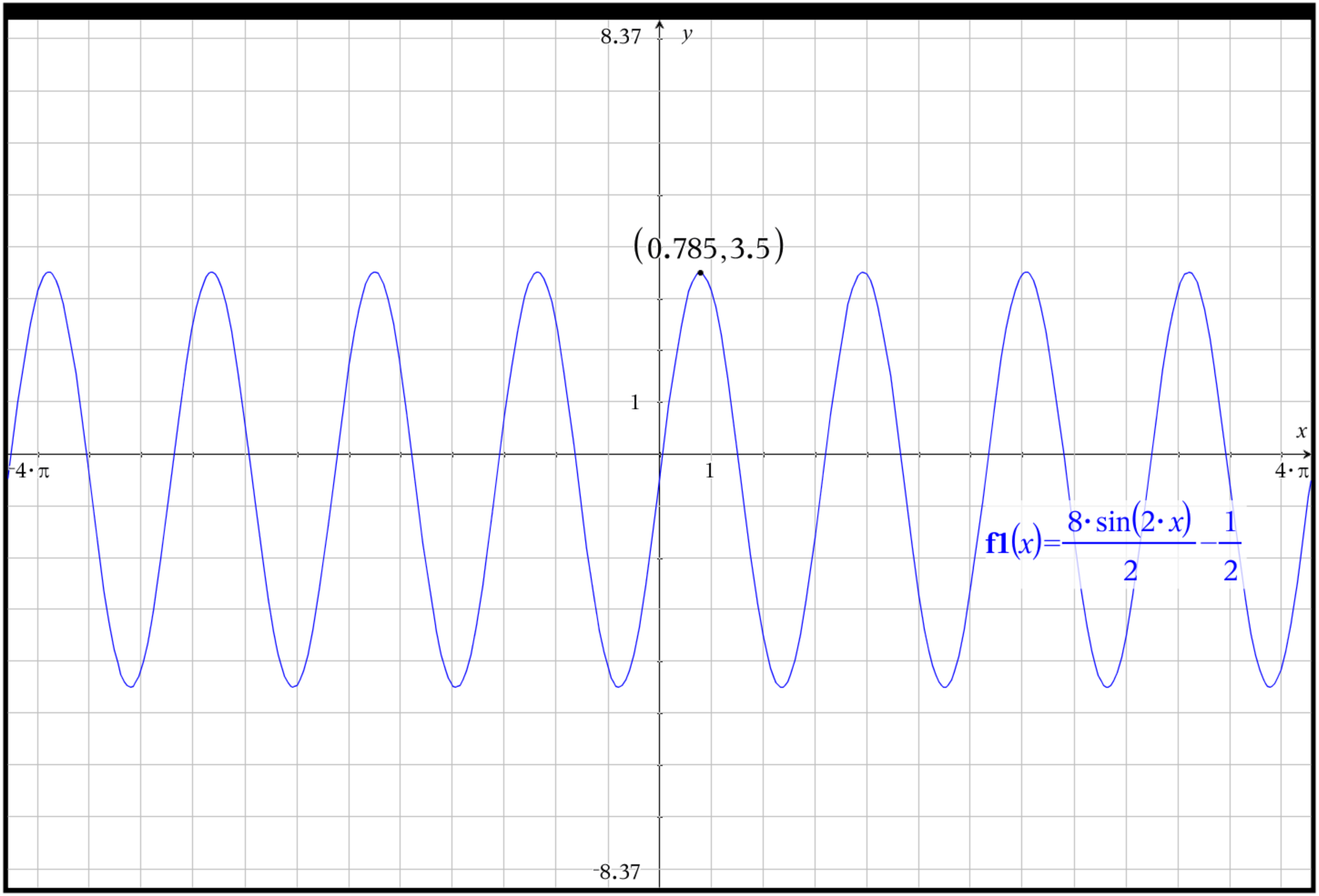
Problem 1

What is the greatest possible value of f if $f(x) = \frac{8\sin 2x}{2} - \frac{1}{2}$?

1. **7/2 or 3.5** The discussion in Lesson 9 about the definition of the sine function and the unit circle made it clear that the value of the sine function ranges from -1 to 1. Therefore, the maximum value of $\frac{8\sin 2x}{2}-\frac{1}{2}$ is $\frac{8(1)}{2}-\frac{1}{2}=\frac{7}{2}$ or 3.5.


8.37
y
(0.785,3.5)
1
x
-4·π
1
4·π
$f1(x)=\frac{8\cdot\sin(2\cdot x)}{2}-\frac{1}{2}$
-8.37

If $\cos\left(\frac{\pi}{3}\right) = a$, what is the value of $\left(\frac{a}{3}\right)^2$?

2. **1/36 or .027 or .028** An radian measure of $\pi/3$ is equivalent to $60°$. If you haven't memorized the fact that $\cos(60°) = ½$, you can derive it from the Reference Information at the beginning of every SAT Math section, which includes the $30°$-$60°$-$90°$ special right triangle. Since $a = ½$, $(a/3)^2 = (1/6)^2 = 1/36$.

If $(\sin x - \cos x)^2 = 0.83$, what is the value of $(\sin x + \cos x)^2$?

3. **1.17**

$$(\sin x - \cos x)^2 = 0.83$$

FOIL: $\sin^2 x - 2\sin x \cos x + \cos^2 x = 0.83$

Regroup: $\sin^2 x + \cos^2 x - 2\sin x \cos x = 0.83$

Simplify: $1 - 2\sin x \cos x = 0.83$

Subtract 1: $-2\sin x \cos x = -0.17$

Multiply by -1: $2\sin x \cos x = 0.17$

Evaluate this expression: $(\sin x + \cos x)^2$

FOIL: $\sin^2 x + 2\sin x \cos x + \cos^2 x$

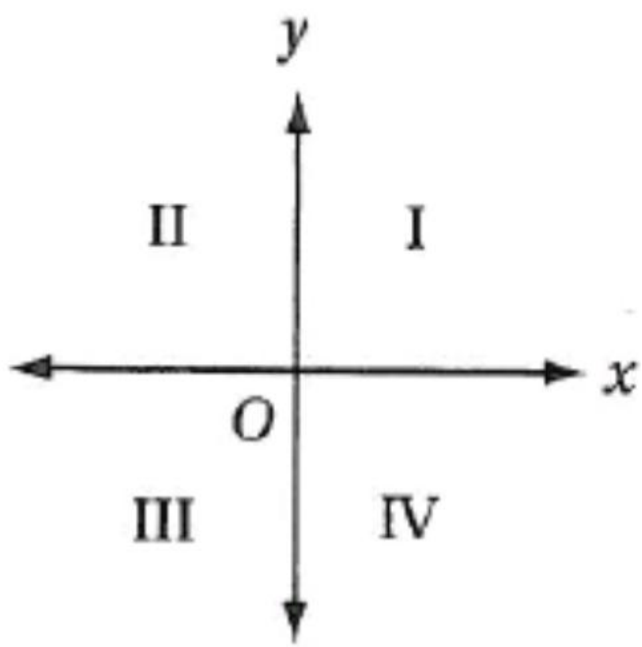
Regroup: $\sin^2 x + \cos^2 x + 2\sin x \cos x$

Substitute: $1 + 0.17 = 1.17$

Which of the following is equivalent to $\dfrac{\sin\left(\dfrac{\pi}{6}\right)}{\cos\left(\dfrac{\pi}{3}\right)}$?

A) $\dfrac{1}{\sqrt{6}}$ B) $\dfrac{1}{\sqrt{3}}$ C) $\dfrac{\sqrt{3}}{\sqrt{2}}$ D) 1

4. **D** $\sin(\pi/6) = ½$ and $\cos(\pi/3) = ½$, so $\sin(\pi/6)/\cos(\pi/3) = 1$.



If $\sin\theta < 0$ and $\sin\theta\cos\theta < 0$, then θ must be in which quadrant of the figure above?

A) I B) II C) III D) IV

5. **D** If $\sin \theta < 0$, then θ must be either in quadrant III or in quadrant IV. (Remember that sine corresponds to the y-coordinates on the unit circle, so it is negative in those quadrants where the y-coordinates are negative.) If $\sin \theta \cos \theta < 0$, then $\cos \theta$ must be positive (because

a negative times a positive is a negative). Since $\cos \theta$ is only positive in quadrants I and IV (because cosine corresponds to the x-coordinates on the unit circle), θ must be in quadrant IV

If $\sin x = \frac{a}{b}$ and $0 < x < \frac{\pi}{2}$, which of the following expressions is equal to $\frac{b}{a}$?

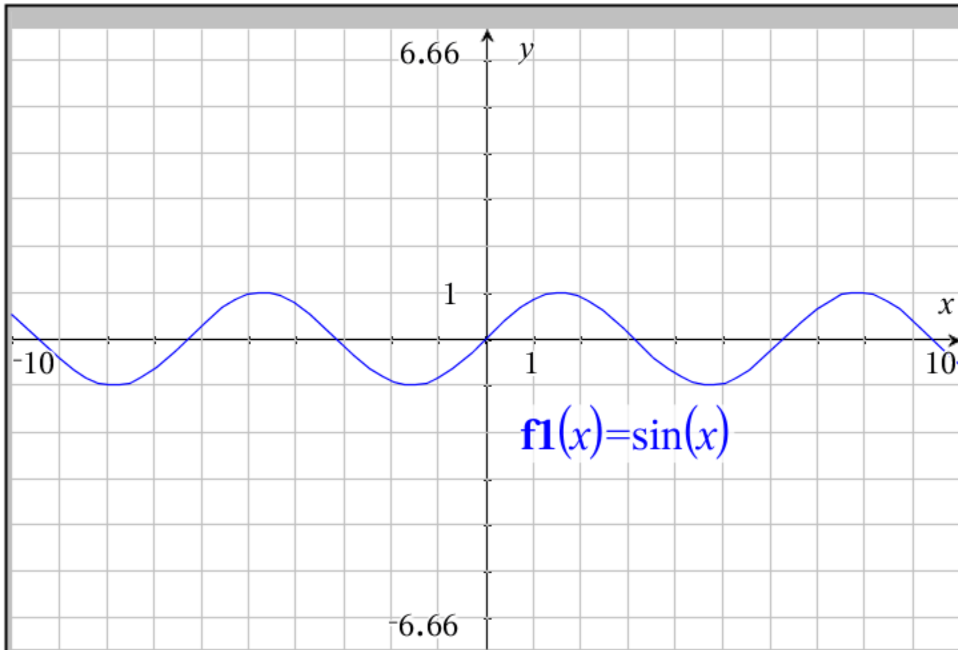
A) $\sin\left(\frac{1}{x}\right)$

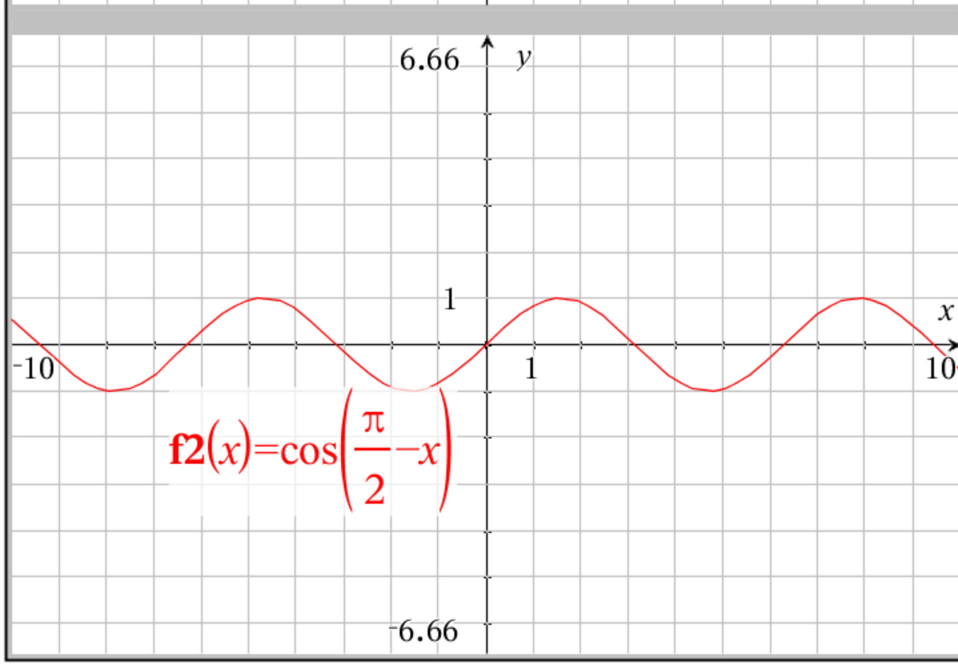
B) $\dfrac{1}{\cos\left(\frac{\pi}{2} - x\right)}$

C) $1 - \sin^2 x$

D) $\sin\left(\frac{\pi}{2} - x\right)$

6. **B** First, notice that a/b and b/a are reciprocals. Next, we can use the identity in Lesson 10 that $\sin x = \cos\left(\frac{\pi}{2} - x\right)$ to see that choice (B) is just the reciprocal of $\sin x$. Alternately, we can just choose a value of x, like $x = 1$, and evaluate $\sin 1 = 0.841$. The correct answer is the expression that gives a value equal to the reciprocal of 0.841, which is $1/0.841 = 1.19$. Plugging in $x = 1$ gives (A) 0.841, (B) 1.19, (C) 0.292, (D) 0.540.





These are the same function

If $\sin x = \frac{a}{b}$ and $0 < x < \frac{\pi}{2}$, which of the following expressions is equal to $\frac{b}{a}$?

A) $\sin\left(\frac{1}{x}\right)$

B) $\dfrac{1}{\cos\left(\frac{\pi}{2} - x\right)}$

C) $1 - \sin^2 x$

D) $\sin\left(\frac{\pi}{2} - x\right)$

If $\sin b = a$, which of the following could be the value of $\cos(b + \pi)$?

A) $\sqrt{a^2 - 1}$

B) $a^2 - 1$

C) $-\sqrt{1 - a^2}$

D) $1 - a^2$

7. **C** Recall from the Pythagorean Identity that $\cos b = \pm\sqrt{1-\sin^2 b}$. Substituting $\sin b = a$ gives $\cos b = \pm\sqrt{1-a^2}$. The angle $b + \pi$ is the reflection of angle b through the origin, so $\cos(b + \pi)$ is the opposite of $\cos b$, which means that $\cos(b+\pi) = \pm\sqrt{1-a^2}$.

If $0 < x < \frac{\pi}{2}$ and $\frac{\cos x}{1 - \sin^2 x} = \frac{3}{2}$, what is the value of $\cos x$?

A) $\frac{1}{9}$

B) $\frac{1}{3}$

C) $\frac{4}{9}$

D) $\frac{2}{3}$

8. **D** Recall from the Pythagorean Identity that $\cos^2 x = 1 - \sin^2 x$.

$$\frac{\cos x}{1-\sin^2 x} = \frac{3}{2}$$

Substitute $\cos^2 x = 1 - \sin^2 x$:

$$\frac{\cos x}{\cos^2 x} = \frac{3}{2}$$

Cancel common factor:

$$\frac{1}{\cos x} = \frac{3}{2}$$

Reciprocate:

$$\cos x = \frac{2}{3}$$