

$dx = \frac{du}{-2\sin(2x)}$

$\int 3^u \cdot \sin(2x) \cdot \frac{du}{-2\sin(2x)}$

$-\frac{1}{2}\int 3^u\,du$

$-\frac{1}{2}\cdot\frac{3^u}{\ln 3} + C$

$-\frac{1}{2}\cdot\frac{3^{\cos(2x)}}{\ln 3} + C$

8. $\int \frac{e^{4x-2}}{\pi}\,dx$

$\frac{1}{\pi}\int e^{4x-2}\,dx$

$u = 4x-2$

$\frac{du}{dx} = 4$

$dx = \frac{du}{4}$

$\frac{1}{\pi}\int e^u \cdot \frac{du}{4}$

$\frac{1}{4\pi}\int e^u\,du$

$\frac{1}{4\pi}e^u + C$

$\frac{1}{4\pi}e^{4x-2} + C$

9. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$u = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}\,du$

$\int \frac{\sin u}{\sqrt{x}} \cdot 2\sqrt{x}\,du$

$2\int \sin u\,du$

$-2\cos u + C$

$-2\cos\sqrt{x} + C$

10. $\int x\sqrt{2x-1}\,dx$

$u = 2x-1$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$x = \frac{u+1}{2}$

$\int x \cdot u^{1/2} \cdot \frac{du}{2}$

$\frac{1}{2}\int \left(\frac{u+1}{2}\right) u^{1/2}\,du$

$\frac{1}{4}\int u^{3/2} + u^{1/2}\,du$

$\frac{1}{4}\left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C\right]$

$\frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C$