

1.  $\int x^2(x^3 - 1)^4 dx$

$$u = x^3 - 1 \quad \int x^2 u^4 \cdot \frac{du}{3x^2}$$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2 dx$$

$$dx = \frac{du}{3x^2}$$

$$\int \frac{u^4}{3} du$$

$$\frac{u^5}{15} + C$$

$$\frac{(x^3 - 1)^5}{15} + C$$

2.  $\int 5x^3 \sqrt{1 - x^2} dx$

$$u = 1 - x^2 \quad \int 5x u^{1/2} \cdot \frac{du}{-2x}$$

$$\frac{du}{dx} = -2x$$

$$dx = \frac{du}{-2x}$$

$$\int -\frac{5}{2} u^{1/2} du$$

$$-\frac{5}{2} u^{3/2} \cdot \frac{2}{3}$$

$$-\frac{15}{8} (1 - x^2)^{3/2} + C$$

3.  $\int \frac{x^2}{(1+x^3)^2} dx$

$$u = 1 + x^3 \quad \int \frac{x^2}{u^2} \cdot \frac{du}{3x^2}$$

$$\frac{du}{dx} = 3x^2$$

$$dx = \frac{du}{3x^2}$$

$$\int \frac{u^{-2}}{3} du$$

$$-\frac{u^{-1}}{3} + C$$

$$-\frac{1}{3(1+x^3)} + C$$

4.  $\int \sin 2x dx$

$$u = 2x \quad \int \sin u \cdot \frac{du}{2}$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{du}{2}$$

$$\int \frac{1}{2} \sin u du$$

$$-\frac{1}{2} \cos u + C$$

$$-\frac{1}{2} \cos 2x + C$$

5.  $\int \frac{1}{\theta^2} \cos \frac{1}{\theta} d\theta$

$$u = \theta^{-1} \quad \int \frac{1}{\theta^2} \cos u \cdot -\theta^{-2} du$$

$$\frac{du}{d\theta} = -\theta^{-2}$$

$$\frac{du}{d\theta} = -\frac{1}{\theta^2}$$

$$du \cdot \theta^2 = d\theta$$

$$\int -\cos u du$$

$$-\sin u + C$$

$$-\sin \frac{1}{\theta} + C$$

6.  $\int \sin 2x \cos 2x dx$

$$u = \sin 2x \quad \int u \cos u \cdot \frac{du}{2 \cos 2x}$$

$$\frac{du}{dx} = \cos 2x \cdot 2$$

$$dx = \frac{du}{2 \cos 2x}$$

$$\int \frac{1}{2} u du$$

$$\frac{1}{4} u^2 + C$$

$$\frac{\sin^2 2x}{4} + C$$

7.  $\int \frac{3x}{\sqrt[3]{10-x^2}} dx$

$$u = 10 - x^2 \quad \int \frac{3x}{u^{1/3}} \cdot \frac{du}{-2x}$$

$$\frac{du}{dx} = -2x$$

$$dx = \frac{du}{-2x}$$

$$\int -\frac{3}{2} u^{-1/3} du$$

$$-\frac{3}{2} u^{2/3} \cdot \frac{3}{2} + C$$

$$-\frac{9}{4} (10 - x^2)^{2/3} + C$$

8.  $\int x^3 \sqrt{5x^4 + 20} dx$

$$u = 5x^4 + 20 \quad \int x^3 u^{1/2} \cdot \frac{du}{20x^3}$$

$$\frac{du}{dx} = 20x^3$$

$$dx = \frac{du}{20x^3}$$

$$\int \frac{1}{20} u^{1/2} du$$

$$\frac{1}{20} u^{3/2} \cdot \frac{2}{3} + C$$

$$\frac{1}{30} (5x^4 + 20)^{3/2} + C$$

$$9. \int \frac{1}{(x-1)^2} dx$$

$$u = x-1 \quad \int \frac{1}{u^2} \cdot du$$

$$\frac{du}{dx} = 1 \quad \int u^{-2} du$$

$$du = dx \quad -u^{-1} + C$$

$$-\frac{1}{(x-1)} + C$$

$$10. \int (x^2+1)(x^3+3x)^{-5} dx$$

$$u = x^3+3x \quad \int (x^2+1) u^{-5} \cdot \frac{du}{3(x^2+1)}$$

$$\frac{du}{dx} = 3x^2+3 \quad \int \frac{1}{3} u^{-5} du$$

$$dx = \frac{du}{3(x^2+1)} \quad -\frac{1}{12} u^{-4} + C$$

$$-\frac{1}{12} (x^3+3x)^{-4} + C$$

$$11. \int \frac{1}{\sqrt{x}} \sin \sqrt{x} dx$$

$$u = x^{1/2} \quad \int \frac{1}{\sqrt{x}} \sin u \cdot 2\sqrt{x} du$$

$$\frac{du}{dx} = \frac{1}{2} x^{-1/2} \quad \int 2 \sin u du$$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad -2 \cos \sqrt{x} + C$$

$$dx = 2\sqrt{x} du$$

$$12. \int x^2 \sec^2 x^3 dx$$

$$u = x^3 \quad \int x^2 \sec^2 u \cdot \frac{du}{3x^2}$$

$$\frac{du}{dx} = 3x^2 \quad \int \frac{1}{3} \sec^2 u du$$

$$dx = \frac{du}{3x^2} \quad \frac{1}{3} \tan x^3 + C$$

$$13. \int \frac{\cos\left(\frac{3}{x}\right)}{x^2} dx$$

$$u = 3x^{-1} \quad \int \frac{\cos u}{x^2} \cdot \frac{x^2 du}{-3}$$

$$\frac{du}{dx} = -3x^{-2} \quad \int -\frac{1}{3} \cos u du$$

$$\frac{du}{dx} = \frac{-3}{x^2} \quad -\frac{1}{3} \sin\left(\frac{3}{x}\right) + C$$

$$x^2 du = -3 dx$$

$$dx = \frac{x^2 du}{-3}$$

$$14. \int \frac{\sin 2x}{(1-\cos 2x)^3} dx$$

$$u = 1 - \cos 2x \quad \int \frac{\sin 2x}{u^3} \cdot \frac{du}{2 \sin 2x}$$

$$\frac{du}{dx} = \sin 2x \cdot 2 \quad \int \frac{1}{2} u^{-3} du$$

$$dx = \frac{du}{2 \sin 2x} \quad -\frac{1}{4} u^{-2} + C$$

$$-\frac{1}{4} (1-\cos 2x)^{-2} + C$$

$$15. \int \sin(\sin x) \cos x dx$$

$$u = \sin x \quad \int \sin u \cos x \cdot \frac{du}{\cos x}$$

$$\frac{du}{dx} = \cos x \quad \int \sin u du$$

$$du = \frac{du}{\cos x} \quad -\cos(\sin x) + C$$