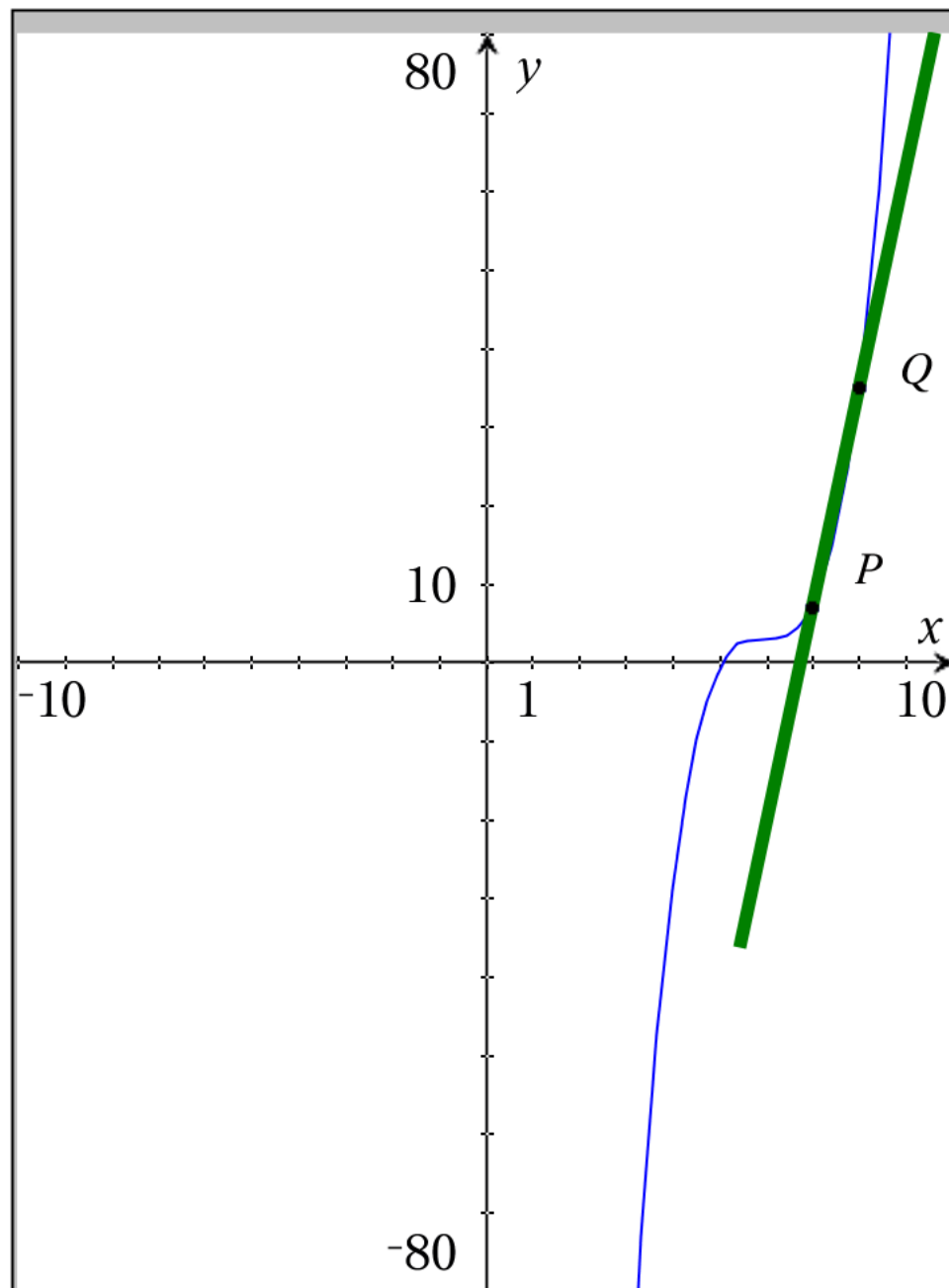


Problem 1



$$f(x) = 4(x-6)^3 + 3$$

$$P = (7, 7)$$

$$Q = (8, 35)$$

$$\text{Slope of } PQ = 28$$

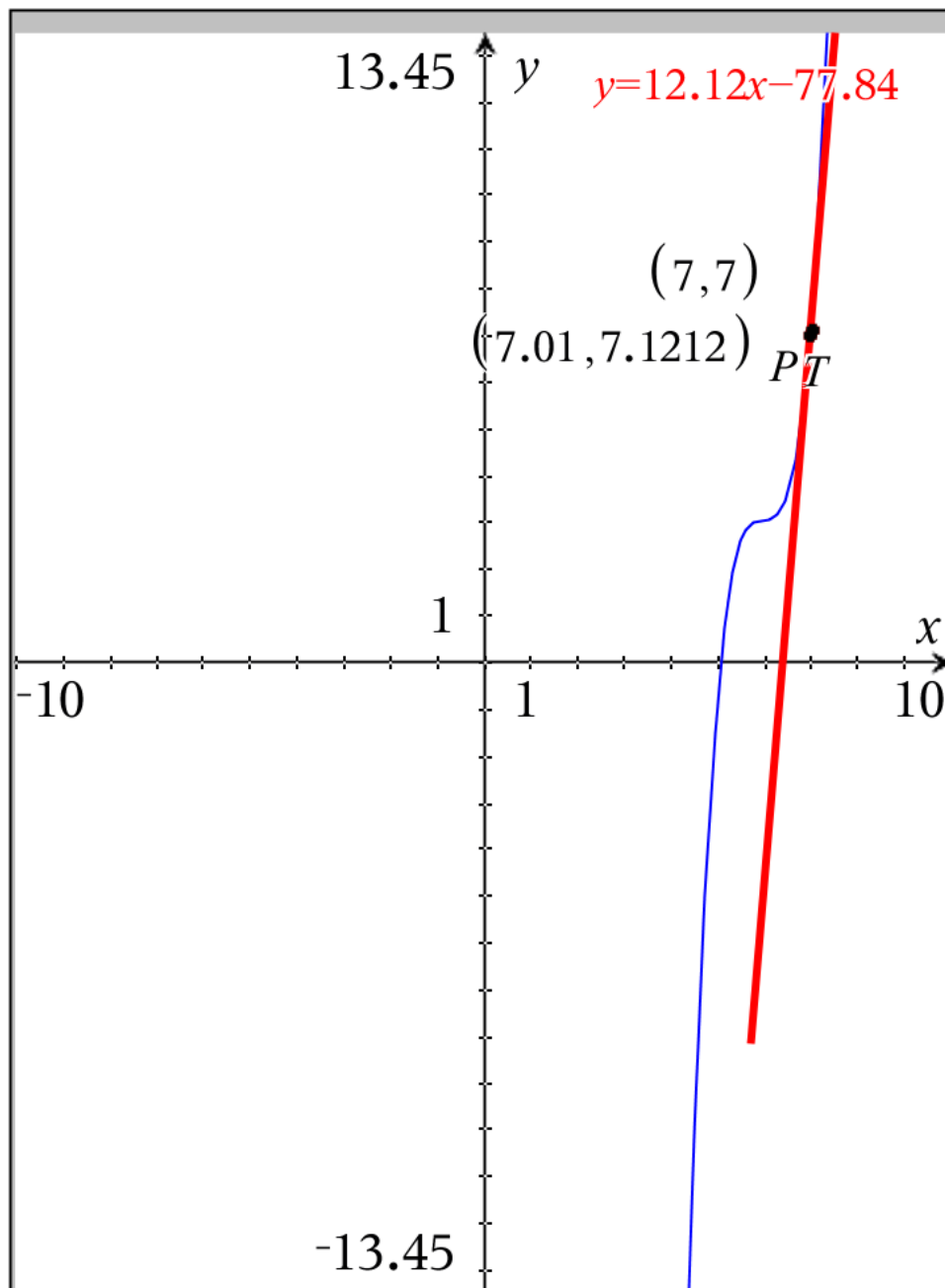
Secant line

Point Slope Form

$$y = 28(x-7) + 7$$

Slope Intercept Form

$$y = 28 \cdot x - 189$$



$$f(x) = 4(x-6)^3 + 3$$

$$P = (7, 7)$$

$$T = (7.01, 7.121204)$$

$$\text{Slope of } PT = 12.1204$$

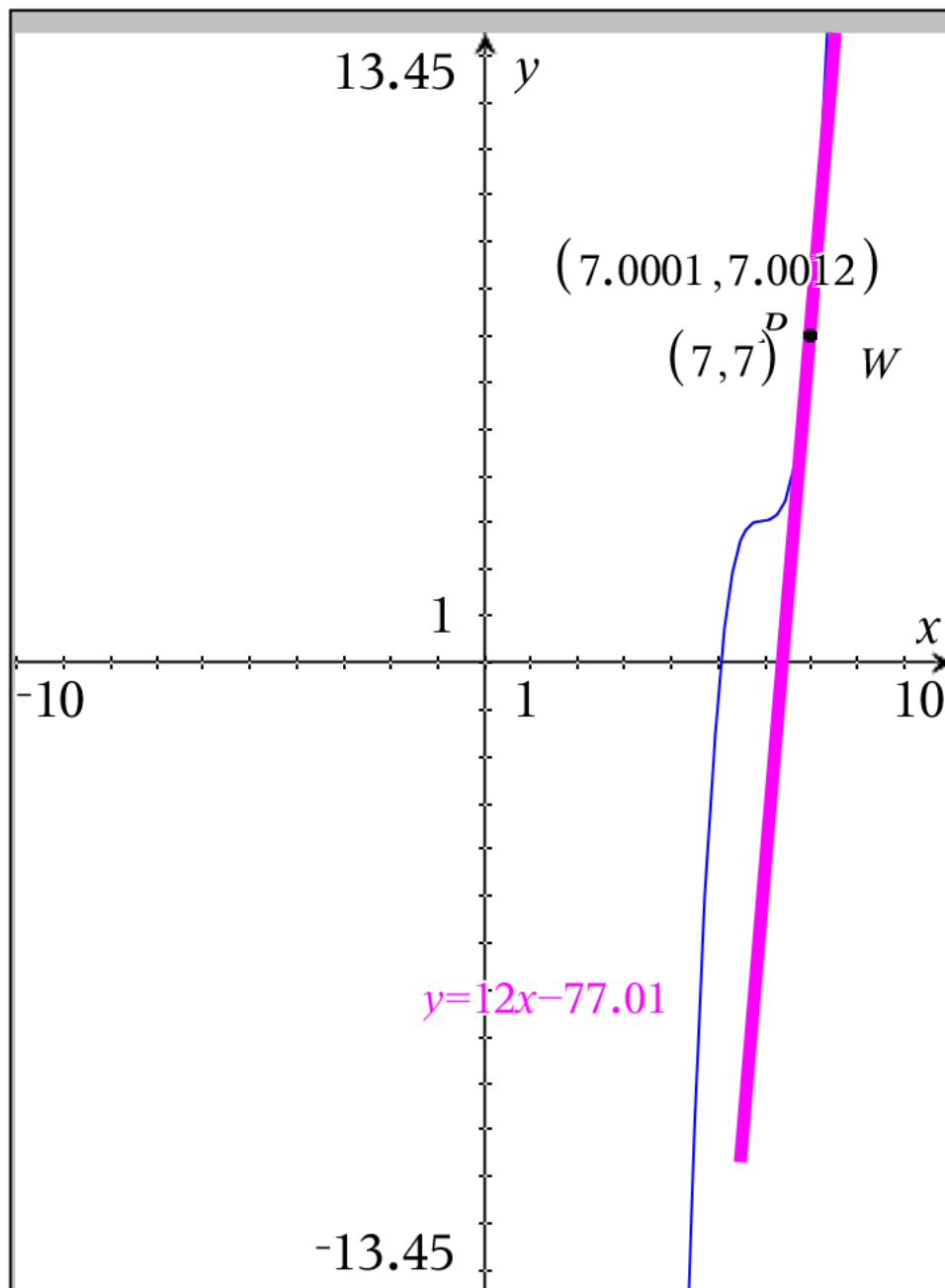
Secant line

Point Slope Form

$$y = 12.1204(x-7) + 7$$

Slope Intercept Form

$$y = 12.1204 \cdot x - 77.8428$$



$$f(x) = 4(x-6)^3 + 3$$

$$P = (7, 7)$$

$$W = (7.0001, 7.00120012)$$

$$\text{Slope of } PW = 12.00120004$$

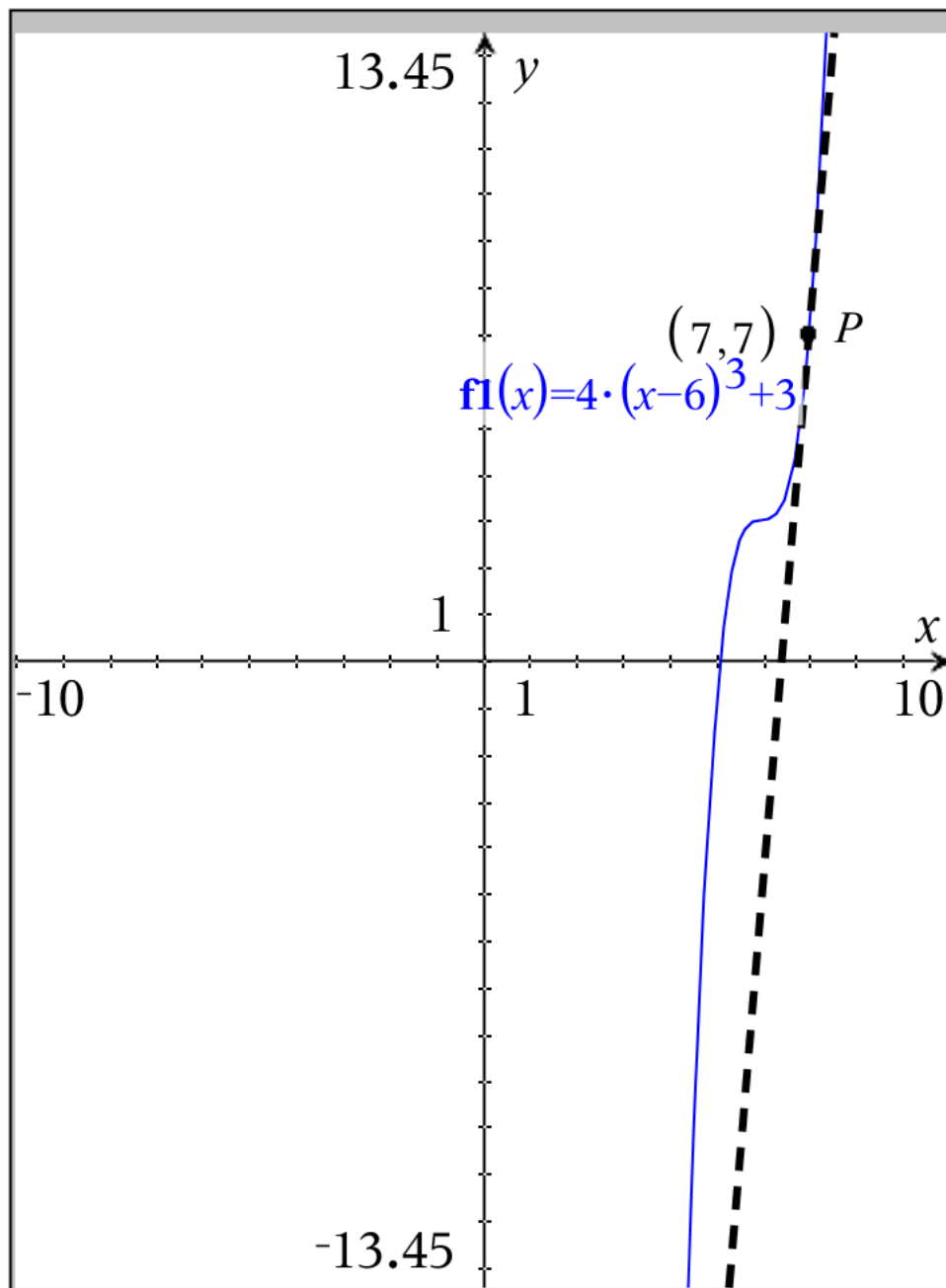
**Secant line**

**Point Slope Form**

$$y = 12.00120004(x-7) + 7$$

**Slope Intercept Form**

$$y = 12.00120004 \cdot x - 77.00840028$$



$$f(x) = 4(x-6)^3 + 3$$

$$P = (7, 7)$$

**ACTUAL Tangent Line**

**Point Slope Form**

$$y = 12(x-7) + 7$$

**Slope Intercept Form**

$$y = 12 \cdot x - 77$$

In this case as we pick values of  $x$  that are getting increasingly closer and closer to P's  $x$  value namely  $x = 7$ ,

In my simple examples Q's  $x$  value was  $x = 8$ ,  
and it led to a slope of secant line through P and Q of  $m = 28$

In my simple examples T's  $x$  value was  $x = 7.01$ ,  
and it led to a slope of secant line through P and T of  $m = 12.1204$

In my simple examples W's  $x$  value was  $x = 7.0001$ ,  
and it led to a slope of secant line through P and W of  $m = 12.00120004$

Say I took the time to find slope between P and one last value of C at  $x = 7.00000001$   
and it led to a slope of secant line through P and that point C of  $m \approx 12$ .  
(accurate to 12 decimals)

It appears that the slopes of the secant lines are pointing towards a slope of the tangent line of  $m = 12$  which is the the slope of the tangent line at  $x = 7$

Steps in this process

1) find an x value that you are planning to use as one of the points in a secant line

2) determine that point's y coordinate

You now know  $(x_1, y_1)$

3) pick an x coordinate that is approaching x from step 1

4) determine that point's y coordinate

You now know  $(x_2, y_2)$

5) find slope of the secant line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

This slope of the secant line is also called the average rate of change from  $x_1$  to  $x_2$

Now if we fix  $x_1$  and let another x coordinate, say  $x_3$ , that gets closer to  $x_1$

and repeat steps 1 through 5 with the "new  $x_3$ ", then the new slope of the secant line will be a better approximation of the slope of the tangent line and ultimately that average rate of change between  $x_1$  and  $x_3$  will get closer to the instantaneous rate of change at  $x_1$

$$f(x) = 4 \cdot x^3 - 72 \cdot x^2 + 432 \cdot x - 861$$

P has x value 7 P has y value  $f(7)=7$  P (7,7)

Q has x value 8 Q has y value  $f(8)=35$  Q (8,35)

m of PQ = 28

T has x value 7.01 T has y value  $f(7.01)=7.121204$  T (7.01,7.121204)

m of PT = 12.1204

W has x value 7.0001 W has y value  $f(7.0001)=7.00120012$  W (7.0001,7.00120012)

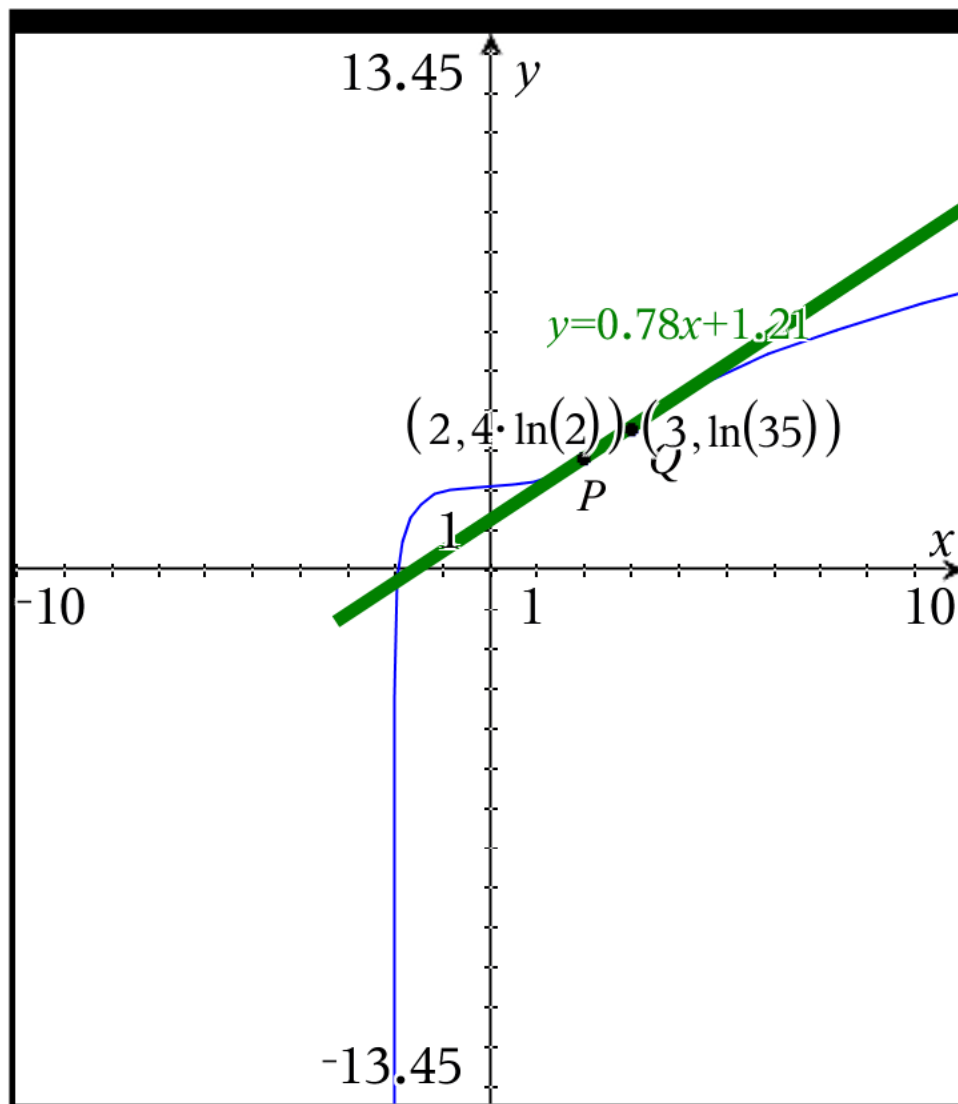
m of PW = 12.00120004

The actual tangent line's slope and the instantaneous rate of change is not found until we can either evaluate a limit properly or we learn how the derivative of a function works, but since I do know how to do these things I'll give you this slope and ultimately the instantaneous rate of change at  $x=7$  m tan = 12 SO as  $x \rightarrow 7$  we can say that the slope of the tangent line and the instantaneous rate of change  $\rightarrow 12$

	A	B	C	D x_1	E y_1	F x_2	G y_2
=							
1	P	7	7	x_p	y_p	x_p	y_p
2	Q	8	35	x_q	y_q	x_t	y_t
3	m_pq	28	28.				
4	change_y	28					
5	change_x	1					
6	T	7.01	7.121204				
7	change_y	0.121204					
8	change_x	0.01					
9	m_pt	12.1204					
10	W	7.0001	7.00120...				
11	change_y	0.001200120004					
A1	"P"						



Problem 1



$$g(x) = \ln(8+x^3)$$

$$P = (2, 4 \cdot \ln(2)) \approx (2, 2.77258872224)$$

$$Q = (3, \ln(35)) \approx (3, 3.55534806149)$$

$$\text{Slope of PQ} = \ln\left(\frac{35}{16}\right)$$

$$\approx 0.78275933925$$

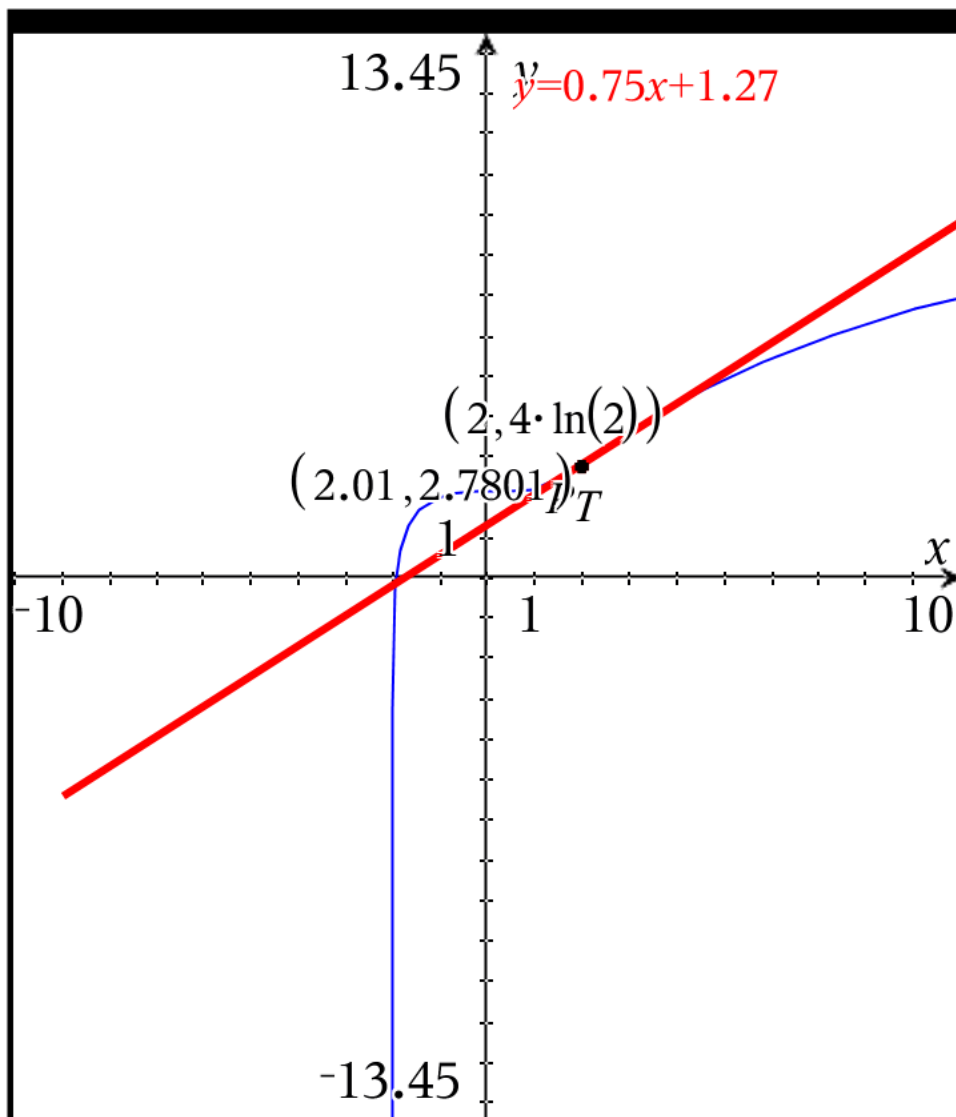
**Secant line**

**Point Slope Form**

$$y = \ln\left(\frac{35}{16}\right)(x-2) + 4 \cdot \ln(2)$$

**Slope Intercept Form**

$$y \approx 0.78275933925 \cdot x + 1.20707004374$$



$$g(x) = \ln(8+x^3)$$

$$P = (2, 4 \cdot \ln(2))$$

$$T = (2.01, 2.78009801926)$$

$$\text{Slope of PT} = 0.75092970221$$

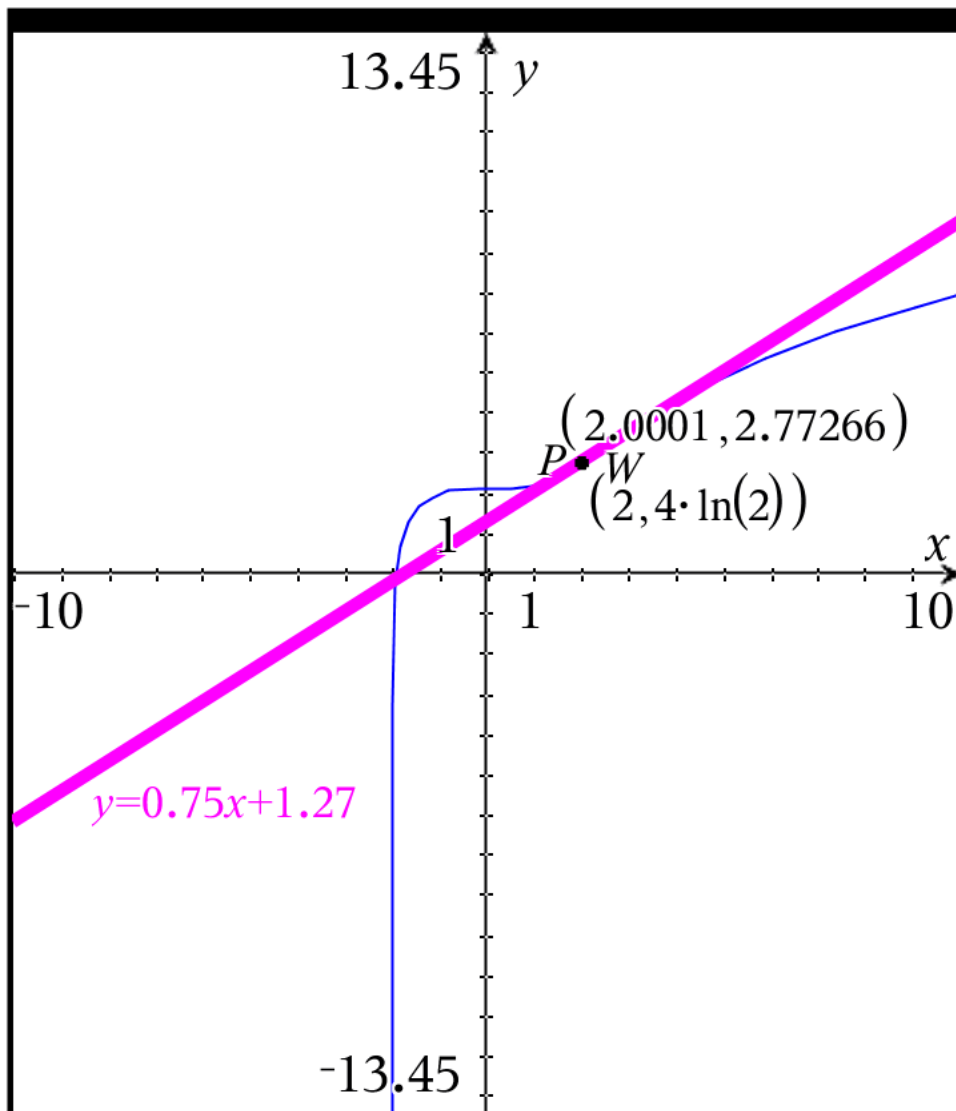
**Secant line**

**Point Slope Form**

$$y = 0.75092970221(x-2) + 4 \cdot \ln(2)$$

**Slope Intercept Form**

$$y = 0.75092970221 \cdot x + 1.27072931782$$



$$P = (2, 4 \cdot \ln(2))$$

$$W = (2.0001, 2.77266372318)$$

$$\text{Slope of PW} = 0.750009374$$

**Secant line**

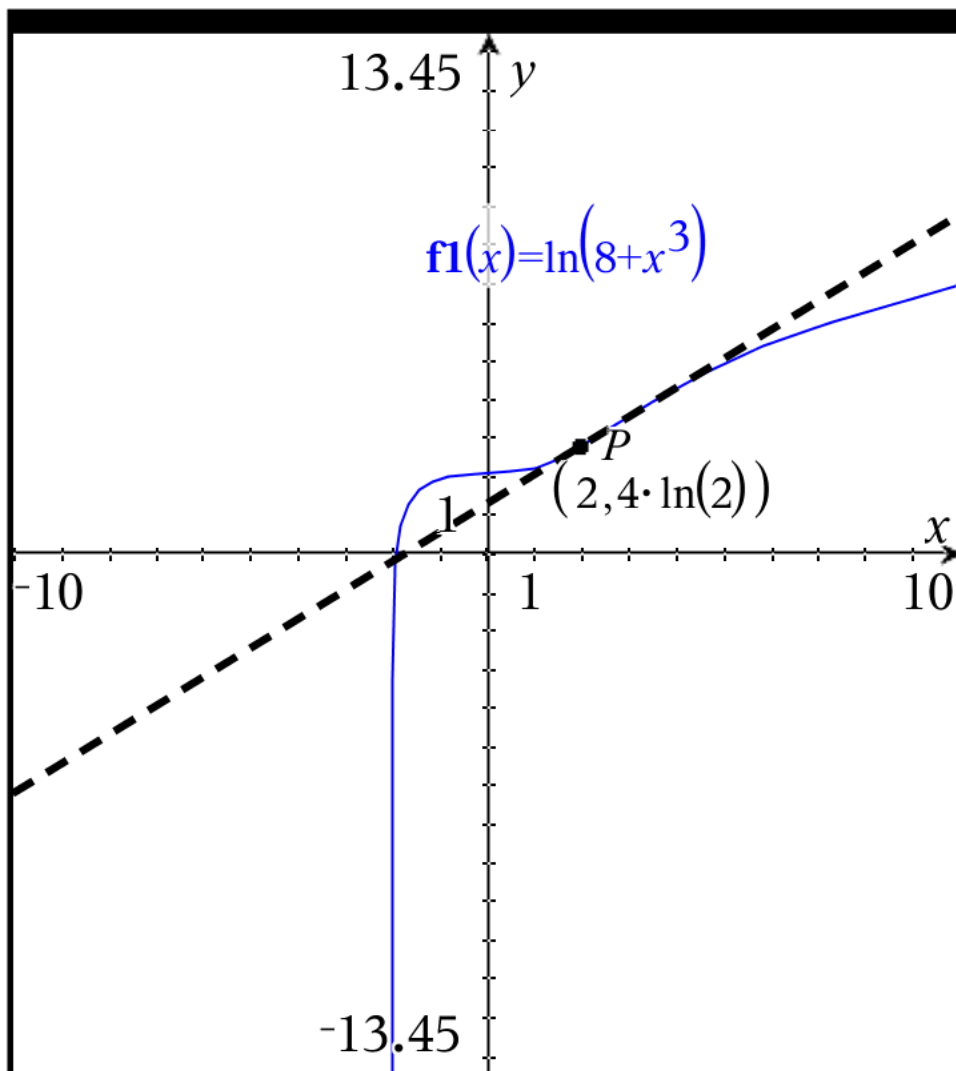
**Point Slope Form**

$$y = 0.750009374(x - 2) + 4 \cdot \ln(2)$$

**Slope Intercept Form**

$$y = 0.750009374 \cdot x + 1.27256997424$$

$$g(x) = \ln(8+x^3)$$



$$g(x) = \ln(8+x^3)$$

$$P = (2, 4 \cdot \ln(2))$$

**ACTUAL Tangent Line**

**Point Slope Form**

$$y = \frac{3}{4}(x-2) + 4 \cdot \ln(2)$$

**Slope Intercept Form**

$$y = \frac{3 \cdot x}{4} + 4 \cdot \ln(2) - \frac{3}{2}$$

In this case as we pick values of  $x$  that are getting increasingly closer and closer to P's  $x$  value namely  $x = 2$ ,

In my simple examples Q's  $x$  value was  $x = 3$ ,

and it led to a slope of secant line through P and Q of  $m = \ln\left(\frac{35}{16}\right) = 0.78275933925$

In my simple examples T's  $x$  value was  $x = 2.01$ ,

and it led to a slope of secant line through P and T of  $m = 0.75092970221$

In my simple examples W's  $x$  value was  $x = 2.0001$ ,

and it led to a slope of secant line through P and W of  $m = 0.750009374$

Say I took the time to find slope between P and one last value of C at  $x = 2.00000001$

and it led to a slope of secant line through P and that point C of  $m \approx 0.75$

(accurate to 12 decimals)

It appears that the slopes of the secant lines are pointing towards a slope of the tangent line of  $m = \frac{3}{4}$

which is the the slope of the tangent line at  $x = 2$

Steps in this process

1) find an x value that you are planning to use as one of the points in a secant line

2) determine that point's y coordinate

You now know  $(x_1, y_1)$

3) pick an x coordinate that is approaching x from step 1

4) determine that point's y coordinate

You now know  $(x_2, y_2)$

5) find slope of the secant line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

This slope of the secant line is also called the average rate of change from  $x_1$  to  $x_2$

Now if we fix  $x_1$  and let another x coordinate, say  $x_3$ , that gets closer to  $x_1$

and repeat steps 1 through 5 with the "new  $x_3$ ", then the new slope of the secant line will be a better approximation of the slope of the tangent line and ultimately that average rate of change between  $x_1$  and  $x_3$  will get closer to the instantaneous rate of change at  $x_1$

$$g(x) = \ln(x^3 + 8)$$

P has x value 2 P has y value  $g(2) = 4 \cdot \ln(2)$      $P(2, 4 \cdot \ln(2)) \approx P(2, 2.77258872224)$

Q has x value 3 Q has y value  $g(3) = \ln(35)$      $Q(3, \ln(35)) \approx Q(3, 3.55534806149)$

$$m \text{ of PQ} = \ln\left(\frac{35}{16}\right) \approx 0.78275933925$$

T has x value 2.01 T has y value  $g(2.01) = 2.78009801926$      $T(2.01, 2.78009801926)$

$$m \text{ of PT} = 0.75092970221$$

W has x value 2.0001 W has y value  $g(2.0001) = 2.77266372318$      $W(2.0001, 2.77266372318)$

$$m \text{ of PW} = 0.750009374$$

The actual tangent line's slope and the instantaneous rate of change is not found until we can either evaluate a limit properly or we learn how the derivative of a function works, but since I do know how to do these things

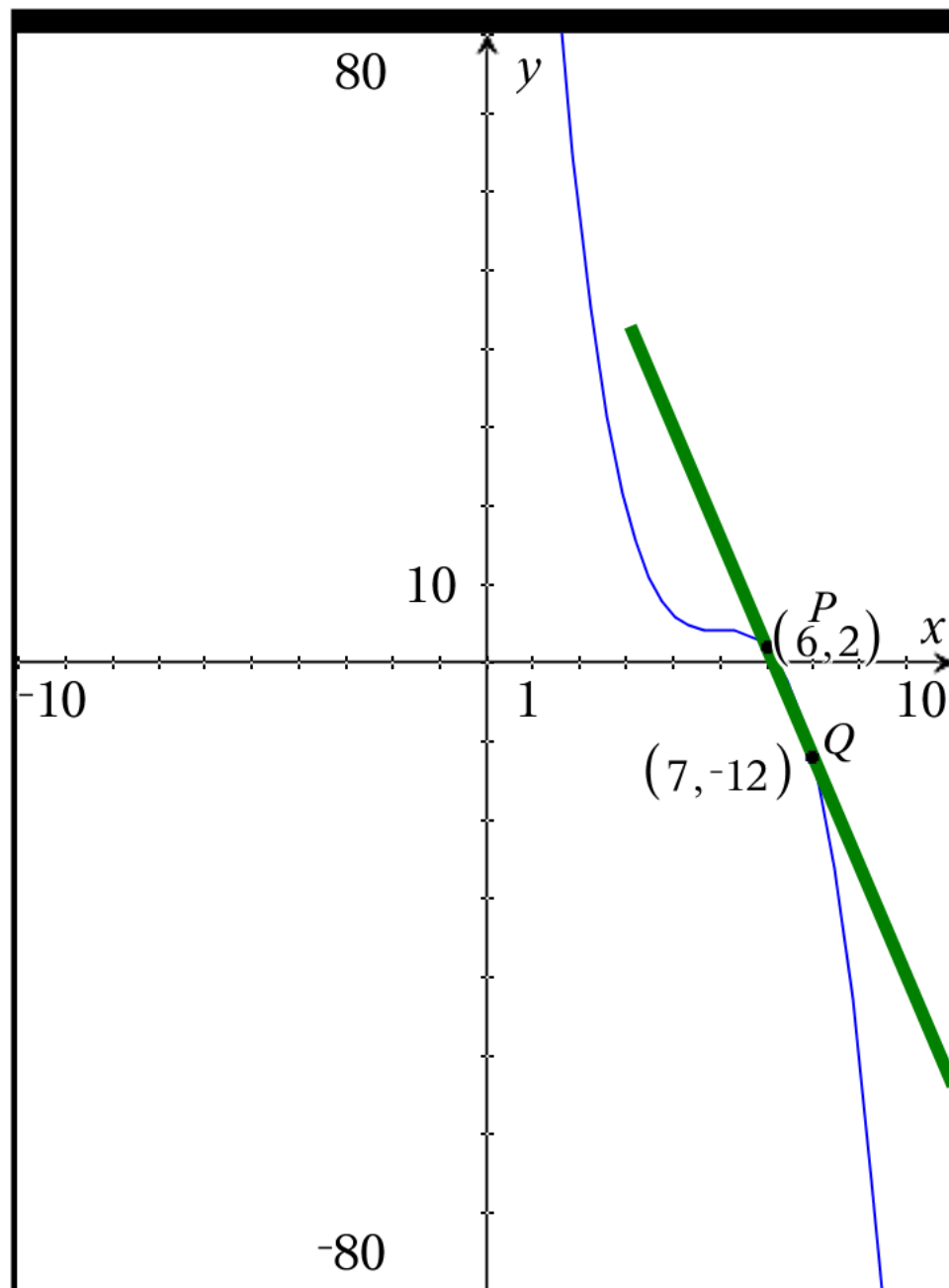
I'll give you this slope and ultimately the instantaneous rate of change at  $x = 2$      $m_{\tan} = \frac{3}{4}$     SO as  $x \rightarrow 2$  we

can say that the slope of the tangent line and the instantaneous rate of change  $\rightarrow \frac{3}{4}$

	A	B	C	D x_1	E y_1	F x_2	G y_2
=							
1	P		2 $4 \cdot \ln(2)$	x_p	y_p	x_p	y_p
2	Q		3 $\ln(35)$	x_q	y_q	x_t	y_t
3	m_pq	$\ln(35/16)$	0.78275...				
4	change_y	$\ln(35/16)$					
5	change_x	1					
6	T	2.01	2.78009...				
7	change_y	0.007509297022					
8	change_x	0.01					
9	m_pt	0.75092970221					
10	W	2.0001	2.77266...				
11	change_y	0.000075000937					
A1	"P"						



Problem 1



$$f(x) = -2(x-5)^3 + 4$$

$$P = (6, 2)$$

$$Q = (7, -12)$$

$$\text{Slope of PQ} = -14$$

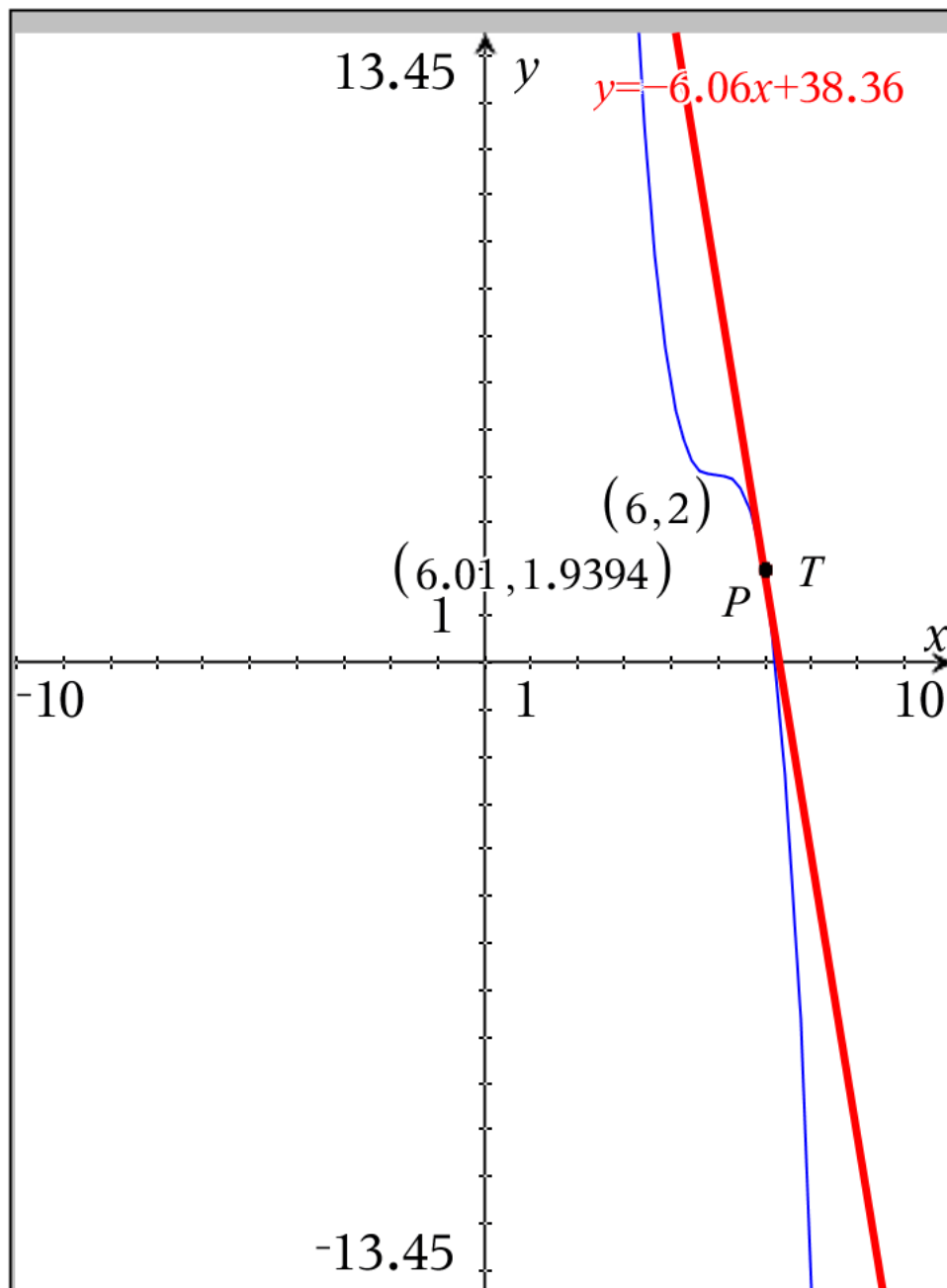
Secant line

Point Slope Form

$$y = -14(x-6) + 2$$

Slope Intercept Form

$$y = 86 - 14 \cdot x$$



$$f(x) = -2(x-5)^3 + 4$$

$$P = (6, 2)$$

$$T = (6.01, 1.939398)$$

$$\text{Slope of } PT = -6.0602$$

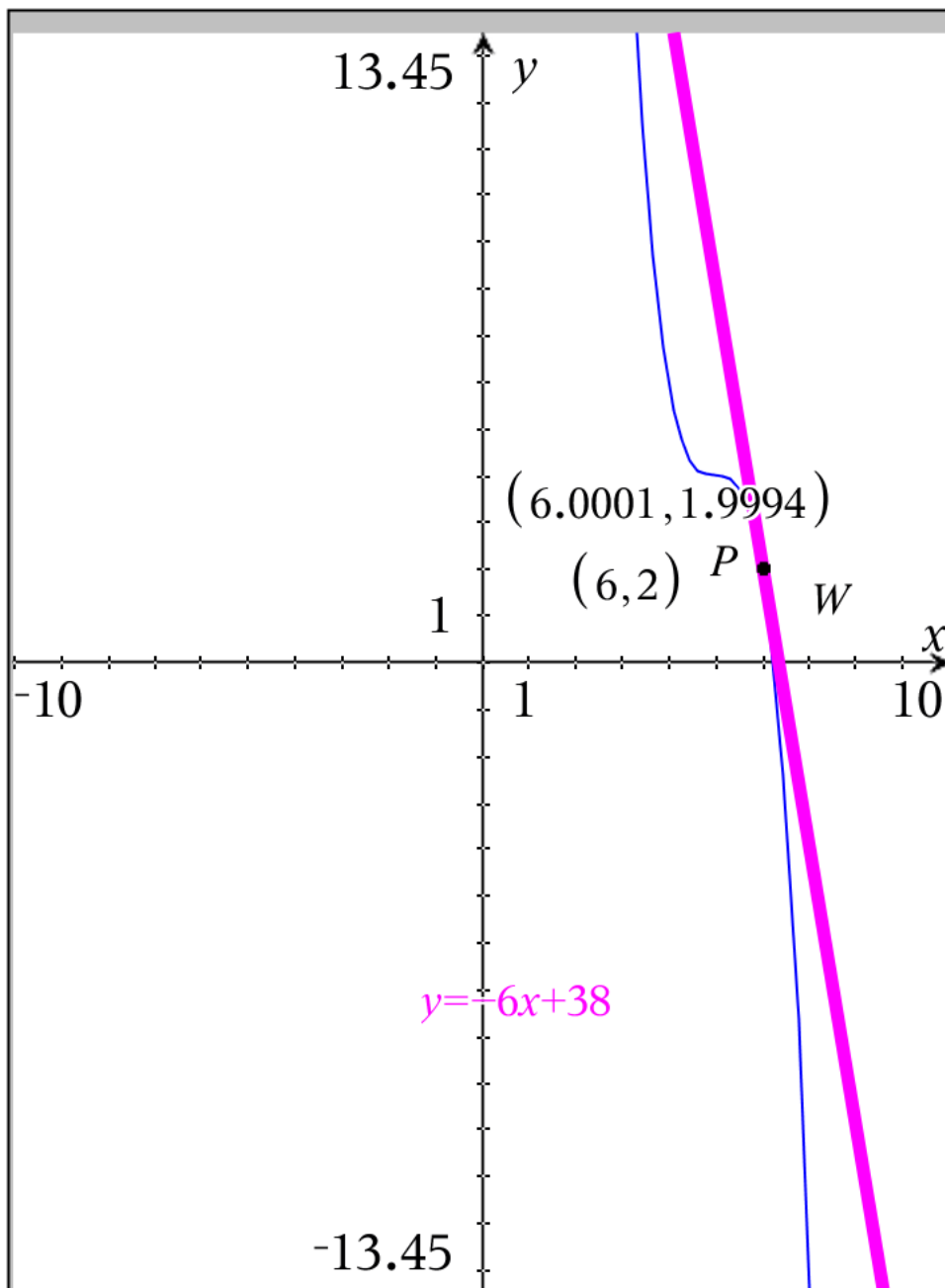
Secant line

Point Slope Form

$$y = -6.0602(x-6) + 2$$

Slope Intercept Form

$$y = 38.3612 - 6.0602 \cdot x$$



$$f(x) = -2(x-5)^3 + 4$$

$$P = (6, 2)$$

$$W = (6.0001, 1.99939994)$$

$$\text{Slope of } PW = -6.00060002$$

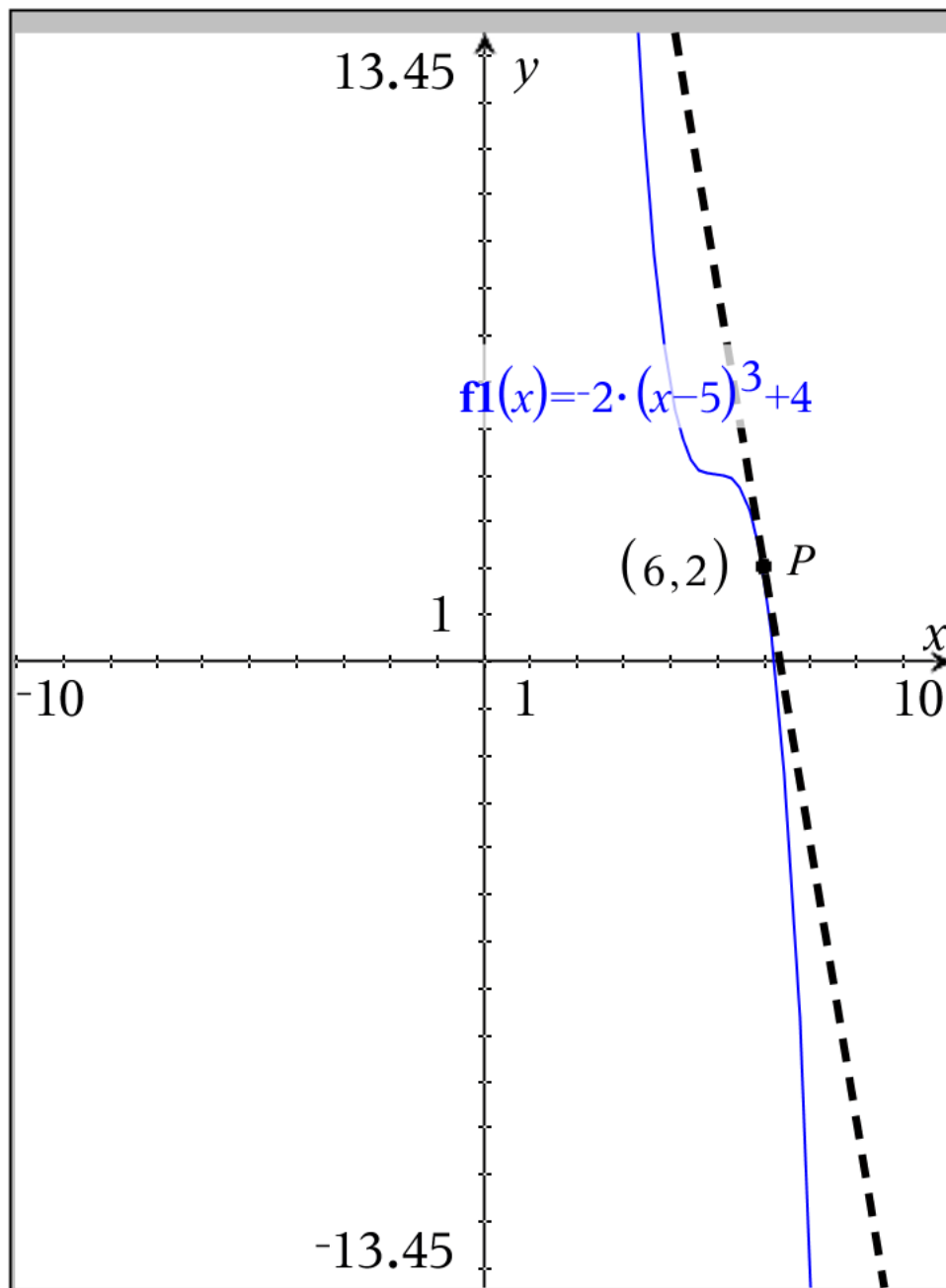
Secant line

Point Slope Form

$$y = -6.00060002(x-6) + 2$$

Slope Intercept Form

$$y = 38.00360012 - 6.00060002 \cdot x$$



$$f(x) = -2(x-5)^3 + 4$$

$$P = (6, 2)$$

**ACTUAL Tangent Line**

**Point Slope Form**

$$y = -6(x-6) + 2$$

**Slope Intercept Form**

$$y = 38 - 6 \cdot x$$

In this case as we pick values of  $x$  that are getting increasingly closer and closer to  $P$ 's  $x$  value namely  $x = 6$ ,

In my simple examples  $Q$ 's  $x$  value was  $x = 7$ ,  
and it led to a slope of secant line through  $P$  and  $Q$  of  $m = -14$

In my simple examples  $T$ 's  $x$  value was  $x = 6.01$ ,  
and it led to a slope of secant line through  $P$  and  $T$  of  $m = -6.0602$

In my simple examples  $W$ 's  $x$  value was  $x = 6.0001$ ,  
and it led to a slope of secant line through  $P$  and  $W$  of  $m = -6.00060002$

Say I took the time to find slope between  $P$  and one last value of  $C$  at  $x = 6.00000001$   
and it led to a slope of secant line through  $P$  and that point  $C$  of  $m \approx -6$ .  
(accurate to 12 decimals)

It appears that the slopes of the secant lines are pointing towards a slope of the tangent line of  $m = -6$  which is the slope of the tangent line at  $x = 6$

Steps in this process

1) find an x value that you are planning to use as one of the points in a secant line

2) determine that point's y coordinate

You now know  $(x_1, y_1)$

3) pick an x coordinate that is approaching x from step 1

4) determine that point's y coordinate

You now know  $(x_2, y_2)$

5) find slope of the secant line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

This slope of the secant line is also called the average rate of change from  $x_1$  to  $x_2$

Now if we fix  $x_1$  and let another x coordinate, say  $x_3$ , that gets closer to  $x_1$

and repeat steps 1 through 5 with the "new  $x_3$ ", then the new slope of the secant line will be a better approximation of the slope of the tangent line and ultimately that average rate of change between  $x_1$  and  $x_3$  will get closer to the instantaneous rate of change at  $x_1$

$$f(x) = -2 \cdot x^3 + 30 \cdot x^2 - 150 \cdot x + 254$$

P has x value 6 P has y value  $f(6)=2$  P (6,2)

Q has x value 7 Q has y value  $f(7)=-12$  Q (7,-12)

m of PQ = -14

T has x value 6.01 T has y value  $f(6.01)=1.939398$  T (6.01 ,1.939398 )

m of PT = -6.0602

W has x value 6.0001 W has y value  $f(6.0001)=1.99939994$  W (6.0001 ,1.99939994 )

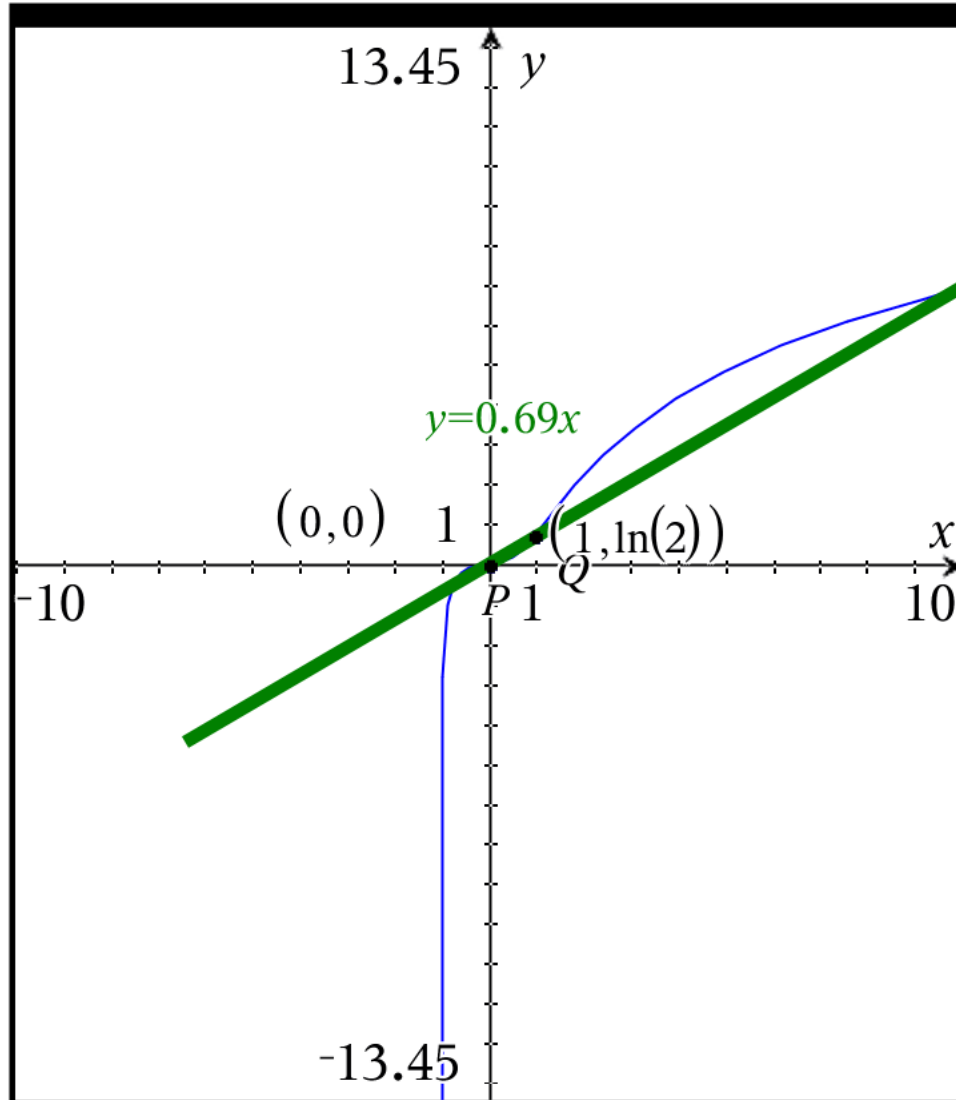
m of PW = -6.00060002

The actual tangent line's slope and the instantaneous rate of change is not found until we can either evaluate a limit properly or we learn how the derivative of a function works, but since I do know how to do these things I'll give you this slope and ultimately the instantaneous rate of change at  $x=6$   $m_{\tan} = -6$  SO as  $x \rightarrow 6$  we can say that the slope of the tangent line and the instantaneous rate of change  $\rightarrow -6$

	A	B	C	D x_1	E y_1	F x_2	G y_2
=							
1	P	6	2	x_p	y_p	x_p	y_p
2	Q	7	-12	x_q	y_q	x_t	y_t
3	m_pq	-14	-14.				
4	change_y	-14					
5	change_x	1					
6	T	6.01	1.939398				
7	change_y	-0.060602					
8	change_x	0.01					
9	m_pt	-6.0602					
10	W	6.0001	1.99939...				
11	change_y	-0.000600060002					
	B2 x_q:=b1+1						



Problem 1



$$P = (0,0) \approx (0.,0.)$$

$$Q = (1, \ln(2)) \approx (1.,0.69314718056)$$

$$\text{Slope of PQ} = \ln(2) \approx 0.69314718056$$

**Secant line**

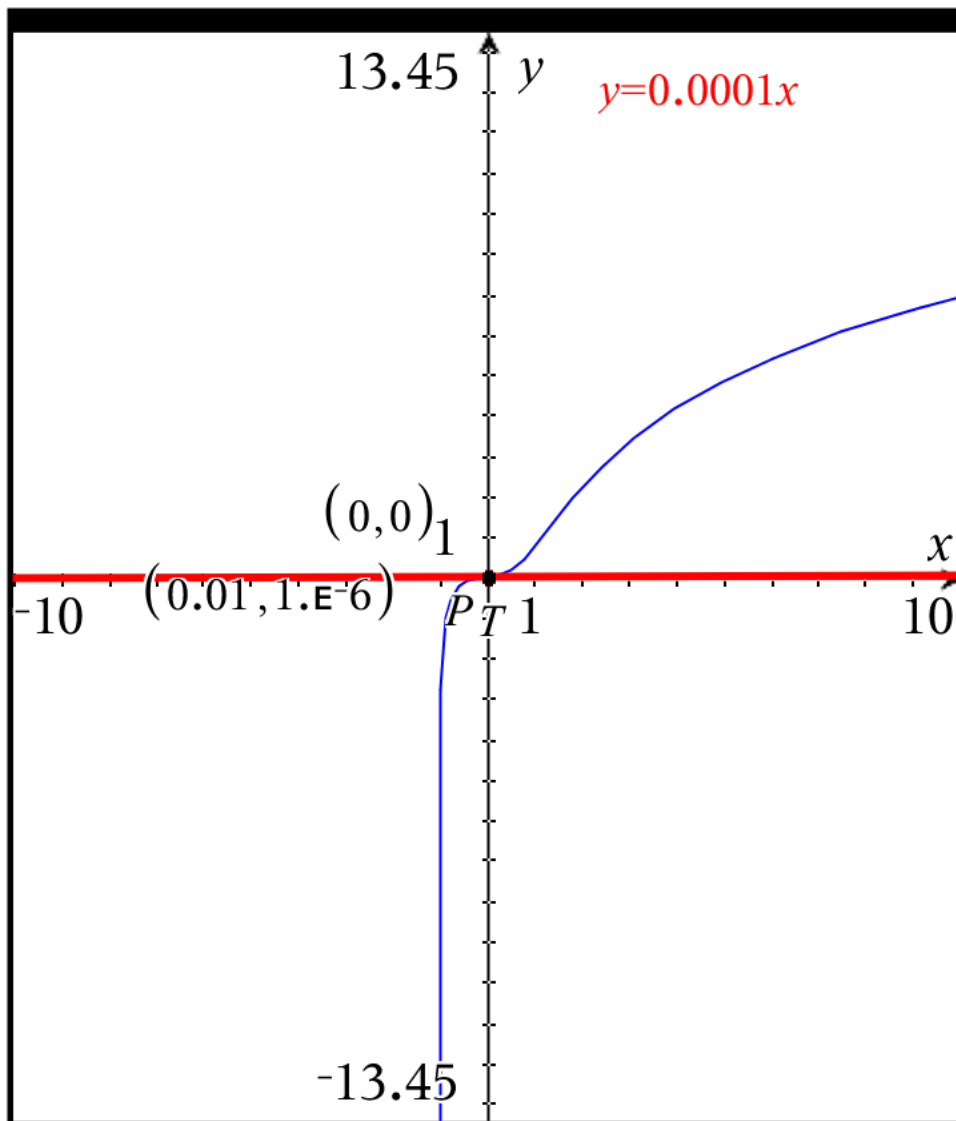
**Point Slope Form**

$$y = \ln(2)(x-0)+0$$

**Slope Intercept Form**

$$y \approx 0.69314718056 \cdot x$$

$$g(x) = \ln(1+x^3)$$



$$P = (0,0)$$

$$T = (0.01, 0.000001)$$

$$\text{Slope of PT} = 0.00009999995$$

Secant line

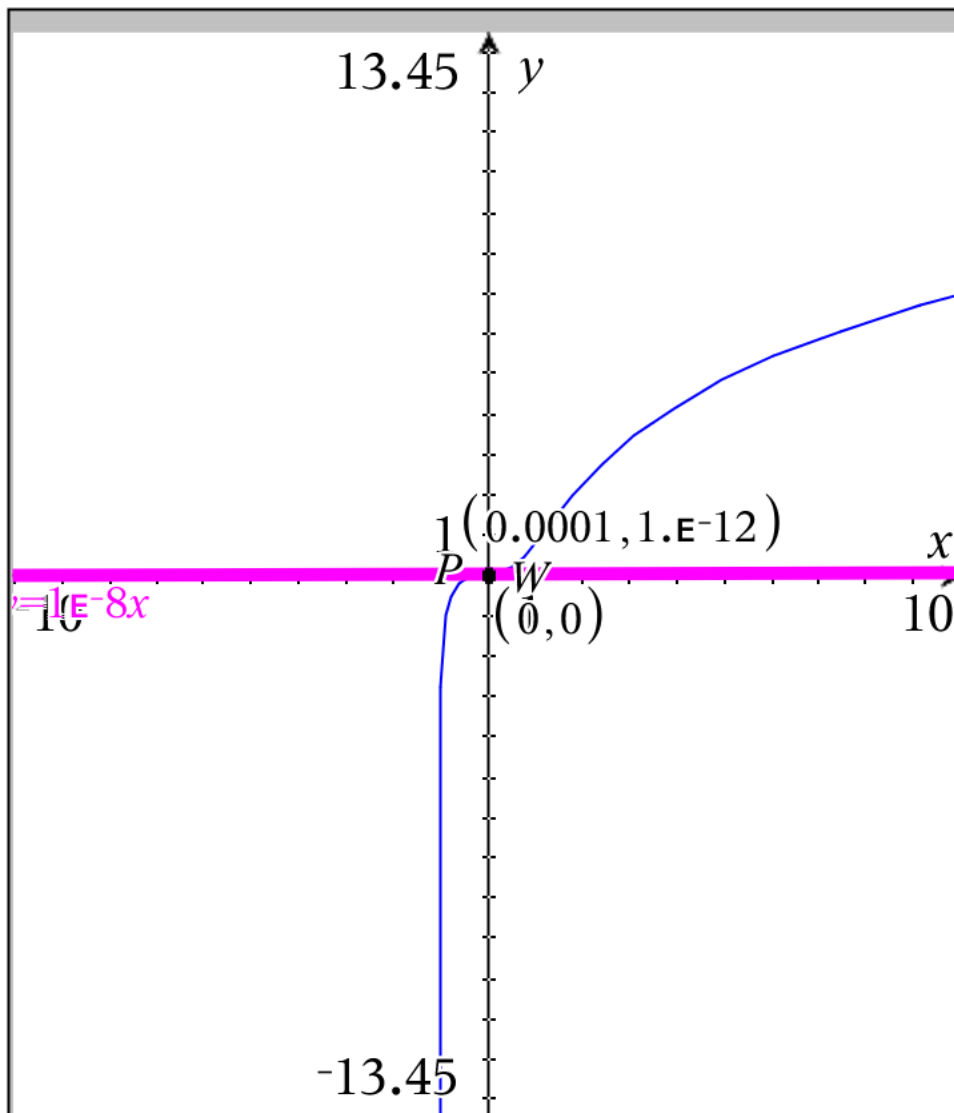
Point Slope Form

$$y = 0.00009999995 (x-0) + 0$$

Slope Intercept Form

$$y = 0.00009999995 \cdot x$$

$$g(x) = \ln(1+x^3)$$



$$P = (0,0)$$

$$W = (0.0001, 1.E-12)$$

$$\text{Slope of PW} = 0.00000001$$

Secant line

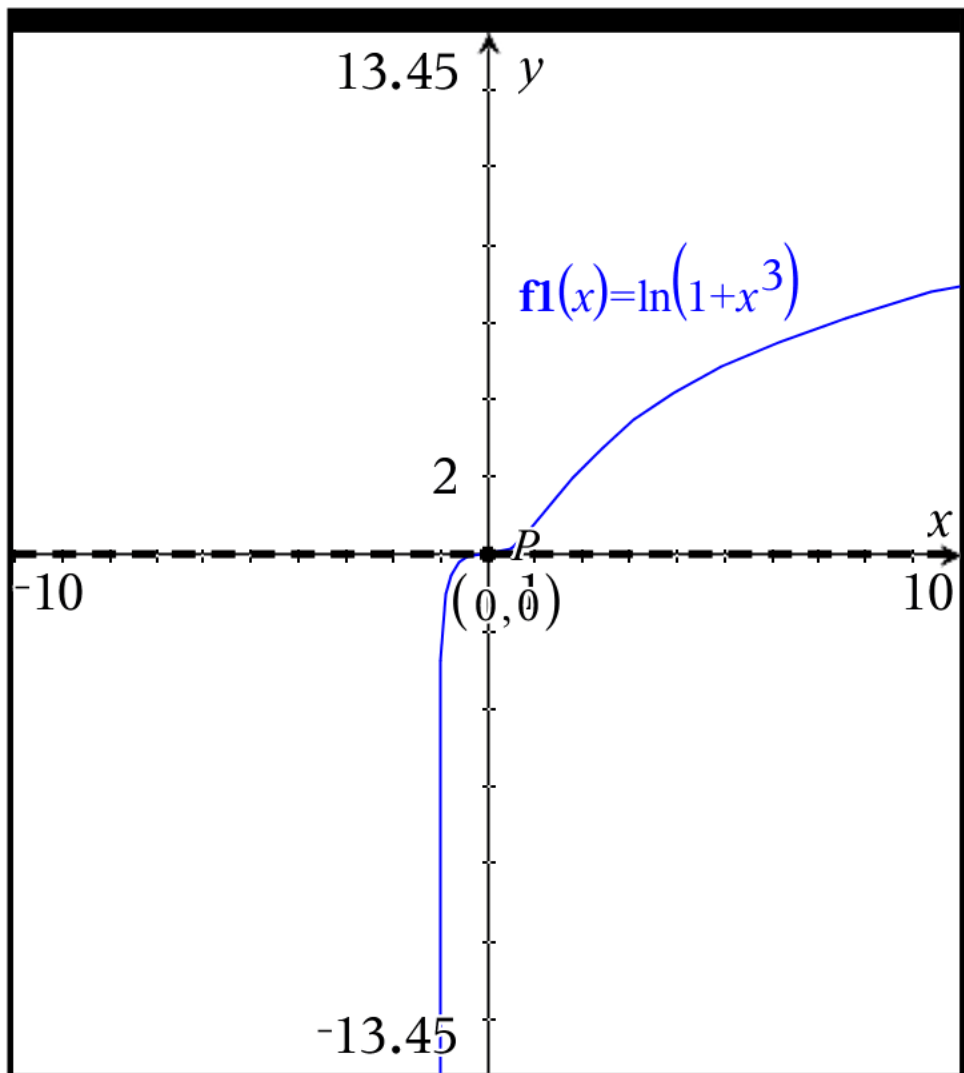
Point Slope Form

$$y = 0.00000001 (x-0) + 0$$

Slope Intercept Form

$$y = 0.00000001 \cdot x$$

$$g(x) = \ln(1+x^3)$$



$$P = (0,0)$$

**ACTUAL Tangent Line**

**Point Slope Form**

$$y = 0(x-0)+0$$

**Slope Intercept Form**

$$y = 0$$

$$g(x) = \ln(1+x^3)$$

In this case as we pick values of  $x$  that are getting increasingly closer and closer to P's  $x$  value namely  $x = 0$ ,

In my simple examples Q's  $x$  value was  $x = 1$ ,

and it led to a slope of secant line through P and Q of  $m = \ln(2) = 0.69314718056$

In my simple examples T's  $x$  value was  $x = 0.01$ ,

and it led to a slope of secant line through P and T of  $m = 0.00009999995$

In my simple examples W's  $x$  value was  $x = 0.0001$ ,

and it led to a slope of secant line through P and W of  $m = 0.00000001$

Say I took the time to find slope between P and one last value of C at  $x = 0.00000001$

and it led to a slope of secant line through P and that point C of  $m \approx 0$ .

(accurate to 12 decimals)

It appears that the slopes of the secant lines are pointing towards a slope of the tangent line of  $m = 0$  which is the the slope of the tangent line at  $x = 0$

Steps in this process

1) find an x value that you are planning to use as one of the points in a secant line

2) determine that point's y coordinate

You now know  $(x_1, y_1)$

3) pick an x coordinate that is approaching x from step 1

4) determine that point's y coordinate

You now know  $(x_2, y_2)$

5) find slope of the secant line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

This slope of the secant line is also called the average rate of change from  $x_1$  to  $x_2$

Now if we fix  $x_1$  and let another x coordinate, say  $x_3$ , that gets closer to  $x_1$

and repeat steps 1 through 5 with the "new  $x_3$ ", then the new slope of the secant line will be a better approximation of the slope of the tangent line and ultimately that average rate of change between  $x_1$  and  $x_3$  will get closer to the instantaneous rate of change at  $x_1$

$$g(x) = \ln(x^3 + 1)$$

P has x value 0 P has y value  $g(0)=0$  P (0,0)  $\approx$  P (0.,0.)

Q has x value 1 Q has y value  $g(1)=\ln(2)$  Q (1, $\ln(2)$ )  $\approx$  Q (1.,0.69314718056)

m of PQ =  $\ln(2) \approx 0.69314718056$

T has x value 0.01 T has y value  $g(0.01)=0.000001$  T (0.01,0.000001)

m of PT = 0.0000999995

W has x value 0.0001 W has y value  $g(0.0001)=1.E-12$  W (0.0001,1.E-12)

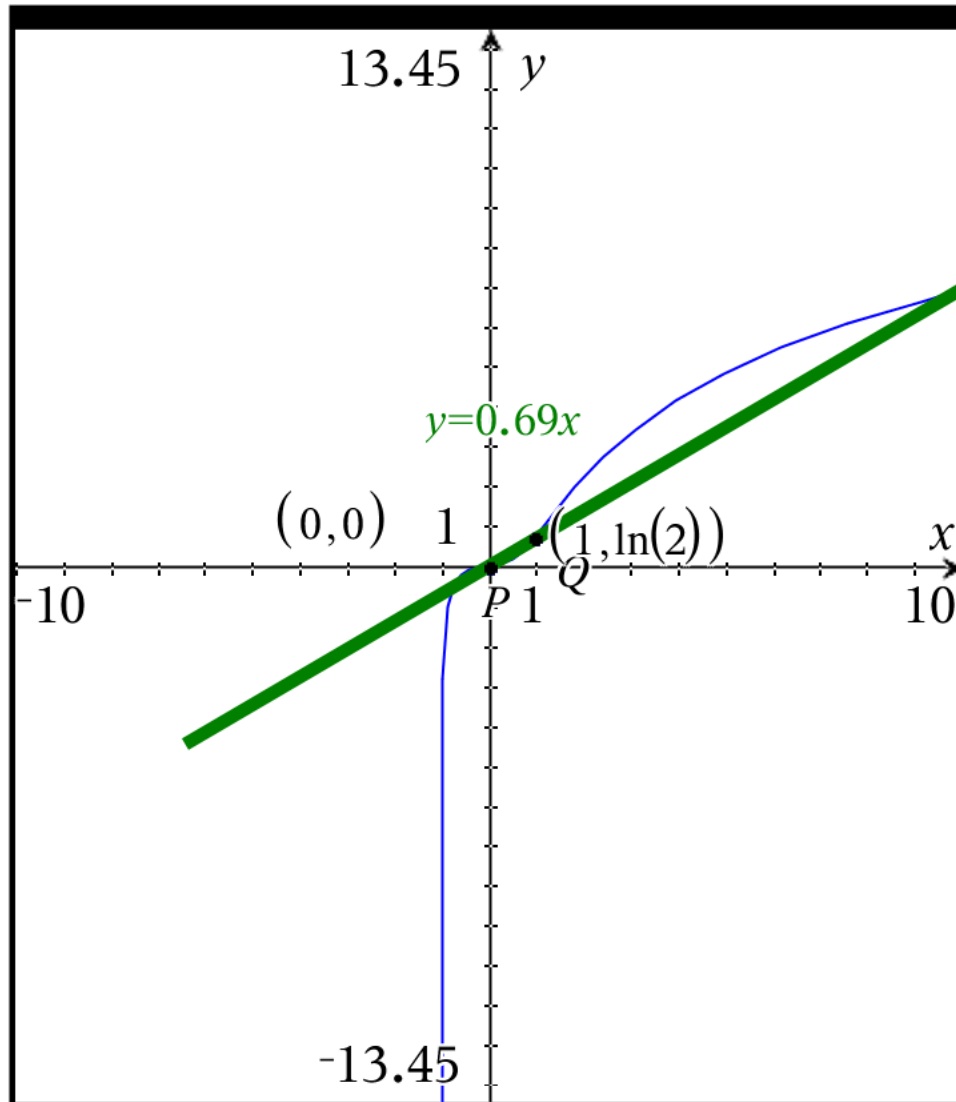
m of PW = 0.00000001

The actual tangent line's slope and the instantaneous rate of change is not found until we can either evaluate a limit properly or we learn how the derivative of a function works, but since I do know how to do these things I'll give you this slope and ultimately the instantaneous rate of change at  $x=0$  m tan = 0 SO as  $x \rightarrow 0$  we can say that the slope of the tangent line and the instantaneous rate of change  $\rightarrow 0$

	A	B	C	D x_1	E y_1	F x_2	G y_
=							
1	P	0	0	x_p	y_p	x_p	y_p
2	Q	1	ln(2)	x_q	y_q	x_t	y_t
3	m_pq	ln(2)	0.69314...				
4	change_y	ln(2)					
5	change_x	1					
6	T	0.01	0.000001				
7	change_y	0.000001					
8	change_x	0.01					
9	m_pt	0.00009999995					
10	W	0.0001	1.E-12				
11	change_y	1.E-12					
B2 x_q:=b1+1							



Problem 1



$$P = (0,0) \approx (0.,0.)$$

$$Q = (1, \ln(2)) \approx (1.,0.69314718056)$$

$$\text{Slope of PQ} = \ln(2) \approx 0.69314718056$$

**Secant line**

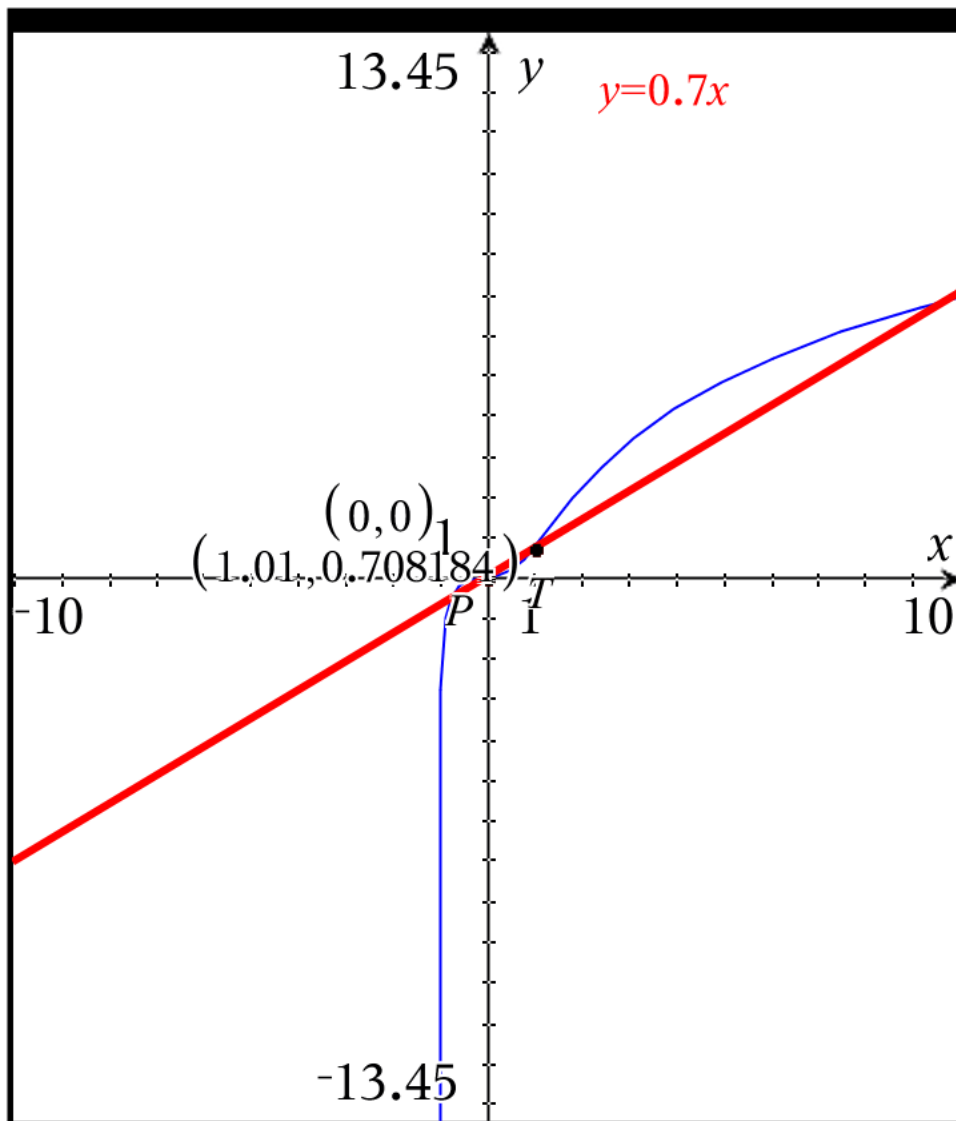
**Point Slope Form**

$$y = \ln(2)(x-0)+0$$

**Slope Intercept Form**

$$y \approx 0.69314718056 \cdot x$$

$$g(x) = \ln(1+x^3)$$



$$P = (0,0)$$

$$T = (1.01, 0.708184057924)$$

$$\text{Slope of PT} = 0.701172334578$$

Secant line

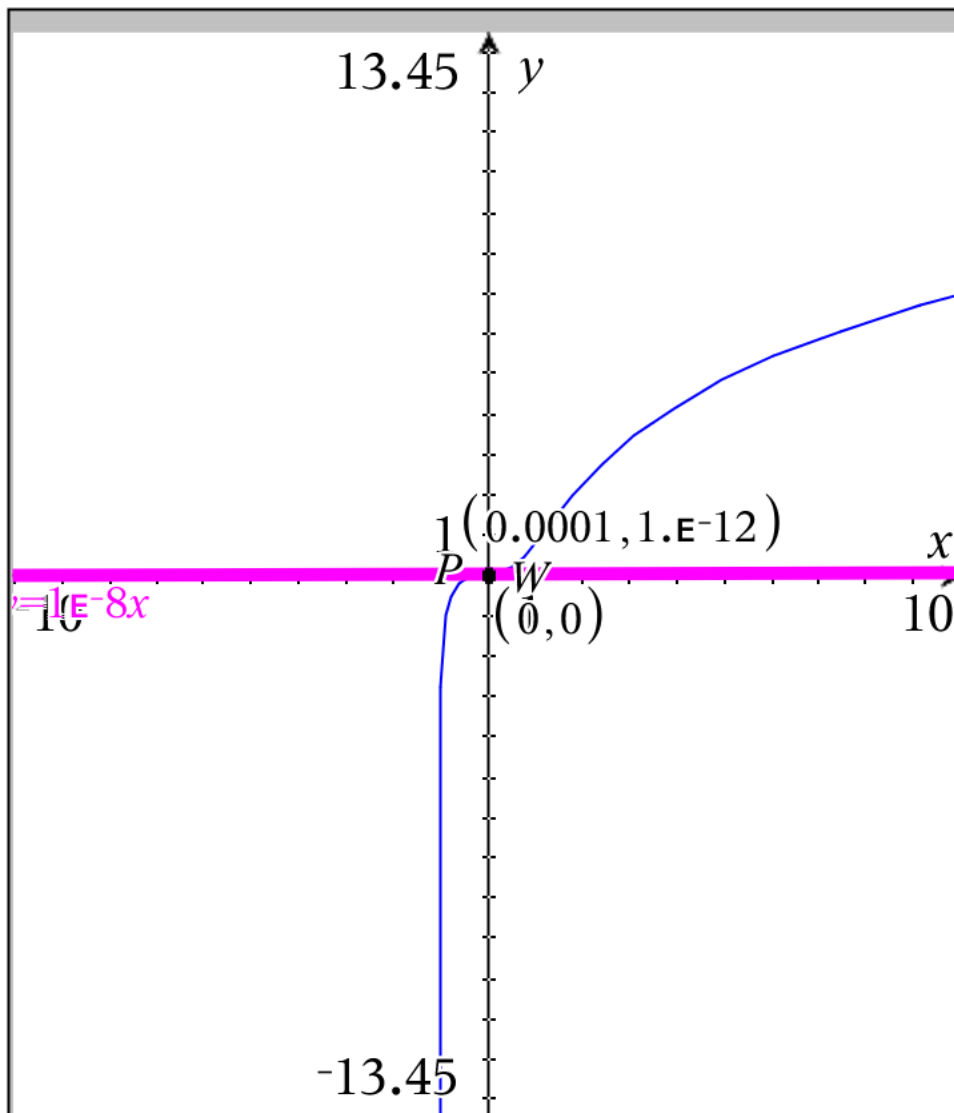
Point Slope Form

$$y = 0.701172334578 (x-0) + 0$$

Slope Intercept Form

$$y = 0.701172334578 \cdot x$$

$$g(x) = \ln(1+x^3)$$



$$g(x) = \ln(1+x^3)$$

$$\mathbf{P} = (0,0)$$

$$\mathbf{W} = (0.0001, 1.E-12)$$

$$\mathbf{Slope\ of\ PW} = 0.00000001$$

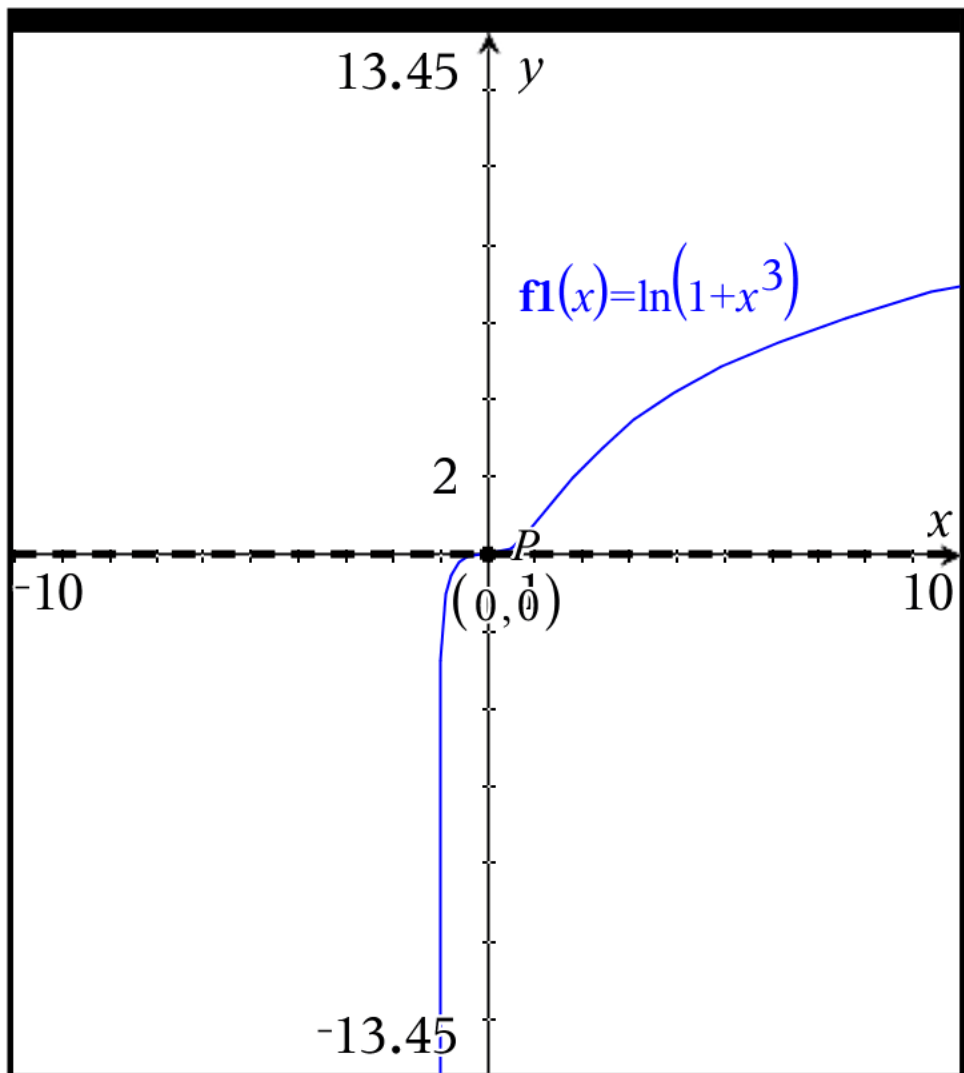
**Secant line**

**Point Slope Form**

$$y = 0.00000001 (x-0) + 0$$

**Slope Intercept Form**

$$y = 0.00000001 \cdot x$$



$$P = (0,0)$$

**ACTUAL Tangent Line**

**Point Slope Form**

$$y = 0(x-0)+0$$

**Slope Intercept Form**

$$y = 0$$

$$g(x) = \ln(1+x^3)$$

In this case as we pick values of  $x$  that are getting increasingly closer and closer to P's  $x$  value namely  $x = 0$ ,

In my simple examples Q's  $x$  value was  $x = 1$ ,

and it led to a slope of secant line through P and Q of  $m = \ln(2) = 0.69314718056$

In my simple examples T's  $x$  value was  $x = 1.01$ ,

and it led to a slope of secant line through P and T of  $m = 0.701172334578$

In my simple examples W's  $x$  value was  $x = 0.0001$ ,

and it led to a slope of secant line through P and W of  $m = 0.00000001$

Say I took the time to find slope between P and one last value of C at  $x = 0.00000001$

and it led to a slope of secant line through P and that point C of  $m \approx 0$ .

(accurate to 12 decimals)

It appears that the slopes of the secant lines are pointing towards a slope of the tangent line of  $m = 0$  which is the the slope of the tangent line at  $x = 0$

Steps in this process

1) find an x value that you are planning to use as one of the points in a secant line

2) determine that point's y coordinate

You now know  $(x_1, y_1)$

3) pick an x coordinate that is approaching x from step 1

4) determine that point's y coordinate

You now know  $(x_2, y_2)$

5) find slope of the secant line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

This slope of the secant line is also called the average rate of change from  $x_1$  to  $x_2$

Now if we fix  $x_1$  and let another x coordinate, say  $x_3$ , that gets closer to  $x_1$

and repeat steps 1 through 5 with the "new  $x_3$ ", then the new slope of the secant line will be a better approximation of the slope of the tangent line and ultimately that average rate of change between  $x_1$  and  $x_3$  will get closer to the instantaneous rate of change at  $x_1$

$$g(x) = \ln(x^3 + 1)$$

P has x value 0 P has y value  $g(0)=0$  P (0,0)  $\approx$  P (0.,0.)

Q has x value 1 Q has y value  $g(1)=\ln(2)$  Q (1, $\ln(2)$ )  $\approx$  Q (1.,0.69314718056)

m of PQ =  $\ln(2) \approx 0.69314718056$

T has x value 1.01 T has y value  $g(1.01)=0.708184057924$  T (1.01,0.708184057924)

m of PT = 0.701172334578

W has x value 0.0001 W has y value  $g(0.0001)=1.E-12$  W (0.0001,1.E-12)

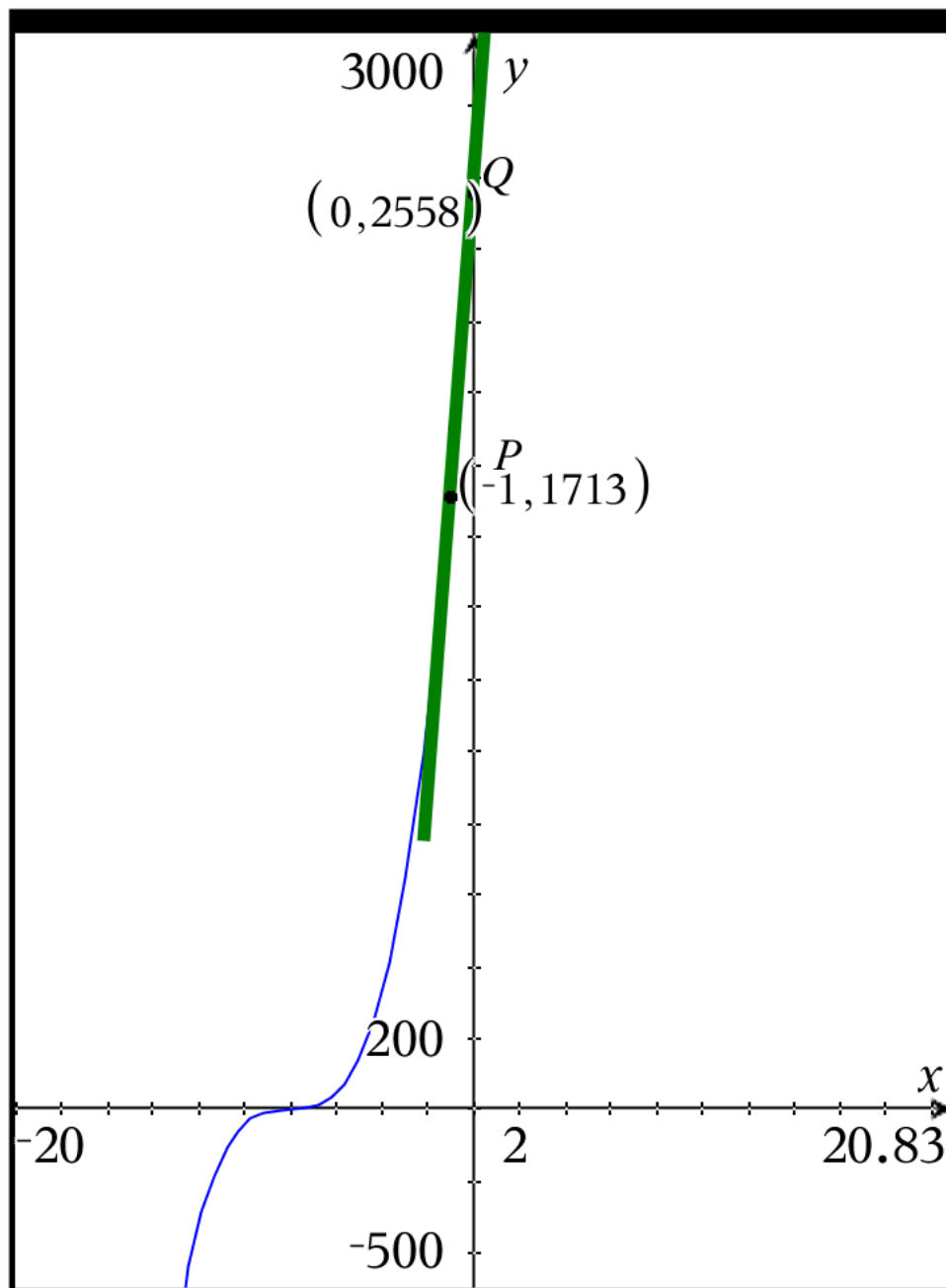
m of PW = 0.00000001

The actual tangent line's slope and the instantaneous rate of change is not found until we can either evaluate a limit properly or we learn how the derivative of a function works, but since I do know how to do these things I'll give you this slope and ultimately the instantaneous rate of change at  $x=0$  m tan = 0 SO as  $x \rightarrow 0$  we can say that the slope of the tangent line and the instantaneous rate of change  $\rightarrow 0$

	A	B	C	D x_1	E y_1	F x_2	G y_
=							
1	P	0	0	x_p	y_p	x_p	y_p
2	Q	1	ln(2)	x_q	y_q	x_t	y_t
3	m_pq	ln(2)	0.69314...				
4	change_y	ln(2)					
5	change_x	1					
6	T	1.01	0.70818...				
7	change_y	0.708184057924					
8	change_x	1.01					
9	m_pt	0.701172334578					
10	W	0.0001	1.E-12				
11	change_y	1.E-12					
B7 change_ypt:=y_t-y_p							



Problem 1



$$f(x) = 5(x+8)^3 - 2$$

$$P = (-1, 1713)$$

$$Q = (0, 2558)$$

$$\text{Slope of PQ} = 845$$

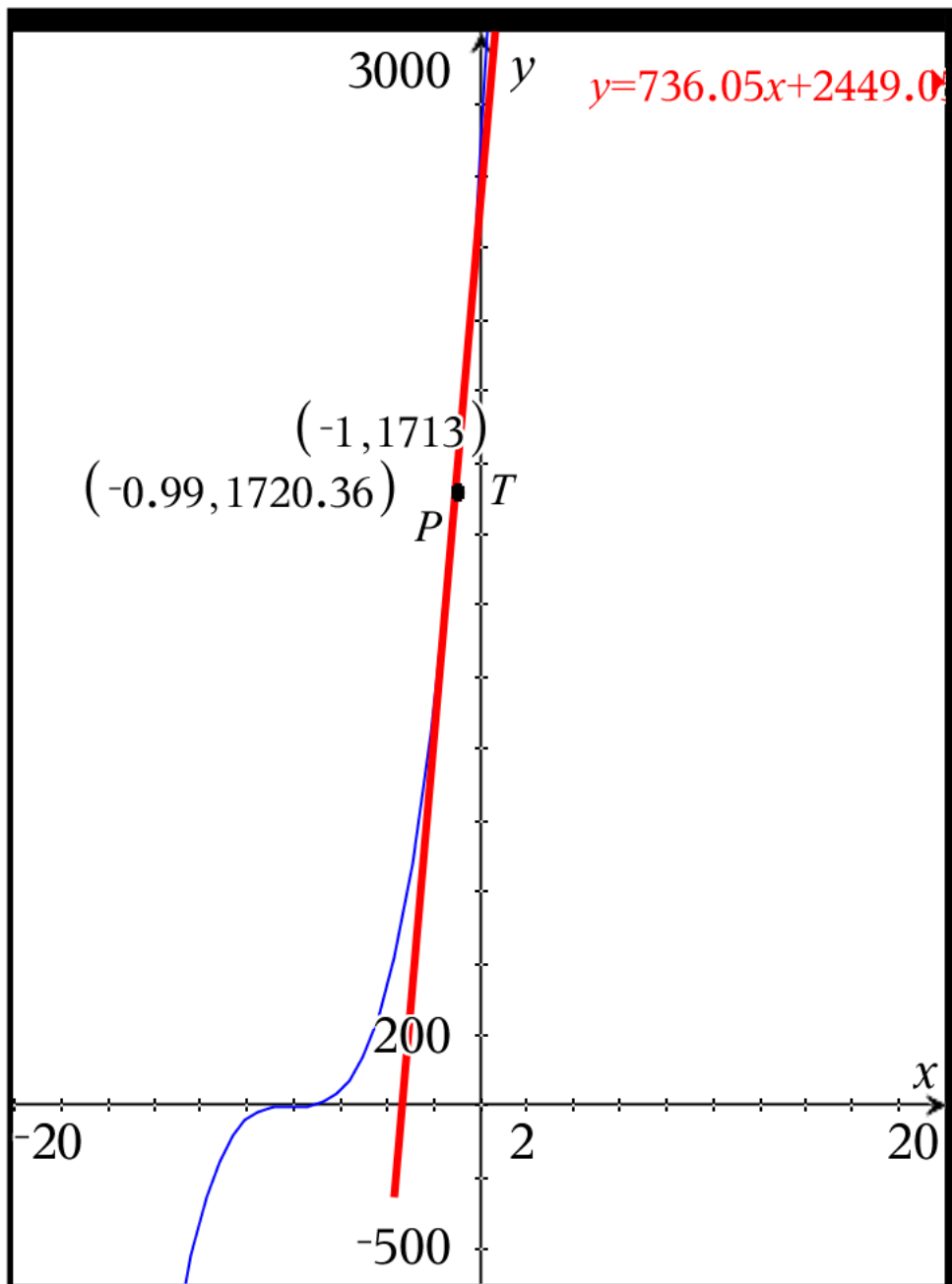
Secant line

Point Slope Form

$$y = 845(x - (-1)) + 1713$$

Slope Intercept Form

$$y = 845 \cdot x + 2558$$



$$f(x) = 5(x+8)^3 - 2$$

$$P = (-1, 1713)$$

$$T = (-0.99, 1720.360505)$$

$$\text{Slope of PT} = 736.0505$$

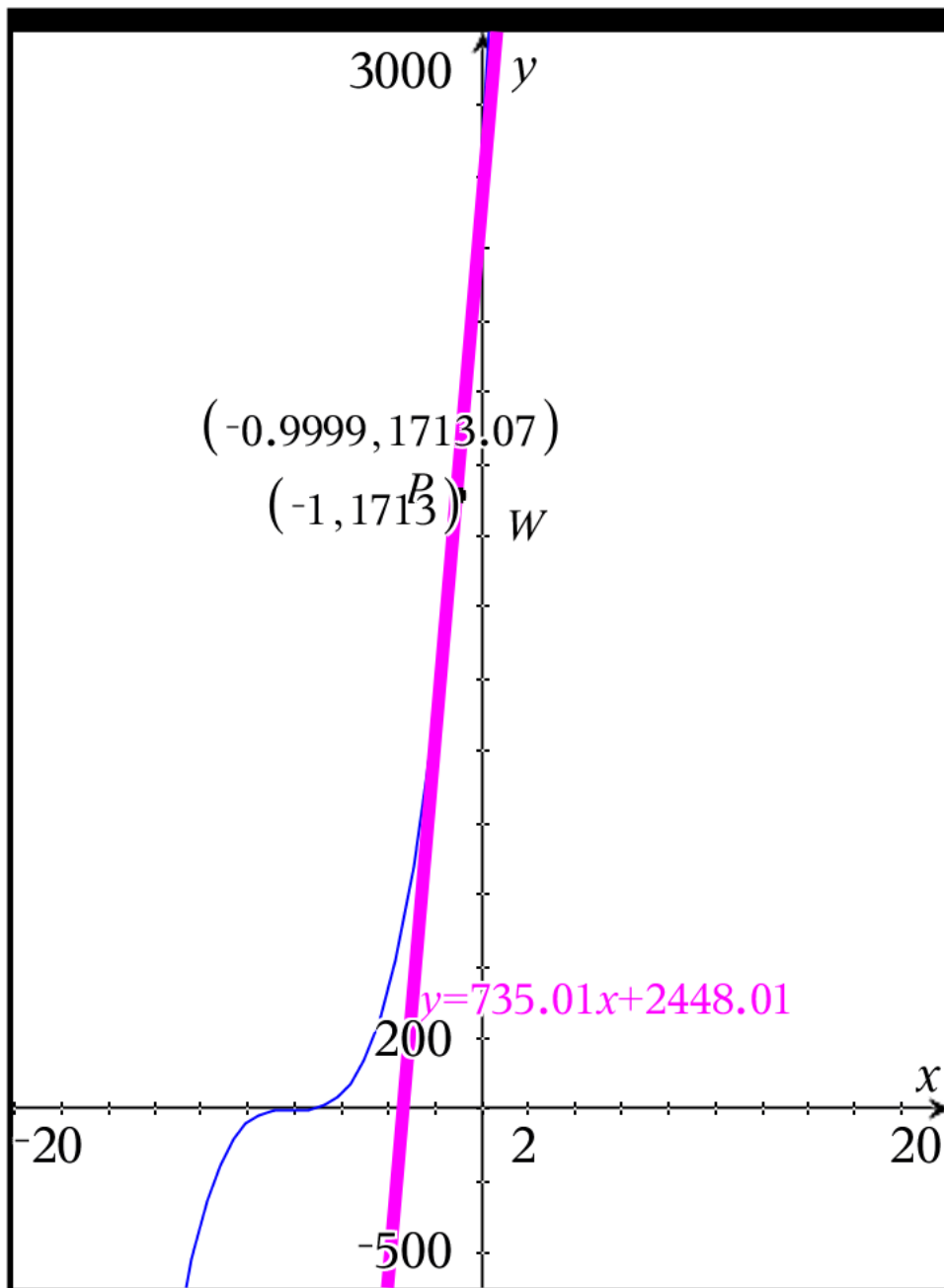
Secant line

Point Slope Form

$$y = 736.0505(x - -1) + 1713$$

Slope Intercept Form

$$y = 736.0505 \cdot x + 2449.0505$$



$$f(x) = 5(x+8)^3 - 2$$

$$P = (-1, 1713)$$

$$W = (-0.9999, 1713.07350105)$$

$$\text{Slope of } PW = 735.0105$$

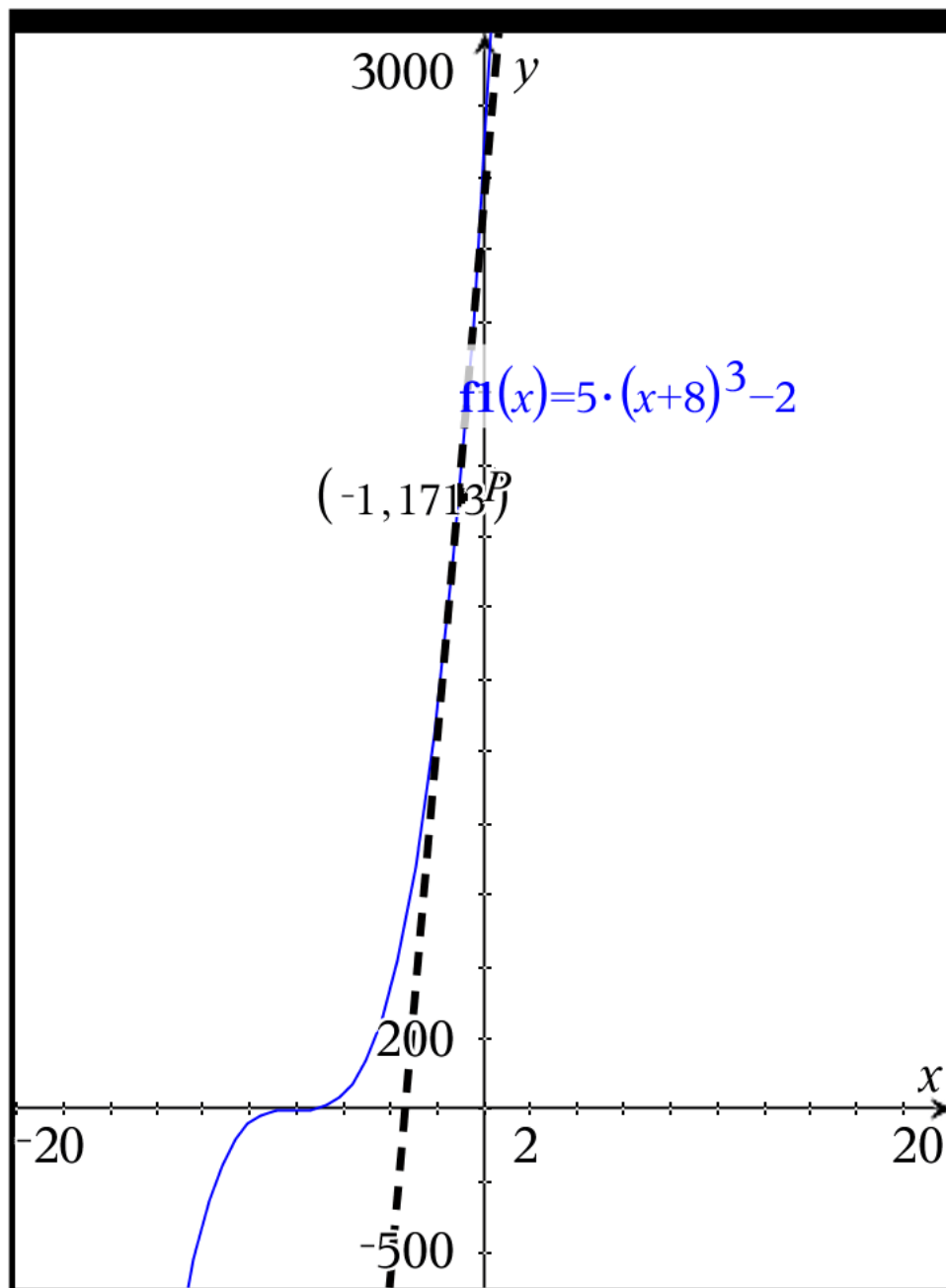
Secant line

Point Slope Form

$$y = 735.0105(x - -1) + 1713$$

Slope Intercept Form

$$y = 735.0105 \cdot x + 2448.0105$$



$$f(x) = 5(x+8)^3 - 2$$

$$P = (-1, 1713)$$

**ACTUAL Tangent Line**

**Point Slope Form**

$$y = 735(x - (-1)) + 1713$$

**Slope Intercept Form**

$$y = 735 \cdot x + 2448$$

In this case as we pick values of  $x$  that are getting increasingly closer and closer to P's  $x$  value namely  $x = -1$  ,

In my simple examples Q's  $x$  value was  $x = 0$  ,  
and it led to a slope of secant line through P and Q of  $m = 845$

In my simple examples T's  $x$  value was  $x = -0.99$  ,  
and it led to a slope of secant line through P and T of  $m = 736.0505$

In my simple examples W's  $x$  value was  $x = -0.9999$  ,  
and it led to a slope of secant line through P and W of  $m = 735.0105$

Say I took the time to find slope between P and one last value of C at  $x = -0.99999999$   
and it led to a slope of secant line through P and that point C of  $m \approx 735$ .  
(accurate to 12 decimals)

It appears that the slopes of the secant lines are pointing towards a slope of the tangent line of  $m = 735$  which is the slope of the tangent line at  $x = -1$

Steps in this process

1) find an x value that you are planning to use as one of the points in a secant line

2) determine that point's y coordinate

You now know  $(x_1, y_1)$

3) pick an x coordinate that is approaching x from step 1

4) determine that point's y coordinate

You now know  $(x_2, y_2)$

5) find slope of the secant line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

This slope of the secant line is also called the average rate of change from  $x_1$  to  $x_2$

Now if we fix  $x_1$  and let another x coordinate, say  $x_3$ , that gets closer to  $x_1$

and repeat steps 1 through 5 with the "new  $x_3$ ", then the new slope of the secant line will be a better approximation of the slope of the tangent line and ultimately that average rate of change between  $x_1$  and  $x_3$  will get closer to the instantaneous rate of change at  $x_1$

$$f(x) = 5 \cdot x^3 + 120 \cdot x^2 + 960 \cdot x + 2558$$

P has x value -1 P has y value  $f(-1) = 1713$  P (-1, 1713 )

Q has x value 0 Q has y value  $f(0) = 2558$  Q (0, 2558 )

m of PQ = 845

T has x value -0.99 T has y value  $f(-0.99) = 1720.360505$  T (-0.99, 1720.360505 )

m of PT = 736.0505

W has x value -0.9999 W has y value  $f(-0.9999) = 1713.07350105$  W (-0.9999, 1713.07350105 )

m of PW = 735.0105

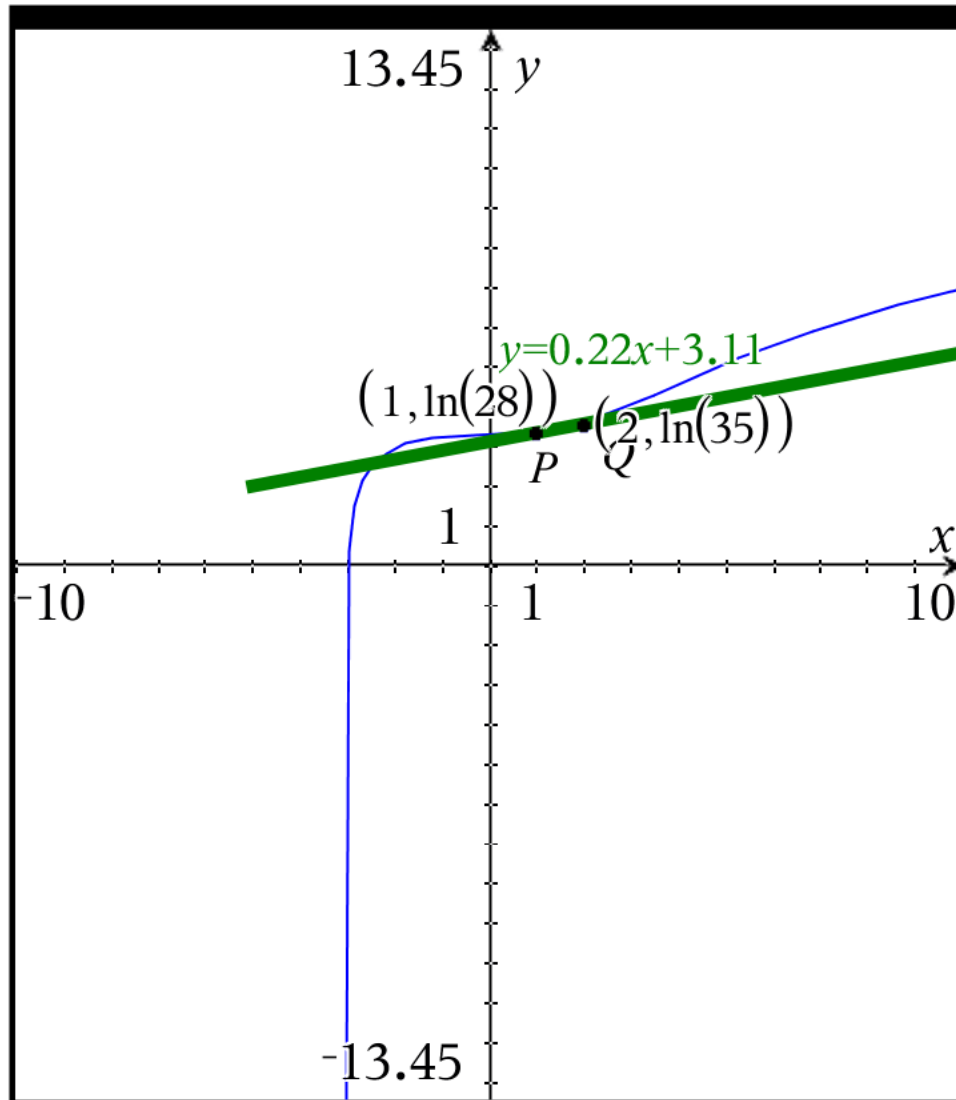
The actual tangent line's slope and the instantaneous rate of change is not found until we can either evaluate a limit properly or we learn how the derivative of a function works, but since I do know how to do these things I'll give you this slope and ultimately the instantaneous rate of change at  $x = -1$   $m_{\tan} = 735$  SO as  $x \rightarrow -1$  we can say that the slope of the tangent line and the instantaneous rate of change  $\rightarrow 735$



	A	B	C	D x_1	E y_1	F x_2	G y_2
	=						
1	P	-1	1713	x_p	y_p	x_p	y_p
2	Q	0	2558	x_q	y_q	x_t	y_t
3	m_pq	845	845.				
4	change_y	845					
5	change_x	1					
6	T	-0.99	1720.36...				
7	change_y	7.360505					
8	change_x	0.01					
9	m_pt	736.0505					
10	W	-0.9999	1713.07...				
11	change_y	0.07350105					
	B2 x_q:=b1+1						



Problem 1



$$g(x) = \ln(27+x^3)$$

$$P = (1, \ln(28)) \approx (1, 3.33220451018)$$

$$Q = (2, \ln(35)) \approx (2, 3.55534806149)$$

$$\text{Slope of PQ} = \ln\left(\frac{5}{4}\right)$$

$$\approx 0.223143551314$$

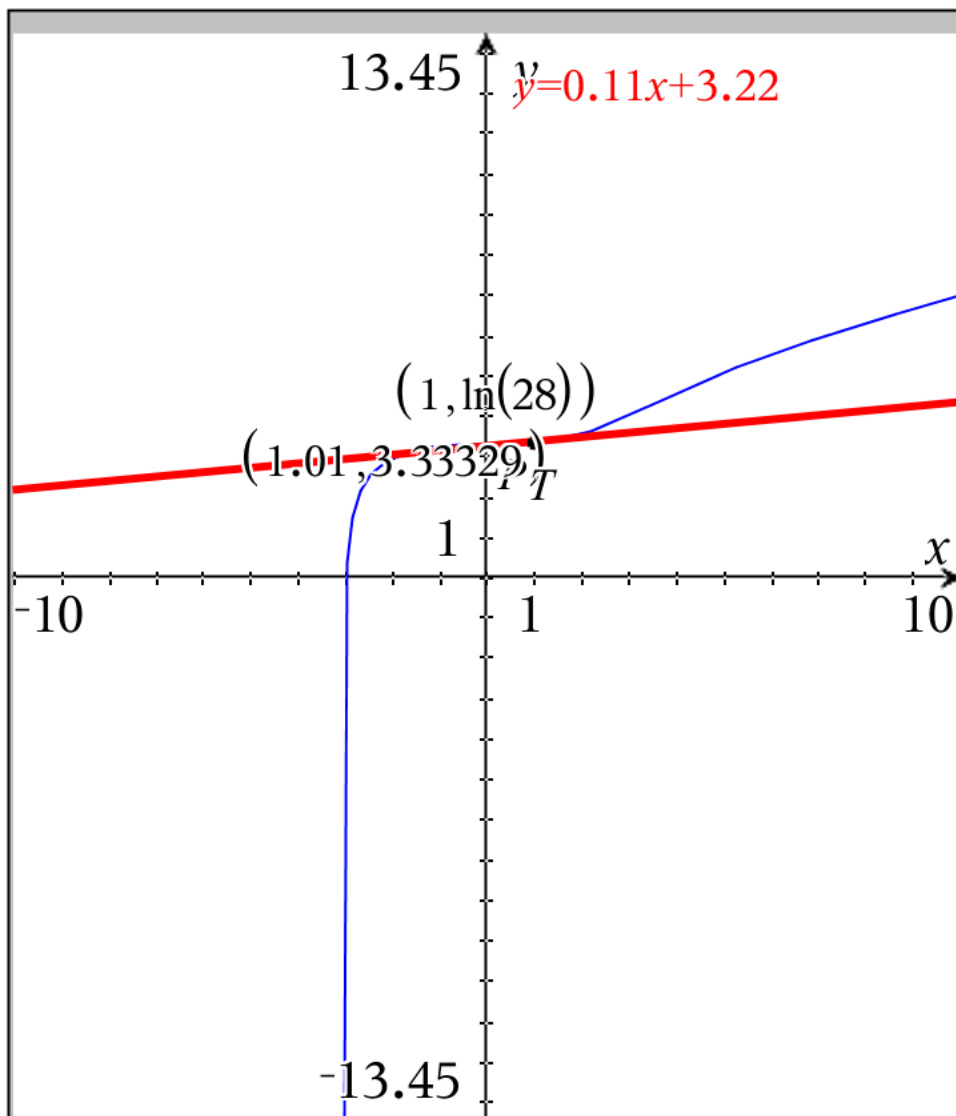
**Secant line**

**Point Slope Form**

$$y = \ln\left(\frac{5}{4}\right)(x-1) + \ln(28)$$

**Slope Intercept Form**

$$y \approx 0.223143551314 \cdot x + 3.10906095886$$



$$\mathbf{P} = (1, \ln(28))$$

$$\mathbf{T} = (1.01, 3.33328610361)$$

$$\mathbf{Slope\ of\ PT} = 0.10815934383$$

**Secant line**

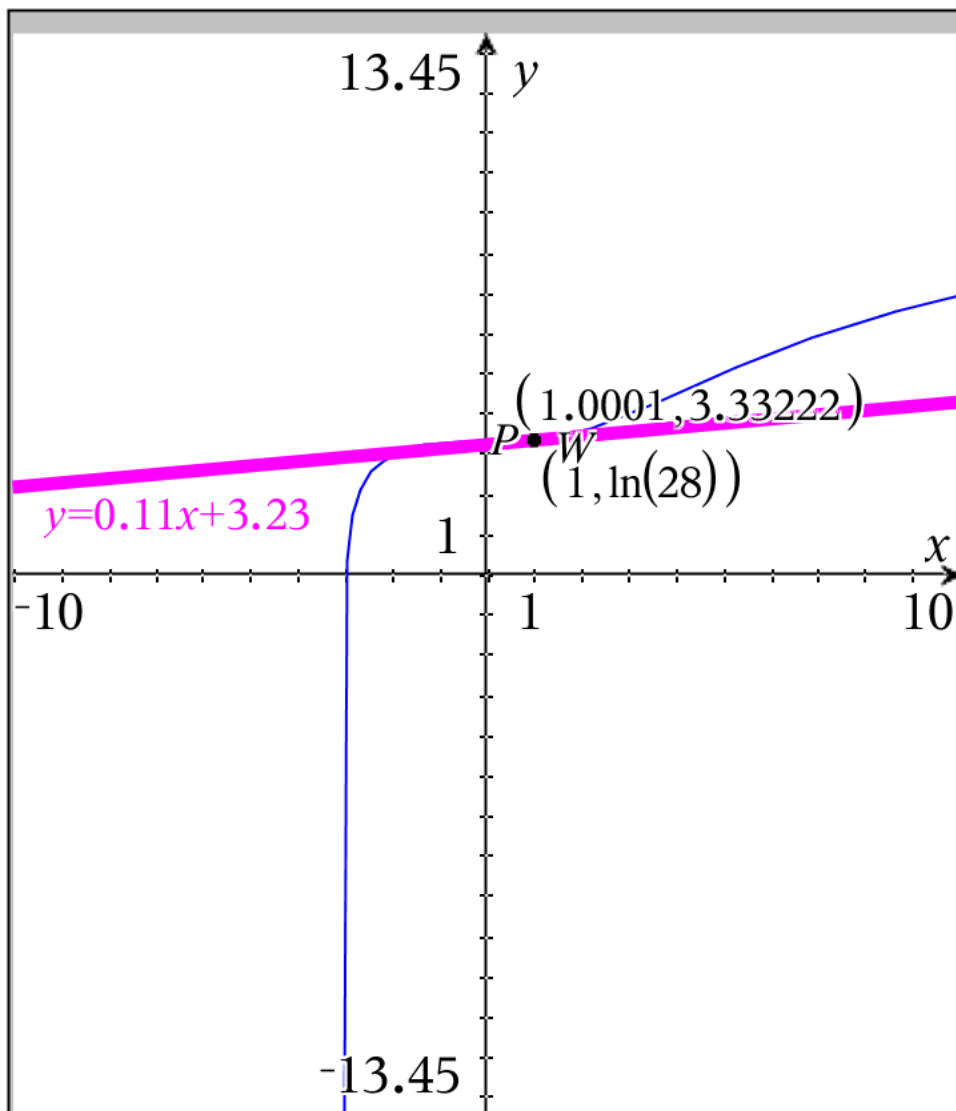
**Point Slope Form**

$$y = 0.10815934383 (x - 1) + \ln(28)$$

**Slope Intercept Form**

$$y = 0.10815934383 \cdot x + 3.22404516635$$

$$g(x) = \ln(27 + x^3)$$



$$P = (1, \ln(28))$$

$$W = (1.0001, 3.33221522548)$$

$$\text{Slope of PW} = 0.107152998$$

**Secant line**

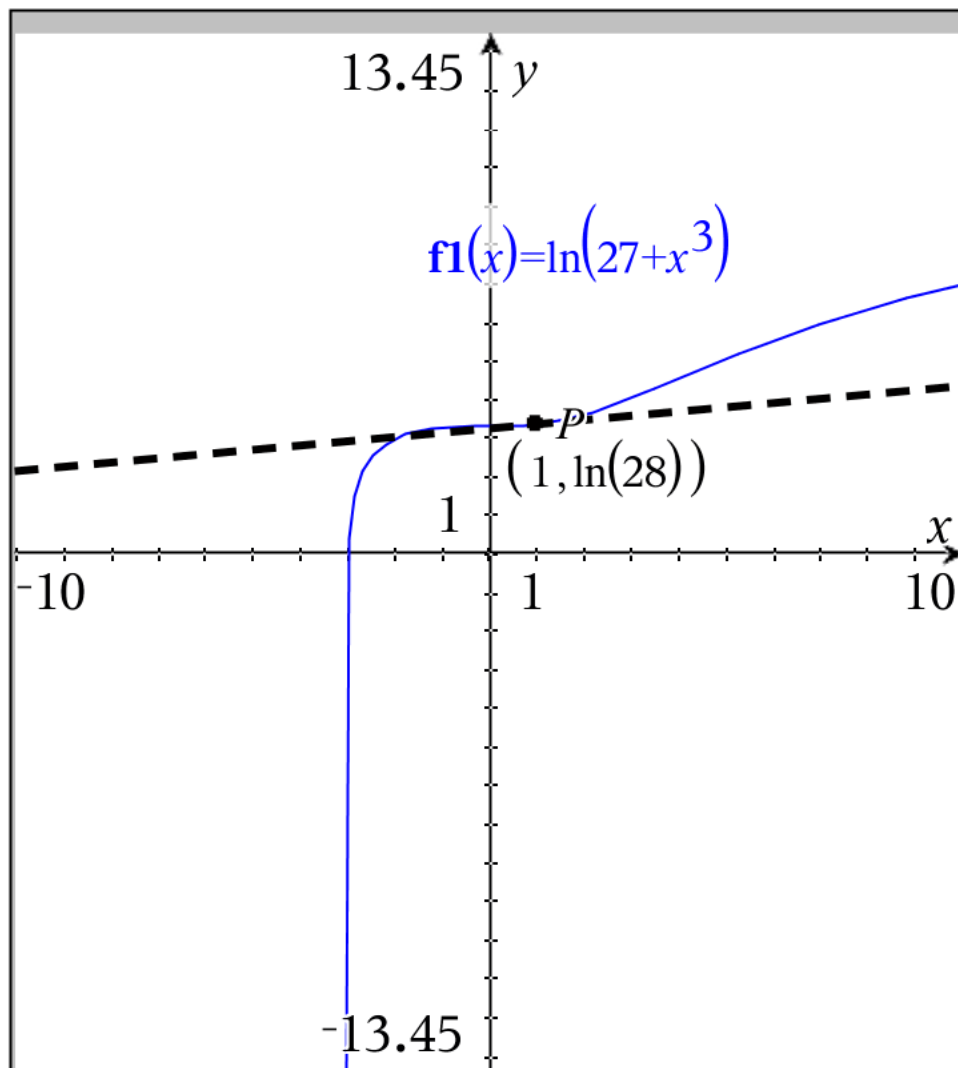
**Point Slope Form**

$$y = 0.107152998(x-1) + \ln(28)$$

**Slope Intercept Form**

$$y = 0.107152998 \cdot x + 3.22505151218$$

$$g(x) = \ln(27+x^3)$$



$$g(x) = \ln(27+x^3)$$

$$P = (1, \ln(28))$$

**ACTUAL Tangent Line**

note  $\frac{3}{28} \approx 0.107142857143$

**Point Slope Form**

$$y = \frac{3}{28}(x-1) + \ln(28)$$

**Slope Intercept Form**

$$y = \frac{3 \cdot x}{28} + \ln(7) + 2 \cdot \ln(2) - \frac{3}{28}$$

In this case as we pick values of  $x$  that are getting increasingly closer and closer to P's  $x$  value namely  $x = 1$ ,

In my simple examples Q's  $x$  value was  $x = 2$ ,

and it led to a slope of secant line through P and Q of  $m = \ln\left(\frac{5}{4}\right) = 0.223143551314$

In my simple examples T's  $x$  value was  $x = 1.01$ ,

and it led to a slope of secant line through P and T of  $m = 0.10815934383$

In my simple examples W's  $x$  value was  $x = 1.0001$ ,

and it led to a slope of secant line through P and W of  $m = 0.107152998$

Say I took the time to find slope between P and one last value of C at  $x = 1.00000001$

and it led to a slope of secant line through P and that point C of  $m \approx 0.10714$

(accurate to 12 decimals)

It appears that the slopes of the secant lines are pointing towards a slope of the tangent line of  $m = \frac{3}{28}$  which is the slope of the tangent line at  $x = 1$

Steps in this process

1) find an x value that you are planning to use as one of the points in a secant line

2) determine that point's y coordinate

You now know  $(x_1, y_1)$

3) pick an x coordinate that is approaching x from step 1

4) determine that point's y coordinate

You now know  $(x_2, y_2)$

5) find slope of the secant line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

This slope of the secant line is also called the average rate of change from  $x_1$  to  $x_2$

Now if we fix  $x_1$  and let another x coordinate, say  $x_3$ , that gets closer to  $x_1$

and repeat steps 1 through 5 with the "new  $x_3$ ", then the new slope of the secant line will be a better approximation of the slope of the tangent line and ultimately that average rate of change between  $x_1$  and  $x_3$  will get closer to the instantaneous rate of change at  $x_1$

$$g(x) = \ln(x^3 + 27)$$

P has x value 1 P has y value  $g(1) = \ln(28)$   $P(1, \ln(28)) \approx P(1, 3.33220451018)$

Q has x value 2 Q has y value  $g(2) = \ln(35)$   $Q(2, \ln(35)) \approx Q(2, 3.55534806149)$

$$m \text{ of } PQ = \ln\left(\frac{5}{4}\right) \approx 0.223143551314$$

T has x value 1.01 T has y value  $g(1.01) = 3.33328610361$   $T(1.01, 3.33328610361)$

$$m \text{ of } PT = 0.10815934383$$

W has x value 1.0001 W has y value  $g(1.0001) = 3.33221522548$   $W(1.0001, 3.33221522548)$

$$m \text{ of } PW = 0.107152998$$

The actual tangent line's slope and the instantaneous rate of change is not found until we can either evaluate a limit properly or we learn how the derivative of a function works, but since I do know how to do these things I'll

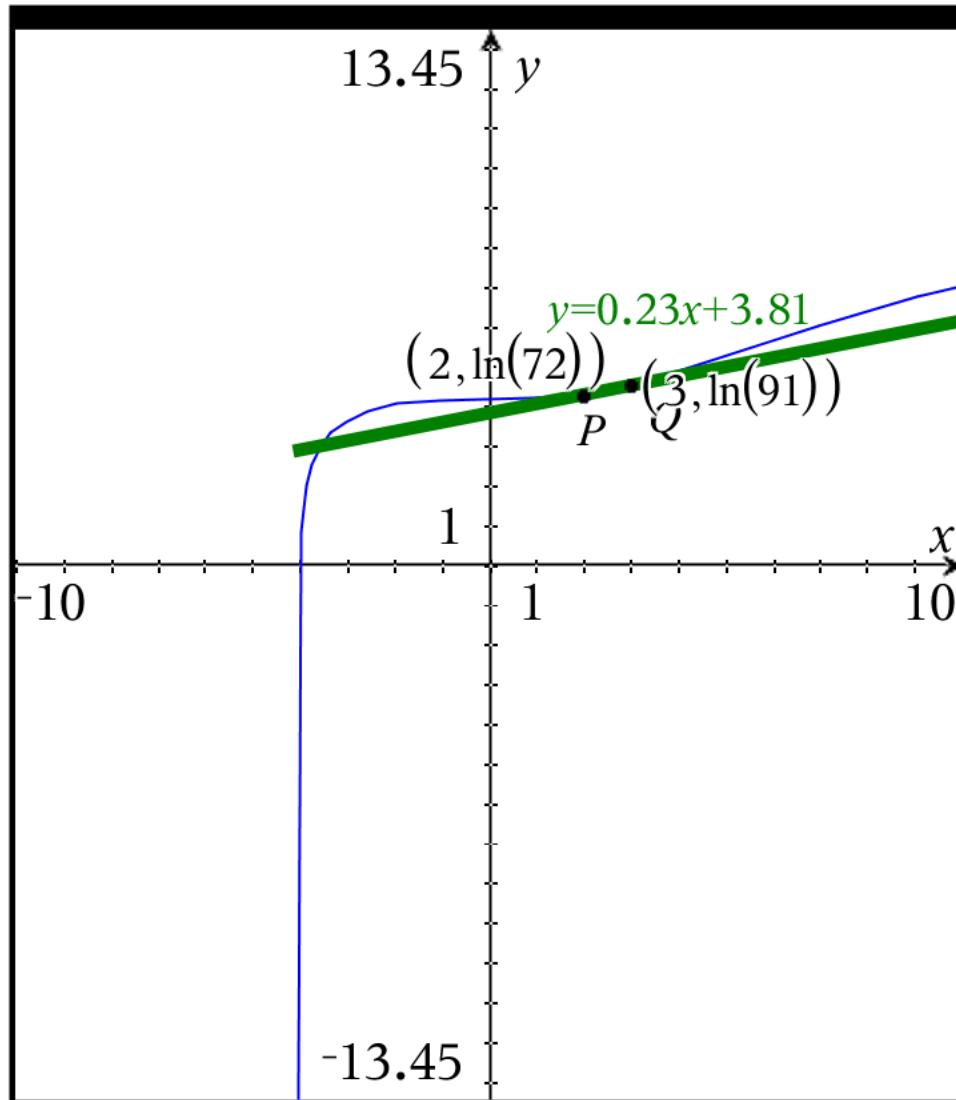
give you this slope and ultimately the instantaneous rate of change at  $x = 1$   $m_{\tan} = \frac{3}{28}$  SO as  $x \rightarrow 1$  we can say

that the slope of the tangent line and the instantaneous rate of change  $\rightarrow \frac{3}{28}$

	A	B	C	D x_1	E y_1	F x_2	G y_2
	=						
1	P	1	$\ln(28)$	x_p	y_p	x_p	y_p
2	Q	2	$\ln(35)$	x_q	y_q	x_t	y_t
3	m_pq	$\ln(5/4)$	0.22314...				
4	change_y	$\ln(5/4)$					
5	change_x	1					
6	T	1.01	3.33328...				
7	change_y	0.001081593438					
8	change_x	0.01					
9	m_pt	0.10815934383					
10	W	1.0001	3.33221...				
11	change_y	0.0000107153					
	B2 x_q:=b1+1						



Problem 1



$$g(x) = \ln(64 + x^3)$$

$$P = (2, \ln(72)) \approx (2, 4.27666611902)$$

$$Q = (3, \ln(91)) \approx (3, 4.51085950652)$$

$$\text{Slope of PQ} = \ln\left(\frac{91}{72}\right)$$

$$\approx 0.234193387501$$

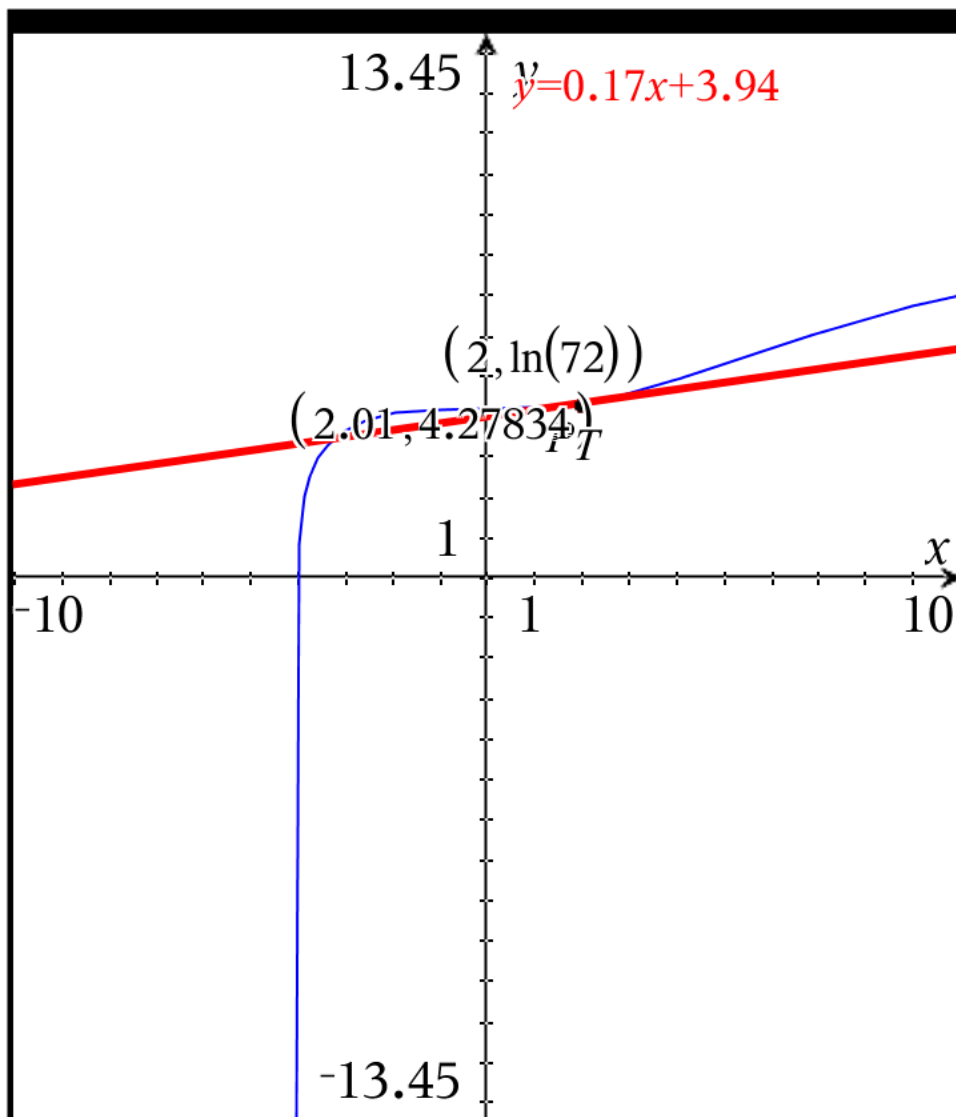
**Secant line**

**Point Slope Form**

$$y = \ln\left(\frac{91}{72}\right)(x - 2) + \ln(72)$$

**Slope Intercept Form**

$$y \approx 0.234193387501 \cdot x + 3.80827934401$$



$$P = (2, \ln(72))$$

$$T = (2.01, 4.27833973163)$$

$$\text{Slope of PT} = 0.16736126176$$

**Secant line**

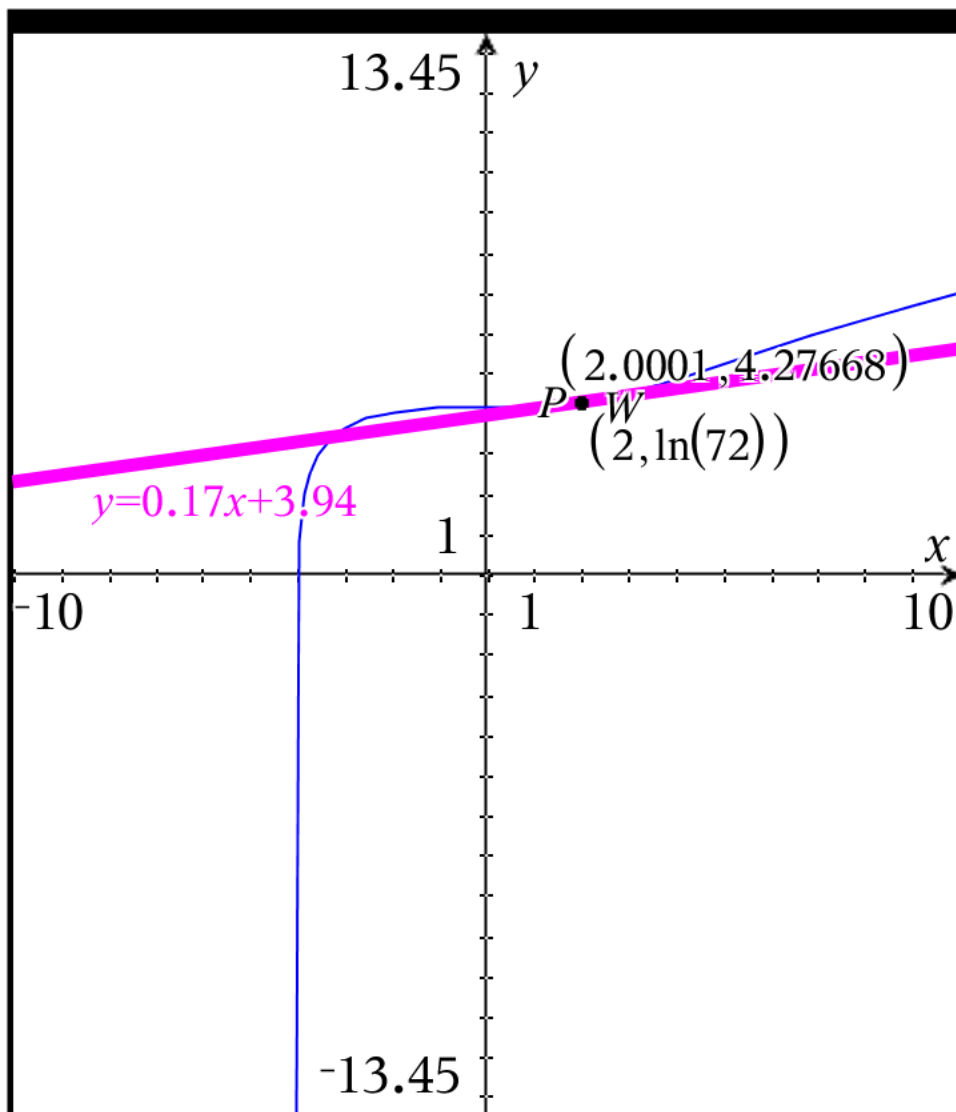
**Point Slope Form**

$$y = 0.16736126176(x - 2) + \ln(72)$$

**Slope Intercept Form**

$$y = 0.16736126176 \cdot x + 3.9419435955$$

$$g(x) = \ln(64 + x^3)$$



$$P = (2, \ln(72))$$

$$W = (2.0001, 4.27668278638)$$

$$\text{Slope of PW} = 0.166673611$$

Secant line

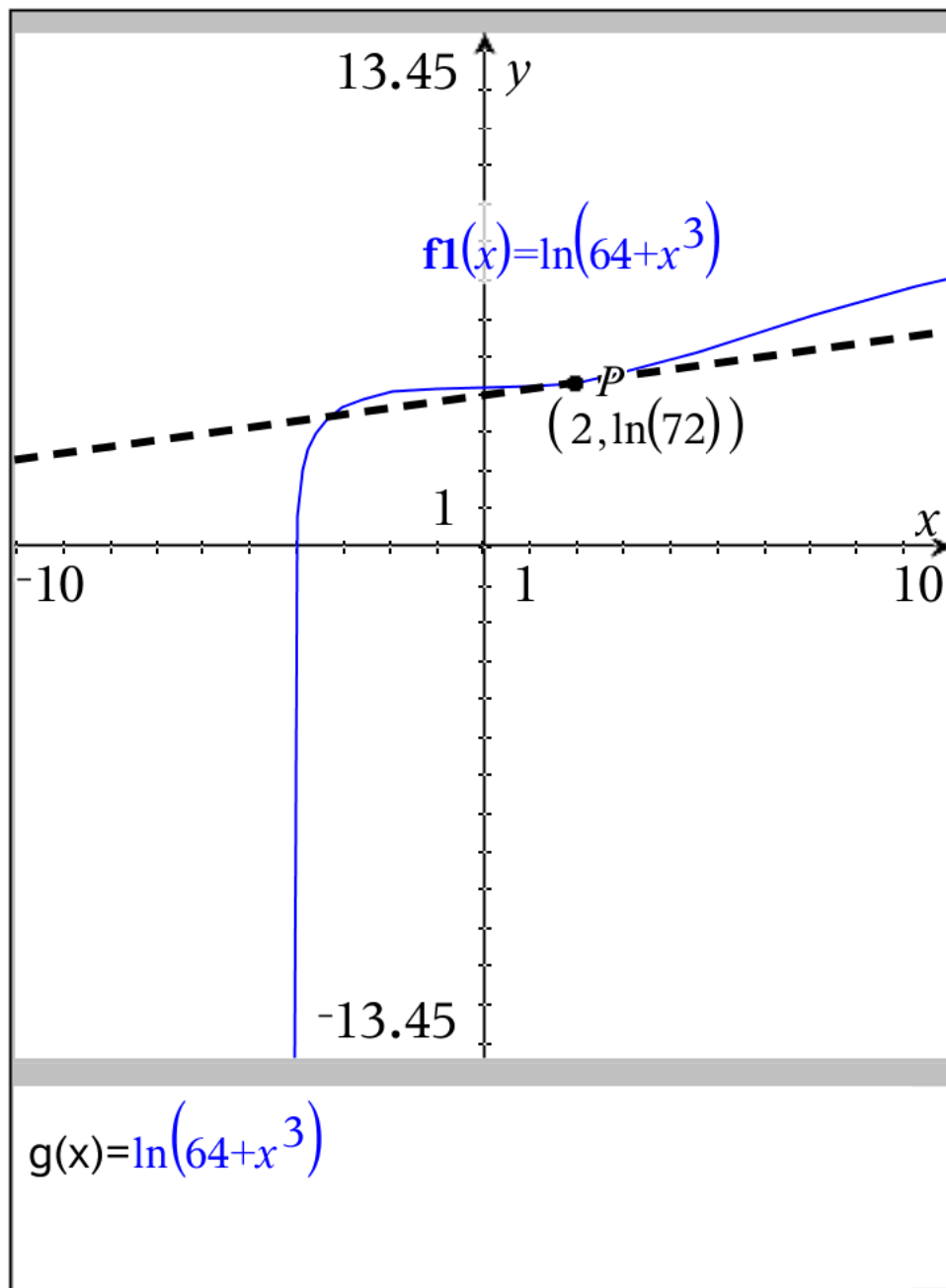
Point Slope Form

$$y = 0.166673611(x - 2) + \ln(72)$$

Slope Intercept Form

$$y = 0.166673611 \cdot x + 3.94331889702$$

$$g(x) = \ln(64 + x^3)$$



$$P = (2, \ln(72))$$

**ACTUAL Tangent Line**

note  $\frac{3}{65} \approx 0.046153846154$

**Point Slope Form**

$$y = \frac{1}{6}(x-2) + \ln(72)$$

**Slope Intercept Form**

$$y = \frac{x}{6} + 2 \cdot \ln(3) + 3 \cdot \ln(2) - \frac{1}{3}$$

In this case as we pick values of  $x$  that are getting increasingly closer and closer to P's  $x$  value namely  $x = 2$ ,

In my simple examples Q's  $x$  value was  $x = 3$ ,

and it led to a slope of secant line through P and Q of  $m = \ln\left(\frac{91}{72}\right) = 0.234193387501$

In my simple examples T's  $x$  value was  $x = 2.01$ ,

and it led to a slope of secant line through P and T of  $m = 0.16736126176$

In my simple examples W's  $x$  value was  $x = 2.0001$ ,

and it led to a slope of secant line through P and W of  $m = 0.166673611$

Say I took the time to find slope between P and one last value of C at  $x = 2.00000001$

and it led to a slope of secant line through P and that point C of  $m \approx 0.16666$

(accurate to 12 decimals)

It appears that the slopes of the secant lines are pointing towards a slope of the tangent line of  $m = \frac{1}{6}$  which is the the slope of the tangent line at  $x = 2$

Steps in this process

1) find an x value that you are planning to use as one of the points in a secant line

2) determine that point's y coordinate

You now know  $(x_1, y_1)$

3) pick an x coordinate that is approaching x from step 1

4) determine that point's y coordinate

You now know  $(x_2, y_2)$

5) find slope of the secant line

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

This slope of the secant line is also called the average rate of change from  $x_1$  to  $x_2$

Now if we fix  $x_1$  and let another x coordinate, say  $x_3$ , that gets closer to  $x_1$

and repeat steps 1 through 5 with the "new  $x_3$ ", then the new slope of the secant line will be a better approximation of the slope of the tangent line and ultimately that average rate of change between  $x_1$  and  $x_3$  will get closer to the instantaneous rate of change at  $x_1$

$$g(x) = \ln(x^3 + 64)$$

P has x value 2 P has y value  $g(2) = \ln(72)$   $P(2, \ln(72)) \approx P(2, 4.27666611902)$

Q has x value 3 Q has y value  $g(3) = \ln(91)$   $Q(3, \ln(91)) \approx Q(3, 4.51085950652)$

$$m \text{ of PQ} = \ln\left(\frac{91}{72}\right) \approx 0.234193387501$$

T has x value 2.01 T has y value  $g(2.01) = 4.27833973163$   $T(2.01, 4.27833973163)$

$$m \text{ of PT} = 0.16736126176$$

W has x value 2.0001 W has y value  $g(2.0001) = 4.27668278638$   $W(2.0001, 4.27668278638)$

$$m \text{ of PW} = 0.166673611$$

The actual tangent line's slope and the instantaneous rate of change is not found until we can either evaluate a limit properly or we learn how the derivative of a function works, but since I do know how to do these things I'll give you this slope and ultimately the instantaneous rate of change at  $x = 2$   $m_{\tan} = \frac{1}{6}$  SO as  $x \rightarrow 2$  we can say

that the slope of the tangent line and the instantaneous rate of change  $\rightarrow \frac{1}{6}$

	A	B	C	D x_1	E y_1	F x_2	G y_2
	=						
1	P	2	$\ln(72)$	x_p	y_p	x_p	y_p
2	Q	3	$\ln(91)$	x_q	y_q	x_t	y_t
3	m_pq	$\ln(91/72)$	0.23419...				
4	change_y	$\ln(91/72)$					
5	change_x	1					
6	T	2.01	4.27833...				
7	change_y	0.001673612618					
8	change_x	0.01					
9	m_pt	0.16736126176					
10	W	2.0001	4.27668...				
11	change_y	0.000016667361					
	B2 x_q:=b1+1						