Calculus 1 - Limits

Worksheet 5

Limits Involving Trig Functions

Calculus 1 - Limits - Worksheet 5 – Limits Involving Trig Functions

1. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\tan x}{3x}$$

2. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin x}{\sin (4x)}$$

3. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{1 - \cos (2x)}{4x}$$

4. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin^2(2x)}{x^2}$$

5. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{1 - \sin\left(\frac{\pi}{2} - x\right)}{2x}$$

6. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{6\cos\left(\frac{\pi}{2} - x\right)}{5x}$$

7. Evaluate this limit using a table of values.

$$\lim_{x \to \frac{\pi}{2}} \ln(\sin x)$$

8. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{x + \sin x}{x}$$

9. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{1 - \cos(2x)}{4x}$$

10. Evaluate this limit using a table of values.

$$\lim_{x \to 0} e^x \cos x$$

11. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin^2 x}{2x}$$

12. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{3\cos(3x)}{5\sin(4x)}$$

13. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin x}{2x^2 - x}$$

14. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin\,(2x)}{(4x)}$$

15. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\cos x - 1}{7x}$$

16. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin(2 + x) - \sin 2}{x}$$

17. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{x^3 - x + \sin x}{x^3}$$

18. Evaluate this limit using a table of values.

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$$

19. Evaluate this limit using a table of values.

$$\lim_{x \to 0} x\cos(2x)$$

20. Evaluate this limit using a table of values.

$$\lim_{x \to 0} e^{-x}\cos\,(3x)$$

Answers - Calculus 1 - Limits - Worksheet 5 – Limits Involving Trig Functions

1. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\tan x}{3x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{\tan x}{3x}$
0.1	0.33445
0.01	0.33334
0.001	0.33333
0	Undefined
-0.001	0.33333
-0.01	0.33334
-0.1	0.33445

The table shows that the limit approaches $\frac{1}{3}$ as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{\tan x}{3x} = \frac{1}{3}$

2. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin x}{\sin\,(4x)}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{\sin x}{\sin\,(4x)}$
0.1	0.25637
0.01	0.25006
0.001	0.25
0	Undefined
-0.001	0.25
-0.01	0.25006
-0.1	0.25637

The table shows that the limit approaches $\frac{1}{4}$ as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{\sin x}{\sin\,(4x)} = \frac{1}{4}$

3. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{1 - \cos(2x)}{4x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{1 - \cos(2x)}{4x}$
0.1	0.04983
0.01	0.005
0.001	0.0005
0	Undefined
-0.001	-0.0005
-0.01	-0.005
-0.1	-0.04983

The table shows that the limit approaches 0 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{1-\cos(2x)}{4x} = 0$

4. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin^2(2x)}{x^2}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{\sin^2(2x)}{x^2}$
0.1	3.947
0.01	3.9995
0.001	4
0	Undefined
-0.001	4
-0.01	3.9995
-0.1	3.947

The table shows that the limit approaches 4 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{\sin^2(2x)}{x^2} = 4$

5. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{1 - \sin\left(\frac{\pi}{2} - x\right)}{2x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{1 - \sin\left(\frac{\pi}{2} - x\right)}{2x}$
0.1	0.02498
0.01	0.0025
0.001	0.00025
0	Undefined
-0.001	-0.00025
-0.01	-0.0025
-0.1	-0.02498

The table shows that the limit approaches 0 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{1 - \sin\left(\frac{\pi}{2} - x\right)}{2x} = 0$

6. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{6\cos\left(\frac{\pi}{2} - x\right)}{5x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{6\cos\left(\frac{\pi}{2} - x\right)}{5x}$
0.1	1.198
0.01	1.2
0.001	1.2
0	Undefined
-0.001	1.2
-0.01	1.2
-0.1	1.198

The table shows that the limit approaches $\frac{6}{5}$ as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{6\cos\left(\frac{\pi}{2} - x\right)}{5x} = \frac{6}{5}$

7. Evaluate this limit using a table of values.

$$\lim_{x \to \frac{\pi}{2}} \ln(\sin x)$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\ln(\sin x)$
2	-0.0951
1.75	-0.0161
1.60	-0.0004
$\pi/2$	0
1.56	-0.00006
1.25	-0.0524
1	-0.1726

The table shows that the limit approaches 0 as $x \to 0$.

Answer: $\lim_{x \to \frac{\pi}{2}} \ln(\sin x) = 0$

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8. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{x + \sin x}{x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{x + \sin x}{x}$
0.1	1.9983
0.01	1.9999
0.001	2
0	Undefined
-0.001	2
-0.01	1.9999
-0.1	1.9983

The table shows that the limit approaches 2 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{x + \sin x}{x} = 2$

9. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{1 - \cos(2x)}{4x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{1 - \cos(2x)}{4x}$
0.1	0.04983
0.01	0.00499
0.001	0.0005
0	Undefined
-0.001	-0.0005
-0.01	-0.00499
-0.1	-0.04983

The table shows that the limit approaches 0 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{1-\cos(2x)}{4x} = 0$

10. Evaluate this limit using a table of values.

$$\lim_{x \to 0} e^x \cos x$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$e^x \cos x$
0.1	1.0996
0.01	1.01
0.001	1.001
0	Undefined
-0.001	0.999
-0.01	0.99
-0.1	0.90032

The table shows that the limit approaches 1 as $x \to 0$.

Answer: $\lim_{x \to 0} e^x \cos x = 1$

11. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin^2 x}{2x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{\sin^2 x}{2x}$
0.1	0.04983
0.01	0.00499
0.001	0.0005
0	Undefined
-0.001	-0.0005
-0.01	-0.00499
-0.1	-0.04983

The table shows that the limit approaches 0 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{\sin^2 x}{2x} = 0$

12. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{3\cos(3x)}{5\sin(4x)}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{3\cos(3x)}{5\sin(4x)}$
0.1	1.4718
0.01	14.997
0.001	150
0	Undefined
-0.001	-150
-0.01	-14.997
-0.1	-1.4719

The table shows that the limit does not approach a value as $x \to 0$. Therefore, the limit does not exist.

Answer: $\lim_{x \to 0} \frac{3\cos(3x)}{5\sin(4x)} = DNE$

13. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin x}{2x^2 - x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{\sin x}{2x^2 - x}$
0.1	-1.24792
0.01	-1.02039
0.001	-1.00200
0	Undefined
-0.001	-0.99800
-0.01	-0.98038
-0.1	-0.93195

The table shows that the limit approaches -1 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{\sin x}{2x^2 - x} = -1$

14. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\sin(2x)}{(4x)}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{\sin(2x)}{(4x)}$
0.1	0.49667
0.01	0.49997
0.001	0.49999
0	Undefined
-0.001	0.49999
-0.01	0.49997
-0.1	0.49667

The table shows that the limit approaches 0.5 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{\sin(2x)}{(4x)} = \frac{1}{2}$

15. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{\cos x - 1}{7x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{\cos x - 1}{7x}$
0.1	-0.00714
0.01	-0.00071
0.001	-0.00007
0	Undefined
-0.001	0.00007
-0.01	0.000714
-0.1	0.00714

The table shows that the limit approaches 0 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{\cos x - 1}{7x} = 0$

16. Evaluate this limit to the nearest three decimal places using a table of values.

$$\lim_{x \to 0} \frac{\sin(2 + x) - \sin 2}{x}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{\sin(2 + x) - \sin 2}{x}$
0.1	-0.46088
0.01	-0.42069
0.001	-0.41660
0	Undefined
-0.001	-0.41569
-0.01	-0.41159
-0.1	-0.37003

The table shows that the limit approaches -0.416 as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{\sin(2+x) - \sin 2}{x} = -0.416$

17. Evaluate this limit using a table of values.

$$\lim_{x \to 0} \frac{x^3 - x + \sin x}{x^3}$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$\frac{x^3 - x + \sin x}{x^3}$
0.1	0.83342
0.01	0.83333
0.001	0.83333
0	Undefined
-0.001	0.83333
-0.01	0.83333
-0.1	0.83342

The table shows that the limit approaches 0.83333333 or $\frac{5}{6}$ as $x \to 0$.

Answer: $\lim_{x \to 0} \frac{x^3 - x + \sin x}{x^3} = \frac{5}{6}$

18. Evaluate this limit using a table of values.

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$x \sin\left(\frac{1}{x}\right)$
0.1	-0.05440
0.01	-0.00506
0.001	0.00083
0	Undefined
-0.001	0.00083
-0.01	-0.00506
-0.1	-0.05440

The table shows that the limit approaches 0 as $x \to 0$.

Answer: $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$

19. Evaluate this limit using a table of values.

$$\lim_{x \to 0} x\cos(2x)$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$x\cos(2x)$
0.1	0.09801
0.01	0.00999
0.001	0.00099
0	Undefined
-0.001	-0.00099
-0.01	-0.00999
-0.1	-0.09801

The table shows that the limit approaches 0 as $x \to 0$.

Answer: $\lim_{x \to 0} x\cos(2x) = 0$

20. Evaluate this limit using a table of values.

$$\lim_{x \to 0} e^{-x} \cos(3x)$$

Solution:

Calculate the value of the limit as the values of x approaches 0.

x	$e^{-x} \cos(3x)$
0.1	0.86442
0.01	0.98960
0.001	0.99899
0	1
-0.001	1.00099
-0.01	1.00959
-0.1	1.05581

The table shows that the limit approaches 1 as $x \to 0$.

Answer: $\lim_{x \to 0} e^{-x} \cos(3x) = 1$