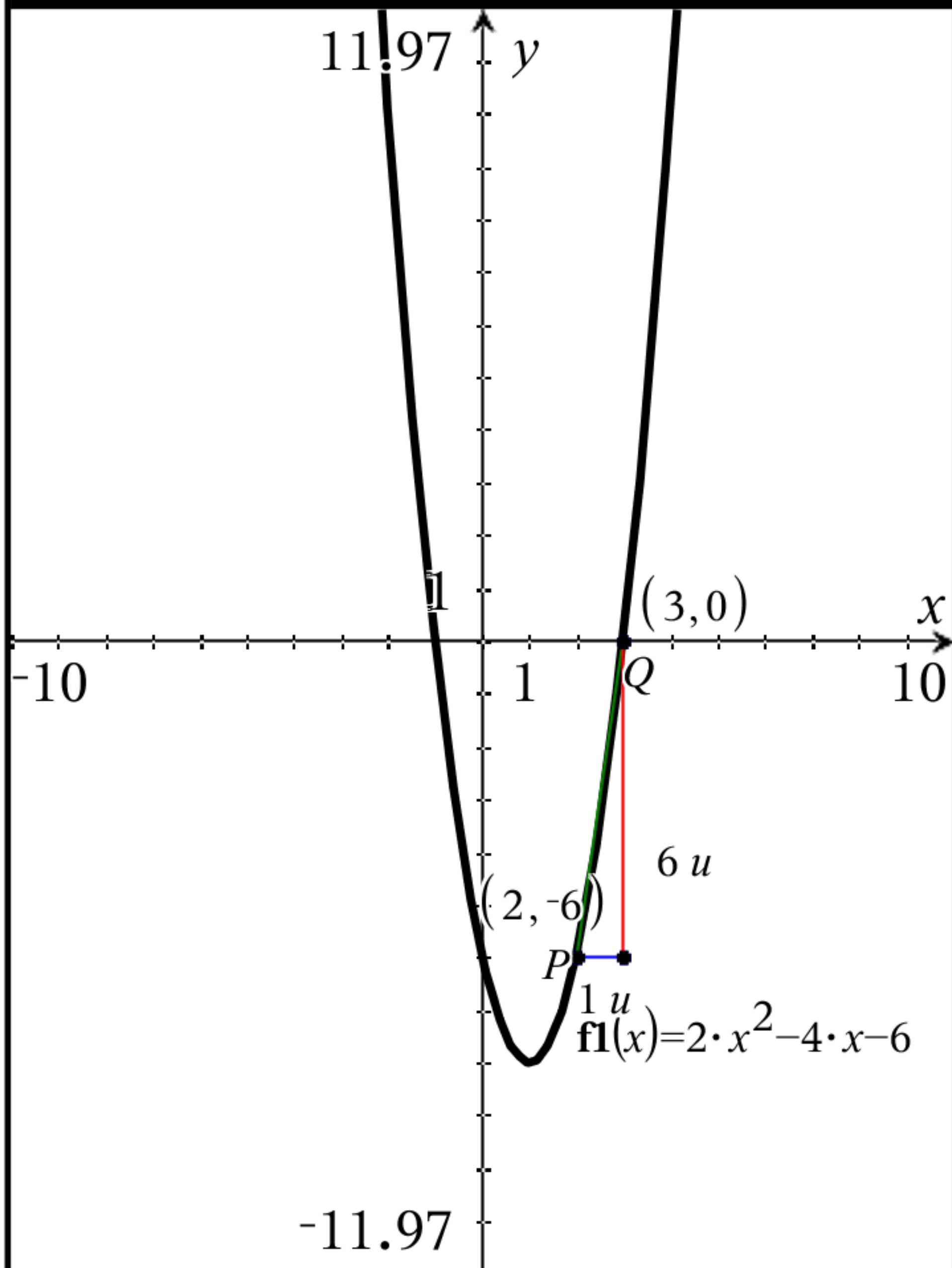


Find the rate of change from

P at x = 2 to Q at x = 3

$f(x)=2\cdot x^2-4\cdot x-6$

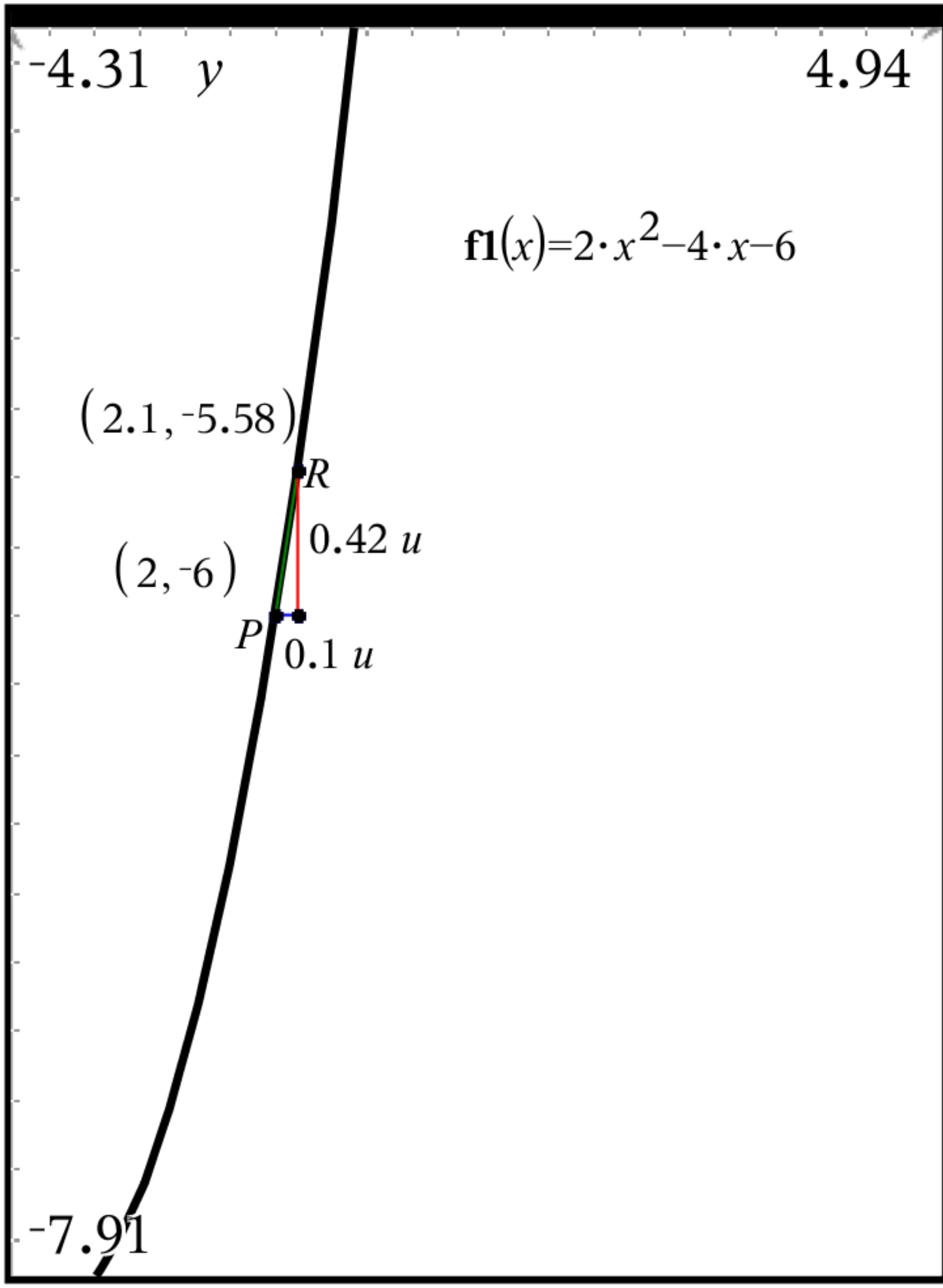
$f(2)=^-6 \rightarrow P = (2, ^-6)$

$f(3)=0 \rightarrow Q = (3, 0)$

$\frac{\Delta y}{\Delta x} = (^-6-0)/(2-3) = 6$

So we can say that the R.O.C.

between x = 2 and x = 3 is $\frac{\Delta y}{\Delta x} = 6$



Find the rate of change from

P at x = 2 to R at x = 2.1

$f(x)=2\cdot x^2-4\cdot x-6$

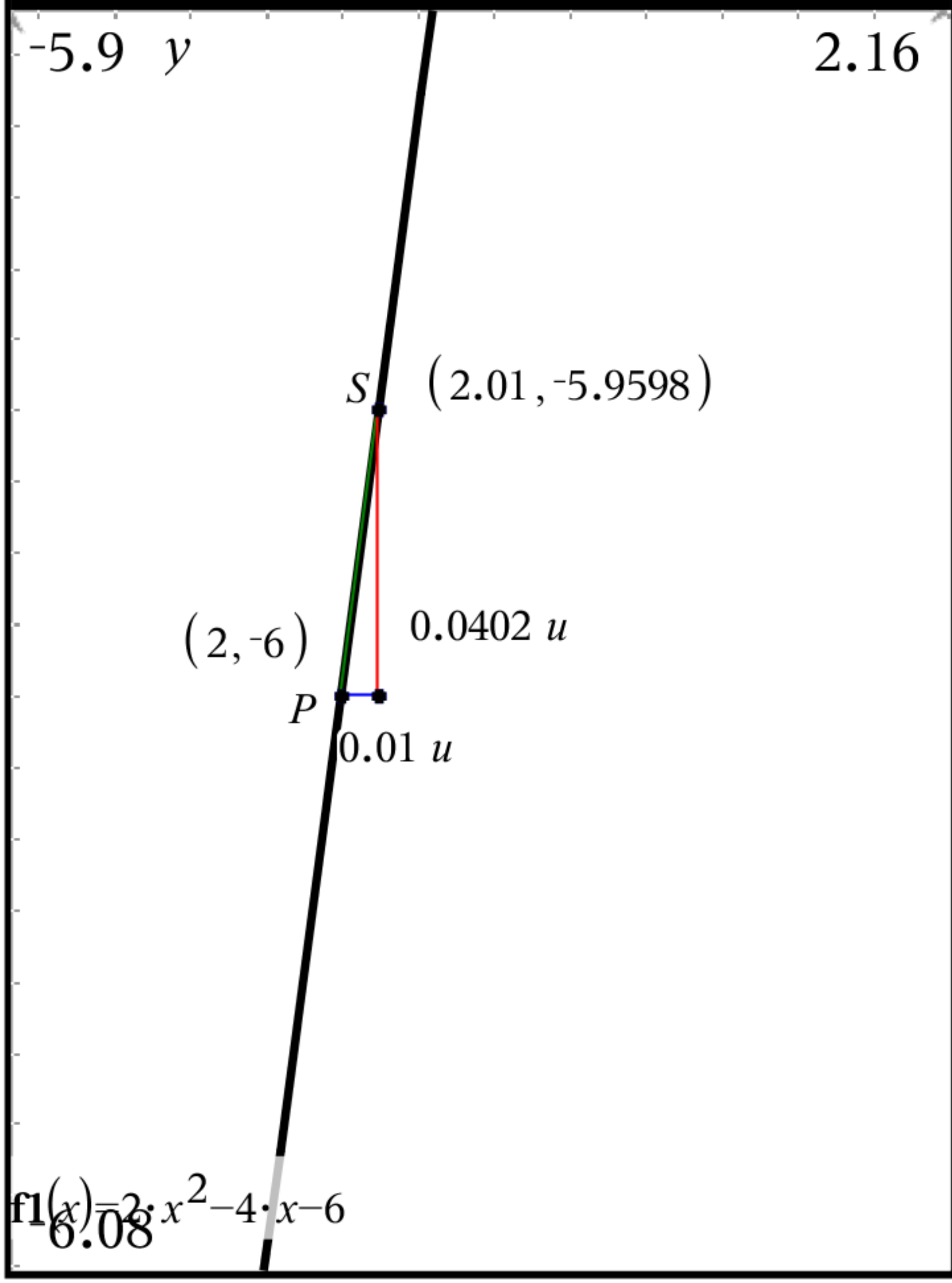
$f(2)={}^{-}6 \rightarrow P = (2, {}^{-}6)$

$f(2.1)={}^{-}5.58 \rightarrow Q = (2.1, {}^{-}5.58)$

$\frac{\Delta y}{\Delta x} = ({}^{-}6 - {}^{-}5.58)/(2-2.1) = 4.2$

So we can say that the R.O.C.

between x = 2 and x = 2.1 is $\frac{\Delta y}{\Delta x} = 4.2$



Find the rate of change from

P at x = 2 to S at x = 2.01

$f(x)=2\cdot x^2-4\cdot x-6$

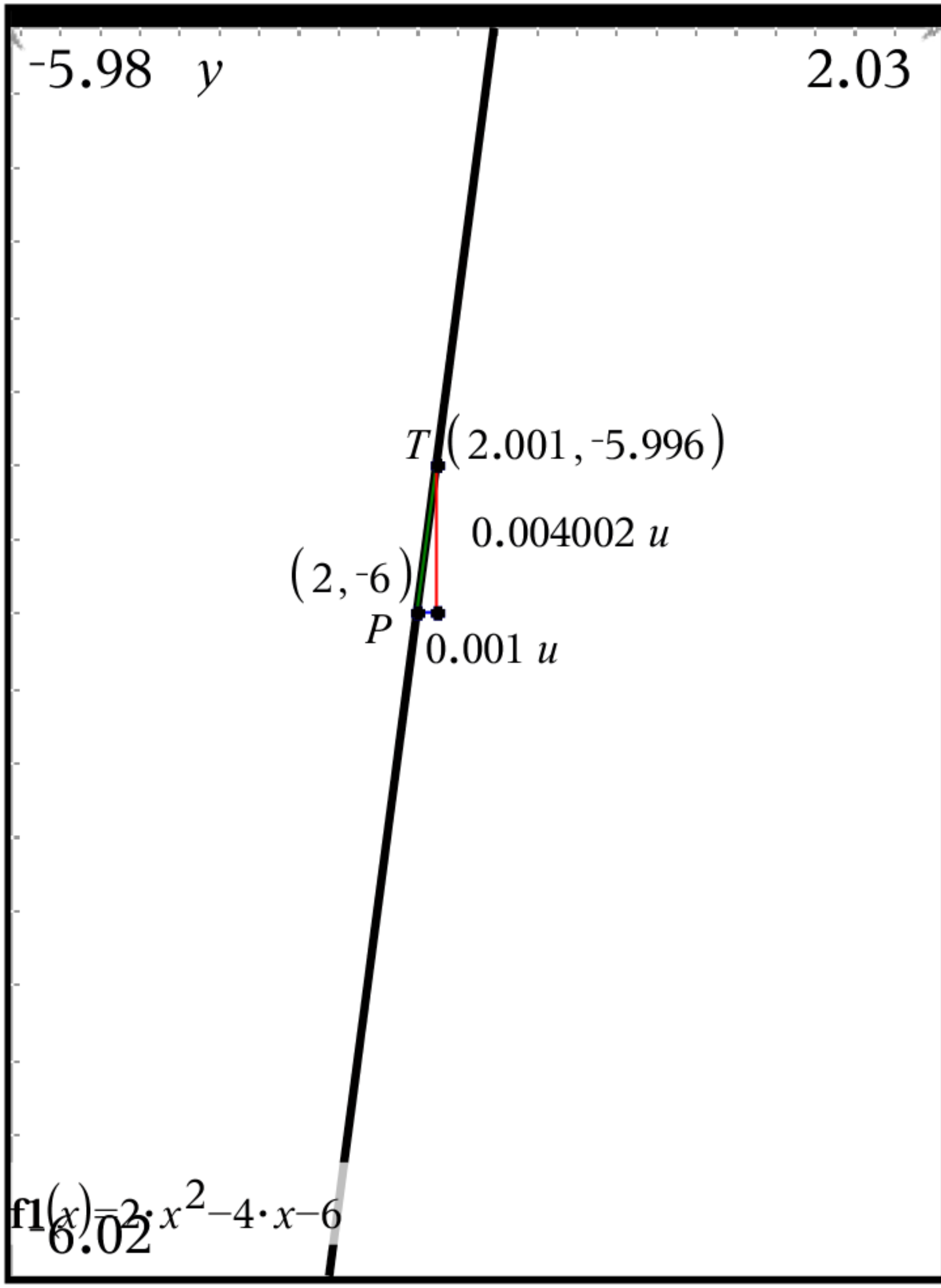
$f(2)={}^-6 \rightarrow P = (2, {}^-6)$

$f(2.01)={}^-5.9598 \rightarrow S = (2.01, {}^-5.9598)$

$\frac{\Delta y}{\Delta x} = ({}^-6 - {}^-5.9598)/(2-2.01) = 4.02$

So we can say that the R.O.C.

between x = 2 and x = 2.01 is $\frac{\Delta y}{\Delta x} = 4.02$



Find the rate of change from

P at x = 2 to T at x = 2.001

f(x)=$2\cdot x^2-4\cdot x-6$

f(2)=$^-6$ → P = (2, $^-6$)

f(2.001)=$^-5.995998$ → T = (2.001 , $^-5.995998$)

$\dfrac{\Delta y}{\Delta x}$ =($^-6$ − $^-5.995998$)/(2 −2.001) =4.002

So we can say that the R.O.C.

between x = 2 and x = 2.001 is $\dfrac{\Delta y}{\Delta x}$ =4.002

Since we calculated the slopes of the secant lines from 2 to a point to the right of 2,

PQ, PR, PS, PT all approximate the slope of the tangent line at x = 2

We approximated the limit at x = 2 from the right side of the domain

$$\lim_{x \to 2^+} \left(\frac{\mathbf{f1}(x) - \mathbf{f1}(2)}{x-2} \right) = 4$$

ROC from x = 2 to x = 3 =Slope of the secant line PQ at x = 2 and x = 3 $= \frac{\mathbf{f1}(3) - \mathbf{f1}(2)}{3-2} = 6$

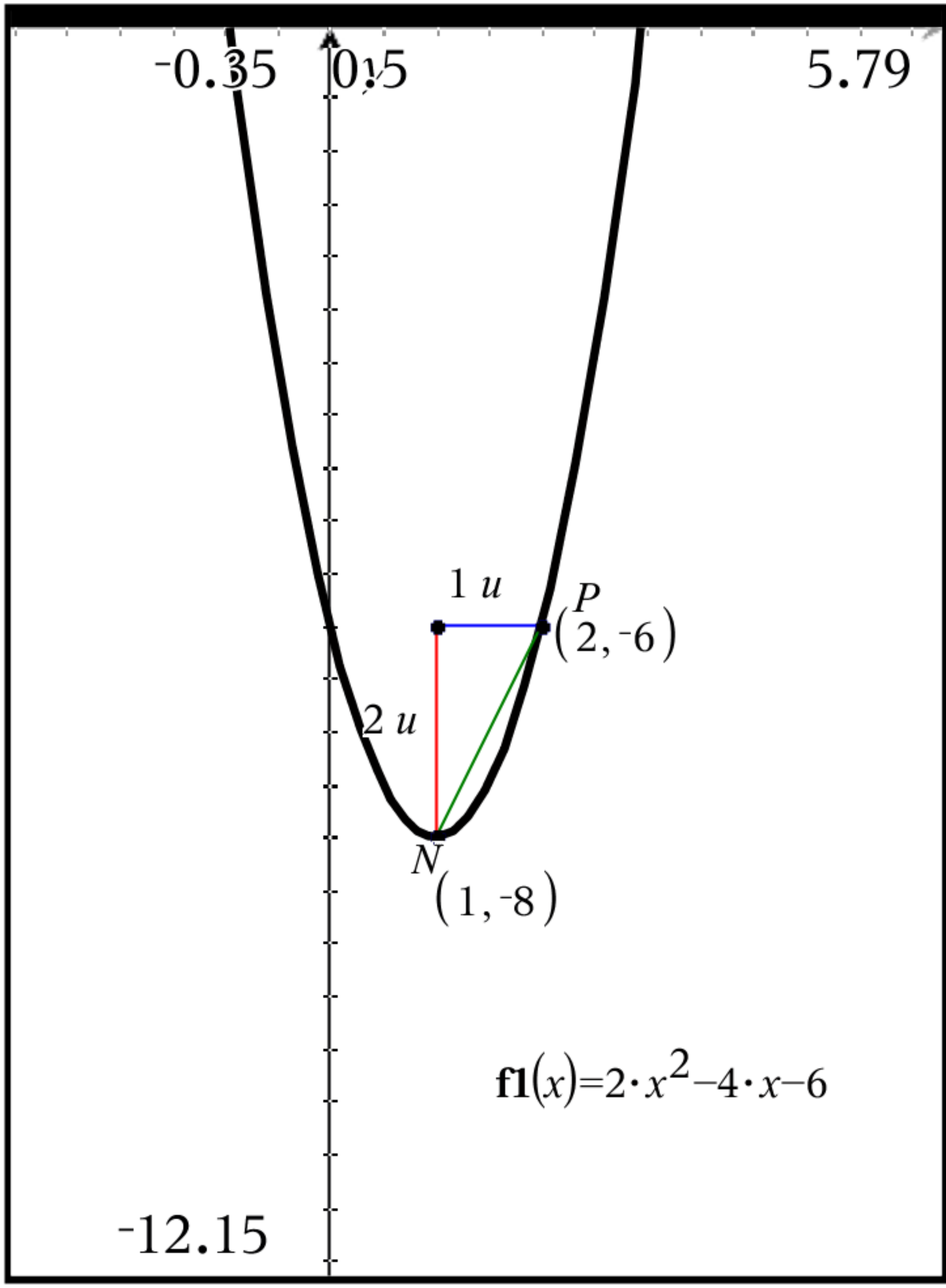
ROC from x = 2 to x = 2.1 =Slope of the secant line PR at x = 2 and x = 2.1 $= \frac{\mathbf{f1}(2.1) - \mathbf{f1}(2)}{2.1-2} = 4.2$

ROC from x = 2 to x = 2.01 =Slope of the secant line PS at x = 2 and x = 2.01

$$= \frac{\mathbf{f1}(2.01) - \mathbf{f1}(2)}{2.01-2} = 4.02$$

ROC from x = 2 to x = 2.001 =Slope of the secant line PT at x = 2 and x = 2.01

$$= \frac{\mathbf{f1}(2.001) - \mathbf{f1}(2)}{2.001-2} = 4.002$$



Find the rate of change from

P at x = 2 to N at x = 1

$f(x)=2\cdot x^2-4\cdot x-6$

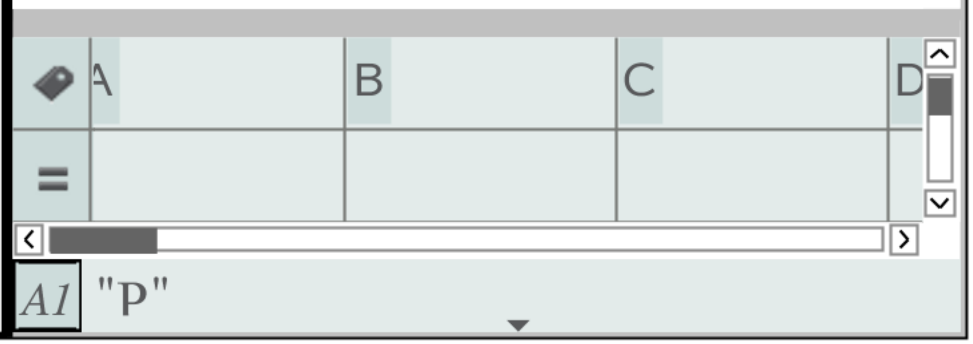
$f(2)=^-6 \rightarrow P=(2,^-6)$

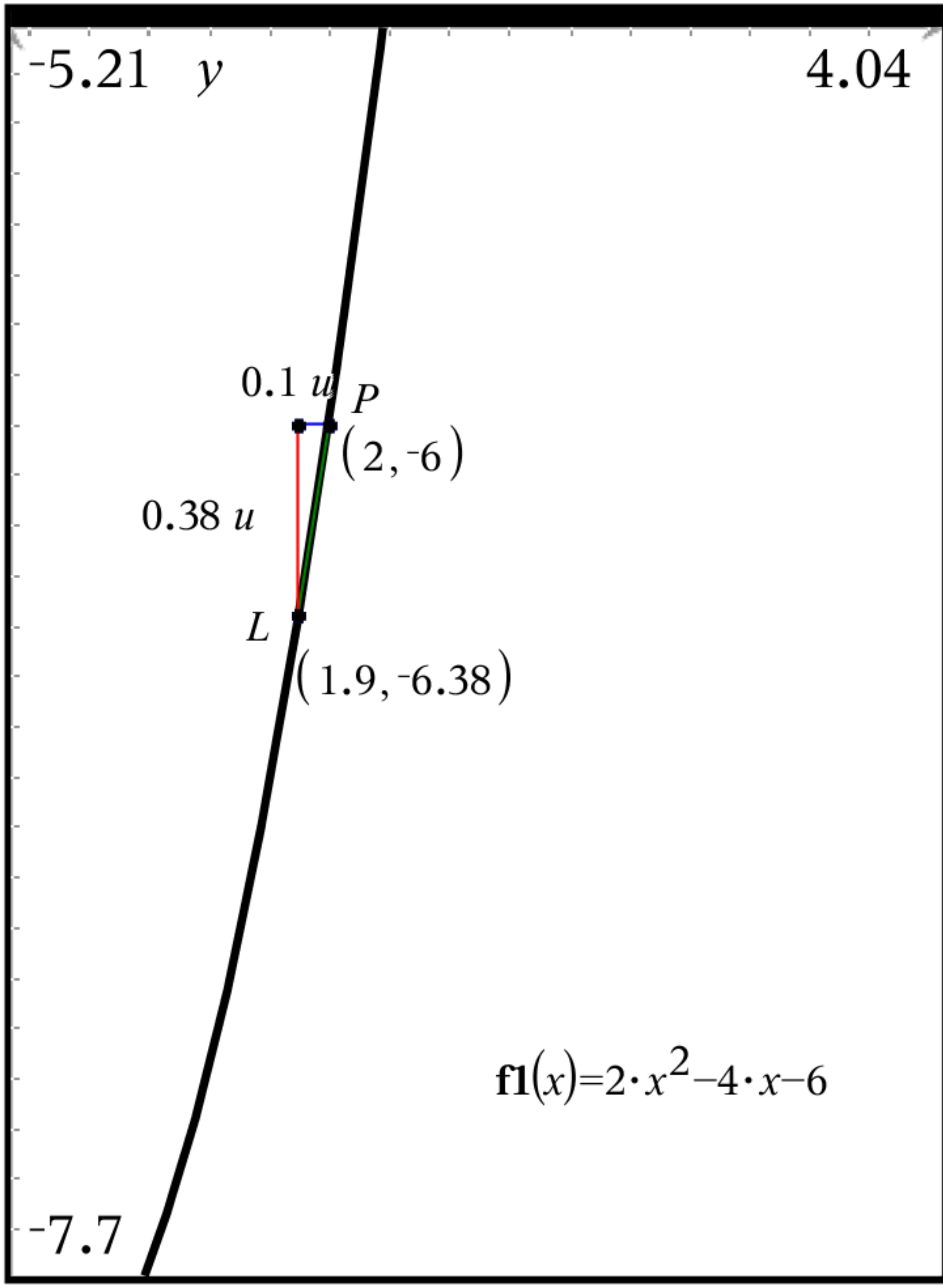
$f(1)=^-8 \rightarrow N=(1,^-8)$

$\frac{\Delta y}{\Delta x}=(^-6-^-8)/(2-1)=2$

So we can say that the R.O.C.

between x = 2 and x = 1 is $\frac{\Delta y}{\Delta x}=2$





Find the rate of change from

P at x = 2 to L at x = 1.9

$f(x)=2\cdot x^2-4\cdot x-6$

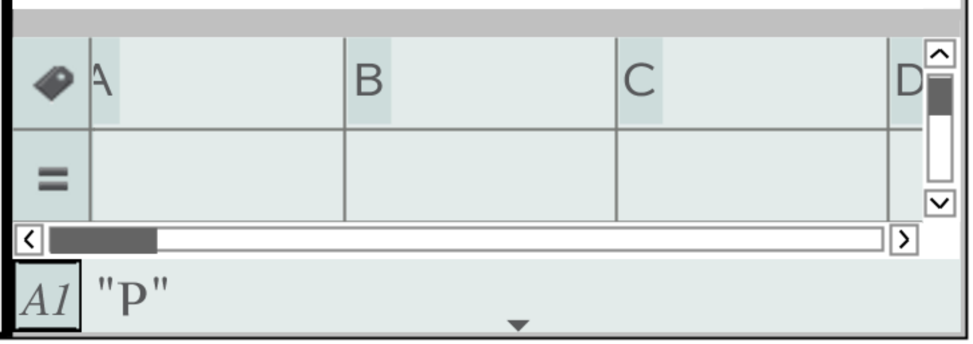
$f(2)=-6 \rightarrow P=(2,-6)$

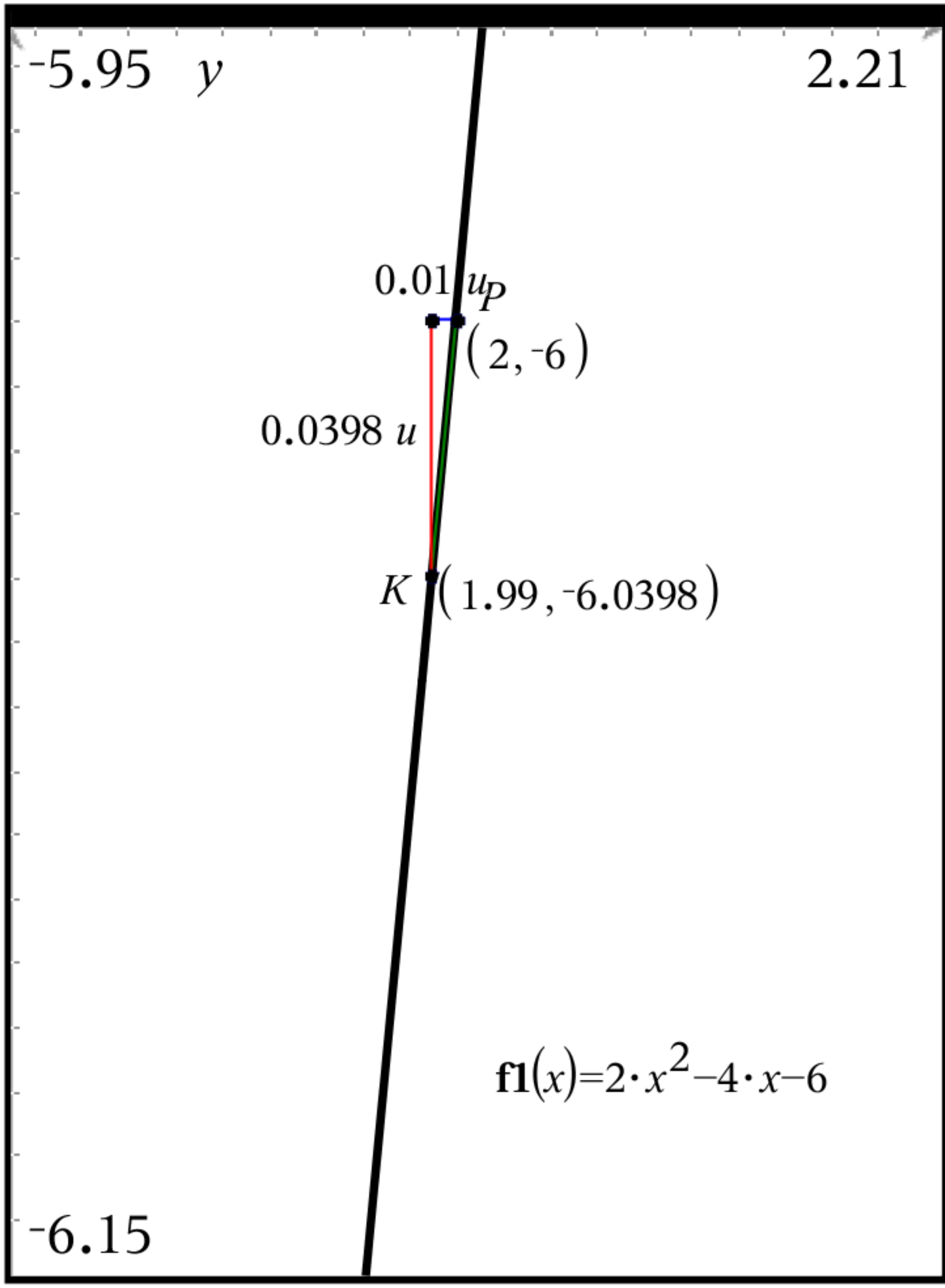
$f(1.9)=-6.38 \rightarrow L=(1.9,-6.38)$

$\frac{\Delta y}{\Delta x}=(-6--6.38)/(2-1.9)=3.8$

So we can say that the R.O.C.

between x = 2 and x = 1.9 is $\frac{\Delta y}{\Delta x}=3.8$





Find the rate of change from

P at x = 2 to K at x = 1.99

$f(x)=2\cdot x^2-4\cdot x-6$

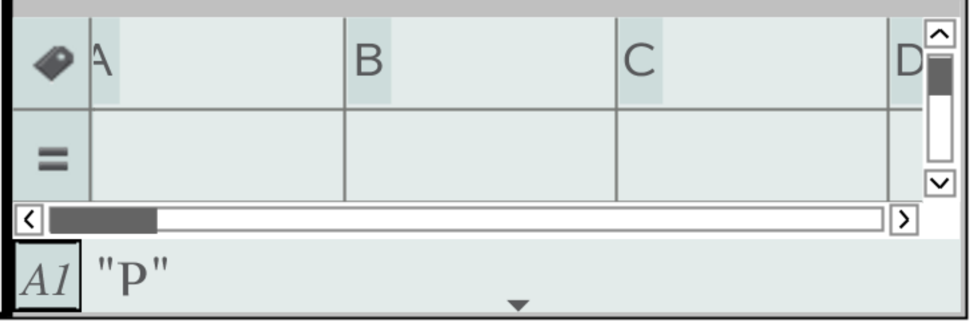
$f(2)={}^-6 \rightarrow P = (2, {}^-6)$

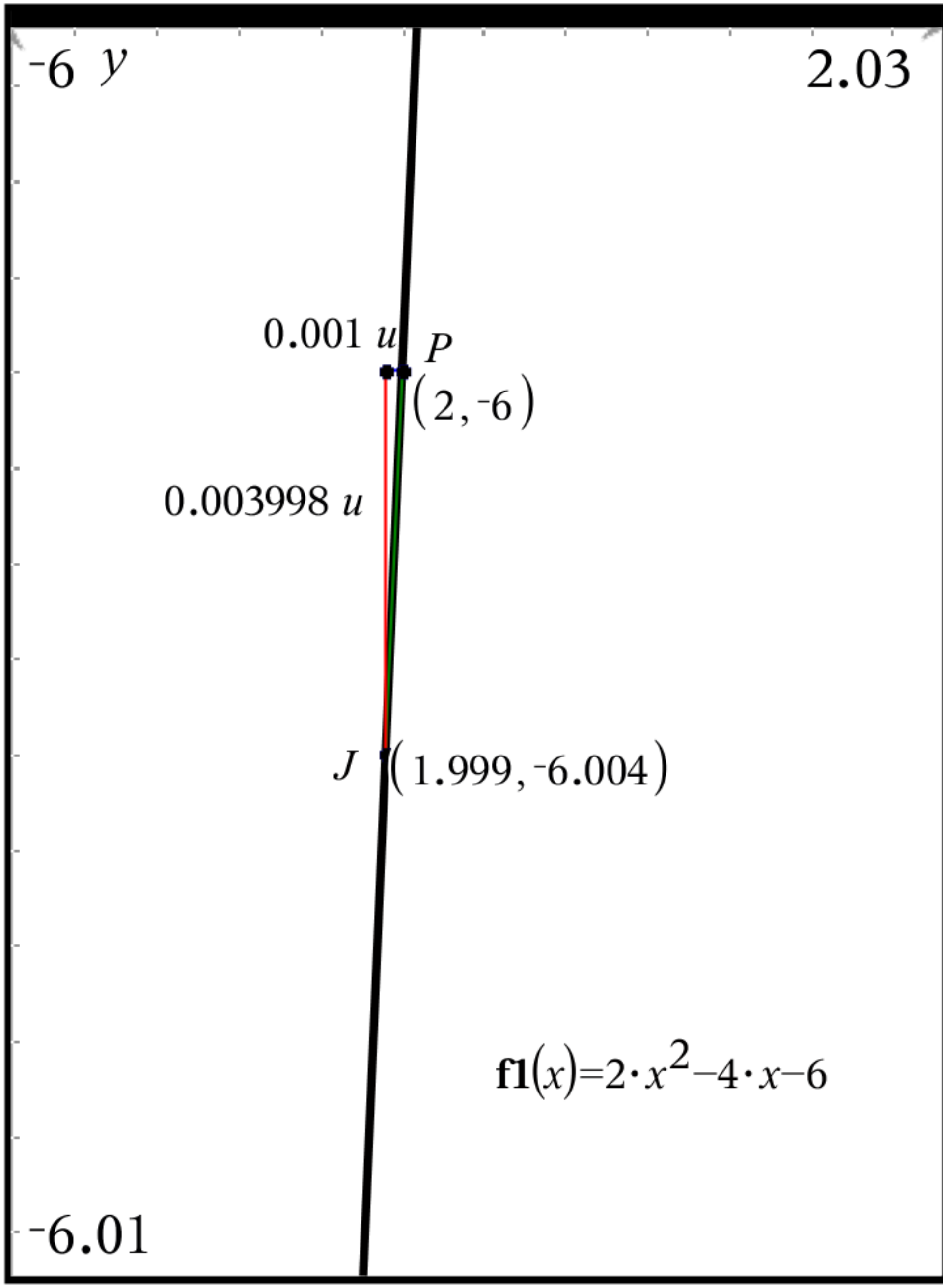
$f(1.99)={}^-6.0398 \rightarrow K = (1.99, {}^-6.0398)$

$\dfrac{\Delta y}{\Delta x} = ({}^-6 - {}^-6.0398)/(2-1.99) = 3.98$

So we can say that the R.O.C.

between x = 2 and x = 1.99 is $\dfrac{\Delta y}{\Delta x} = 3.98$





Find the rate of change from

P at x = 2 to J at x = 1.999

$f(x)=2\cdot x^{2}-4\cdot x-6$

$f(2)={}^{-}6 \rightarrow P = (2, {}^{-}6)$

$f(1.999)={}^{-}6.003998 \rightarrow J = (1.999, {}^{-}6.003998)$

$\frac{\Delta y}{\Delta x}=({}^{-}6-{}^{-}6.003998)/(2-1.999)=3.998$

So we can say that the R.O.C.

between x = 2 and x = 1.999 is $\frac{\Delta y}{\Delta x}=3.998$

	A	B	C	D
=				

A1 "P"

Since we calculated the slopes of the secant lines from 2 to a point to the left of 2,

PN, PL, PK, PJ all approximate the slope of the tangent line at x = 2

We approximated the limit at x = 2 from the left side of the domain

$$\lim_{x \to 2^-} \left(\frac{\mathbf{f1}(x) - \mathbf{f1}(2)}{x-2} \right) = 4$$

ROC from x = 2 to x = 1 =Slope of the secant line PN at x = 2 and x = 1 $= \frac{\mathbf{f1}(1) - \mathbf{f1}(2)}{1-2} = 2$

ROC from x = 2 to x = 1.9 =Slope of the secant line PL at x = 2 and x = 1.9 $= \frac{\mathbf{f1}(1.9) - \mathbf{f1}(2)}{1.9-2} = 3.8$

ROC from x = 2 to x = 1.99 =Slope of the secant line PK at x = 2 and x = 1.99

$$= \frac{\mathbf{f1}(1.99) - \mathbf{f1}(2)}{1.99-2} = 3.98$$

ROC from x = 2 to x = 1.999 =Slope of the secant line PT at x = 2 and x = 1.999

$$= \frac{\mathbf{f1}(1.999) - \mathbf{f1}(2)}{1.999-2} = 3.998$$

In Problems 1−4 we calculated the slopes of the secant lines and determined the ROC between two given x values.

In both of these cases, the slope of the secant lines at the given points and the ROC between the particular values of x

and they gave us:

ROC of 2 to 3.8 to 3.98, to 3.998 from the left as x approached 2 from x = 1, 1.9, 1.99, and 1.999

and ROC of 6, 4.2, 4.02, 4.002 from the right asx approached 2 from x = 3, 2.1, 2.01, and 2.001

This means that it appears as x approaches 2 from either side the difference quotient approaches 4 and it appears that the ROC approaches IROC of 4 at x = 2

Given $\mathbf{f1}(x):=2\cdot x^{2}-4\cdot x-6$

If we use the substitution method of determining a limit of the difference quotient at x = 2 for f(x)

We get $\frac{\mathbf{f1}(2)-\mathbf{f1}(2)}{2-2}=\frac{0}{0}=\text{undef}$

This leads us to the indeterminate form, or like I like to the DO SOMETHING point.

Now, we have an approximation of 4 through 1−6 for this value, but the algebra is possible to actualy find the limit through algebraic methods

Note $\mathbf{f1}(2)=^{-}6$ and $\mathbf{f1}(x)=2\cdot x^{2}-4\cdot x-6$

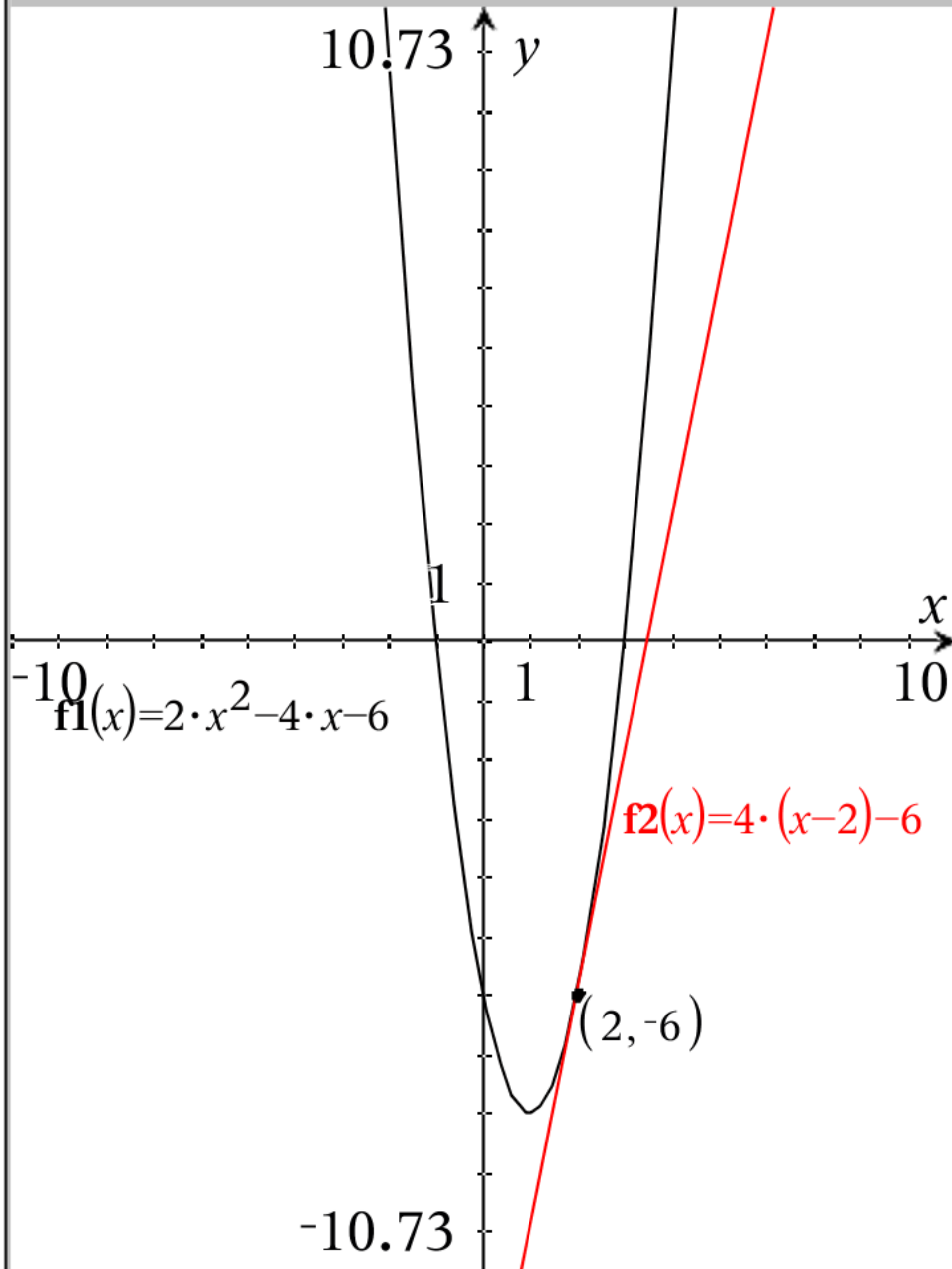
so $\mathbf{f1}(x)-\mathbf{f1}(2)=2\cdot x^{2}-4\cdot x-6-^{-}6=2\cdot x^{2}-4\cdot x$

and $\frac{\mathbf{f1}(x)-\mathbf{f1}(2)}{x-2}=\frac{2x^{2}-4x}{x-2}=\frac{2\cdot x\cdot (x-2)}{x-2}=2x$

so $\lim_{x\to 2}\left(\frac{\mathbf{f1}(x)-\mathbf{f1}(2)}{x-2}\right)=\lim_{x\to 2}(2x)=2\cdot 2=4$

DO NOT WORRY ABOUT PROBLEM #9

(TOO ALGEBRAIC FOR FIRST QUIZ)



Since we found

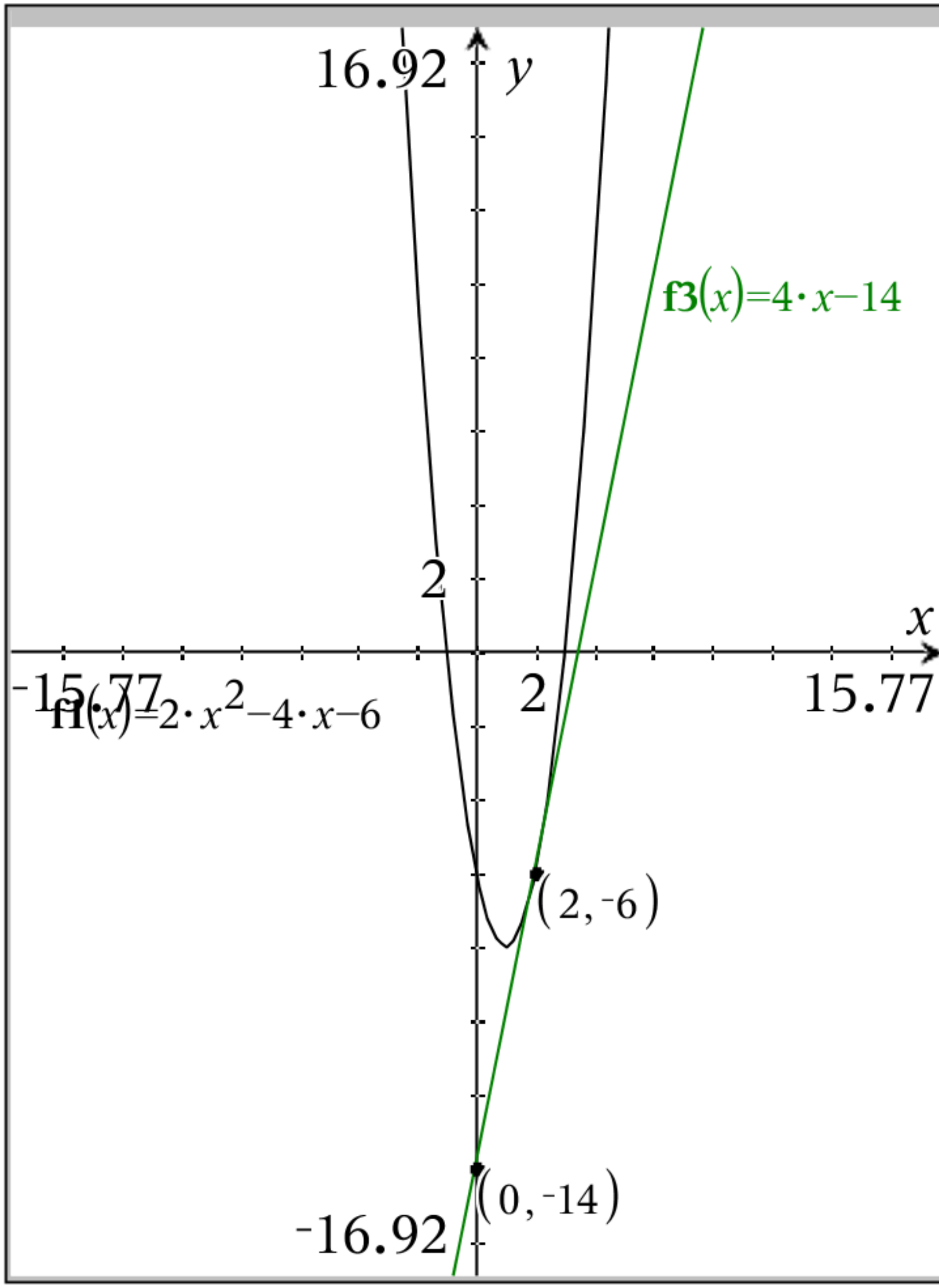
$$\lim_{x \to 2} \left(\frac{\mathbf{f1}(x) - \mathbf{f1}(2)}{x-2} \right) = 4$$

in problem 8

and we know the slope of the tangent is equal to this limit at the x value of x = 2 and we know that the line and the parabola must pass through (2,6)

The equation of the tangent line at x = 2 is

$y = 4(x-2)-6$



Since we found

$$\lim_{x \to 2} \left(\frac{\mathbf{f1}(x) - \mathbf{f1}(2)}{x-2} \right) = 4$$

in problem 8

and we know the slope of the tangent is equal to this limit at the x value of x = 2 and we know that the line and the parabola must pass through (2,6)

The equation of the tangent line at x = 2

can be found by solving

y=mx+b with (x,y) = (2,−6) and m = 4

−6=4(2)+b

−6=8+b

b = −14 so y = 4x−14

$g(x)=\frac{2x^2+4x}{x+2}$ Domain $x \neq -2$

Note: This is a rational function that can be reduced

$g(x)=\frac{2x^2+4x}{x+2}=\frac{2x(x+2)}{x+2}=2x$ except at $x=-2$

To find hole let $x=-2$ in "new" version of $g(x)$

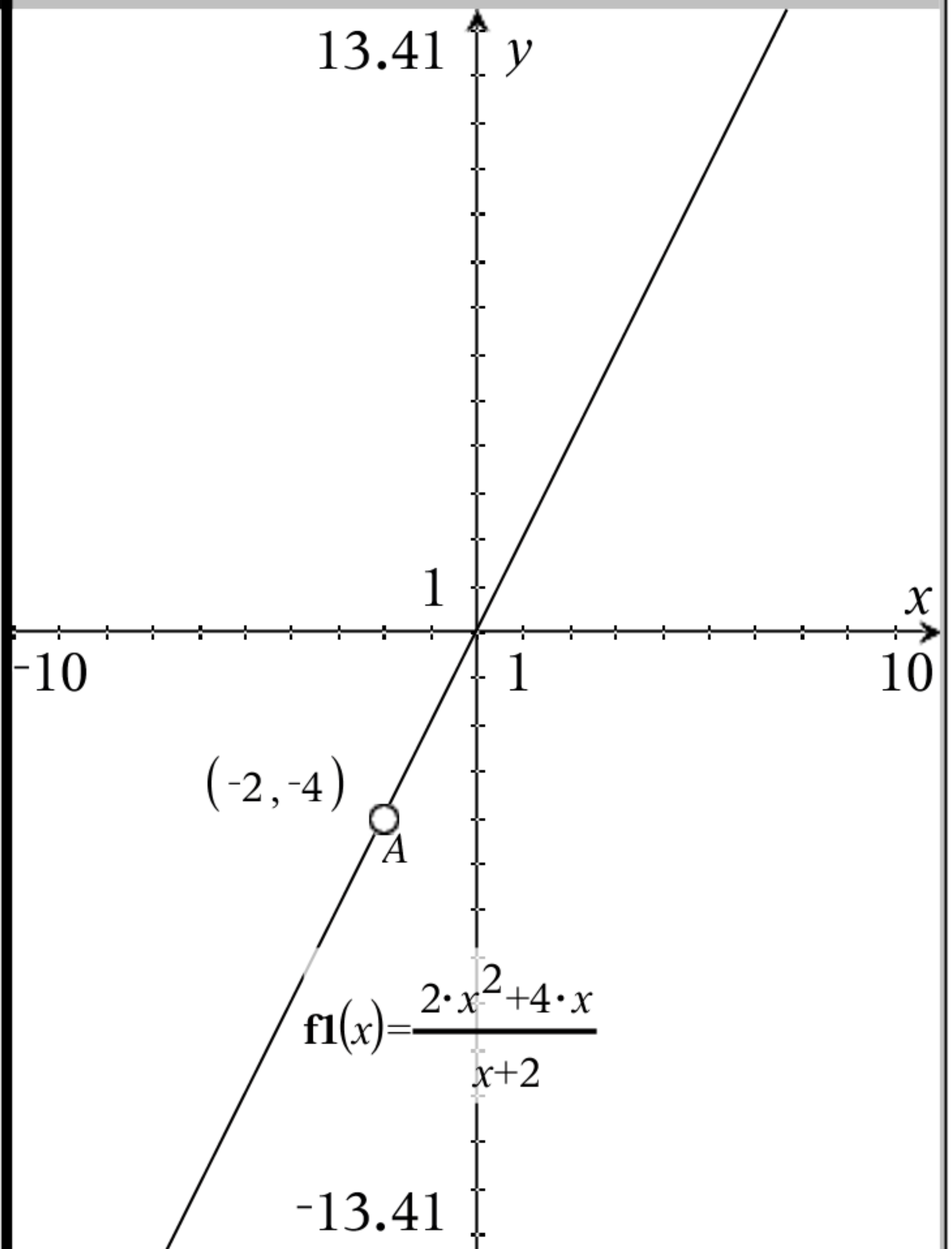
$g(-2)=2(-2)=-4$

So the hole in $g(x)$ exists at $x=-2$

and it is $(-2,-4)$

Since a limit exists from both the left and right sides and it agrees as it occurs at $x=-2$ and the limit is -4

$$\lim_{x\to -2}(g(x))=-4$$

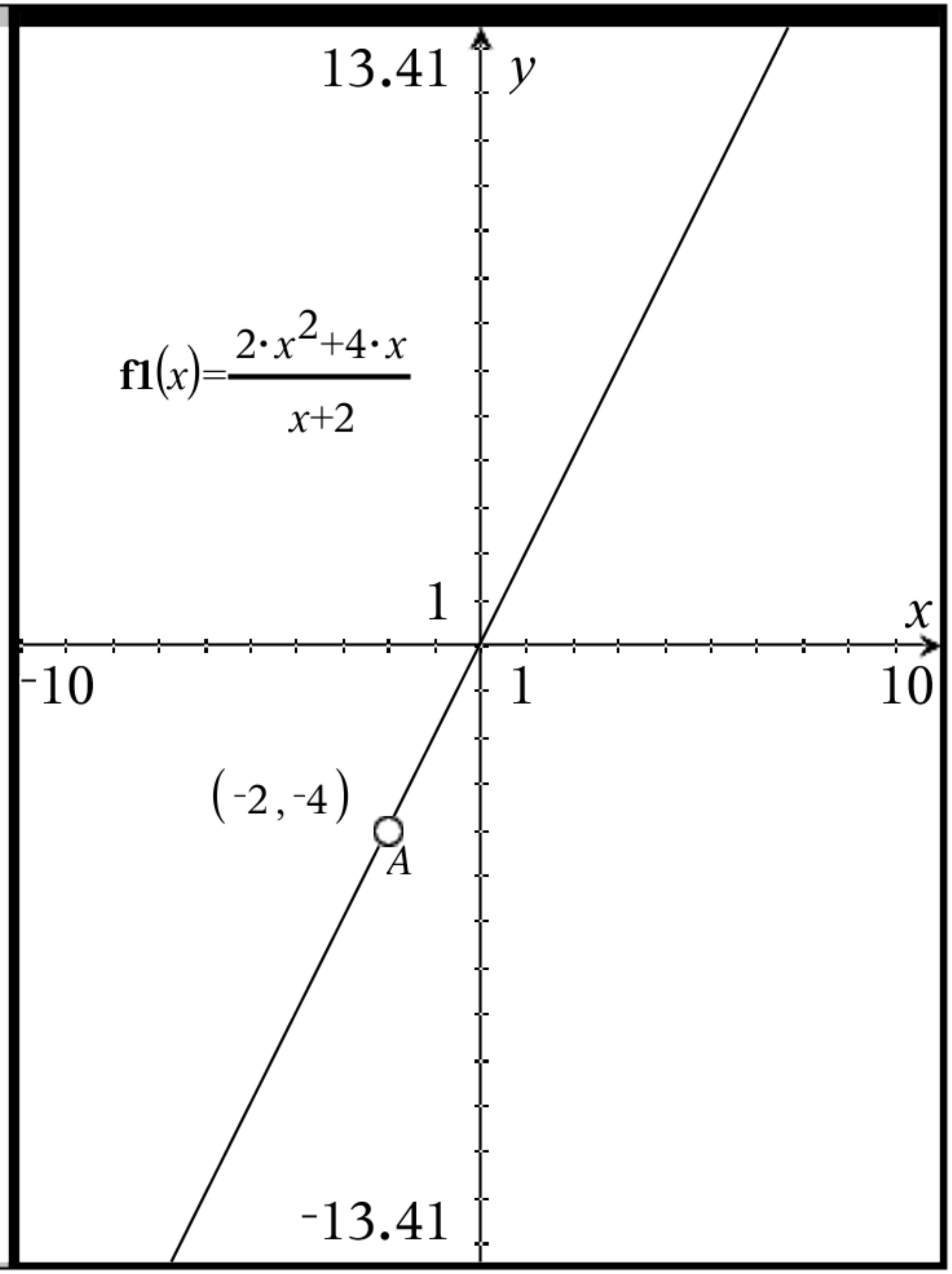


$g(x)=\dfrac{2x^2+4x}{x+2}$

If we attempt to find the limit at x = −2 for g(x) we will lead ourselves to

$g(-2)=\dfrac{2\cdot(^-2)^2+4\cdot{}^-2}{^-2+2}=\dfrac{0}{0}$

This is called indeterminate form, so we must DO SOMETHING (in this case we must simplify through factoring and reduction of the rational function g(x))



$h(x)=\dfrac{2x^2+4x}{x+3}$ Domain $x\neq -3$

Note: This is a rational function that can NOT be reduced, its numerators fatcors are NOT equal to its denominator's factors

$$h(x)=\frac{2x^2+4x}{x+3}=\frac{2x(x+2)}{x+3}$$

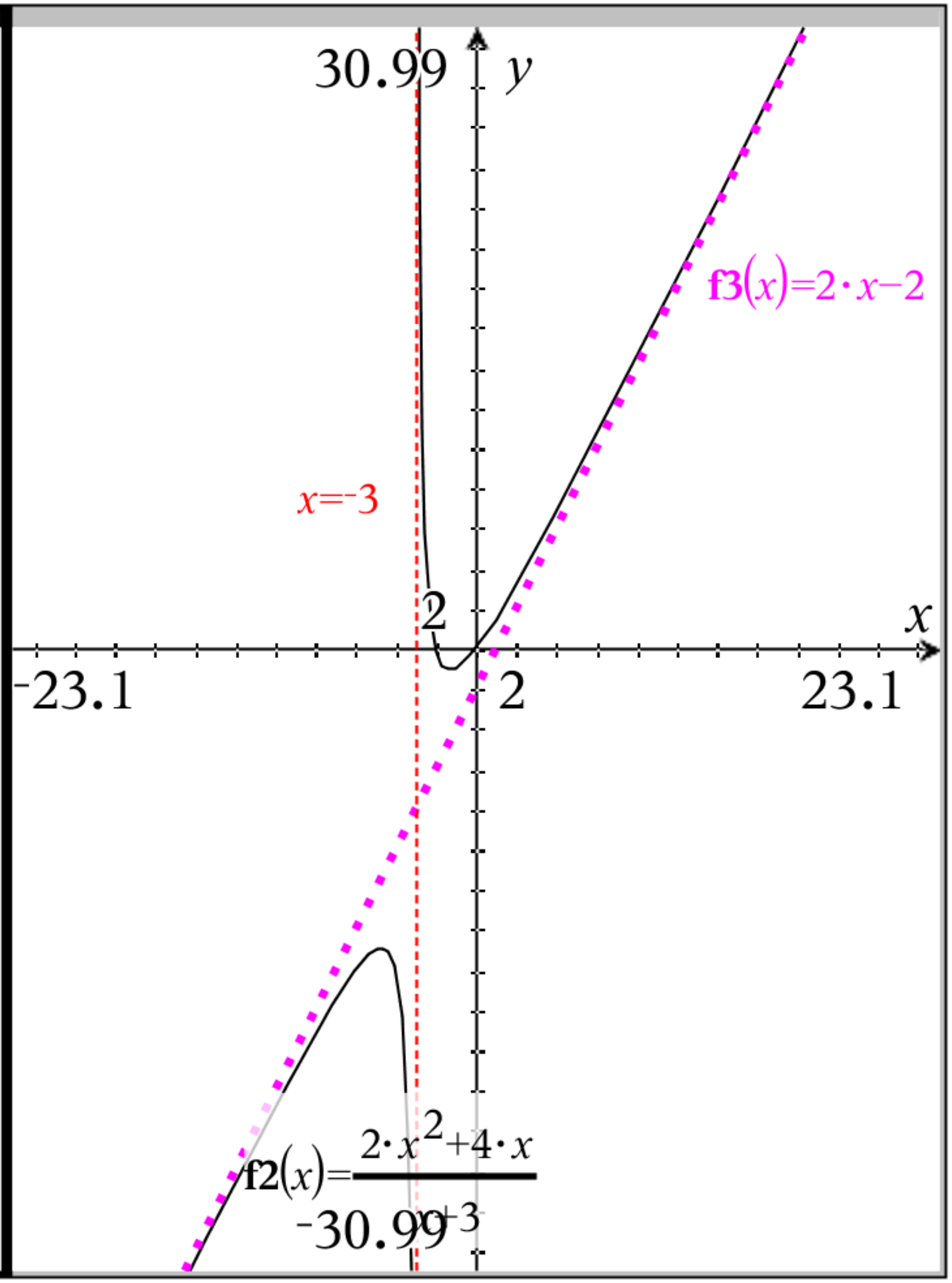
To find vertical asymptote

let x = −3 in "new" version of h(x)

$2(-3)(-3+4)=-6\cdot(1)=-6$

$-3+3 = 0$

$h(-3)=\dfrac{-6}{0}$ which impossible and NOT indeterminate, so a vertical asymptote exists at x = −3 not a hole



$g(x)=\dfrac{2x^2+4x}{x+2}$

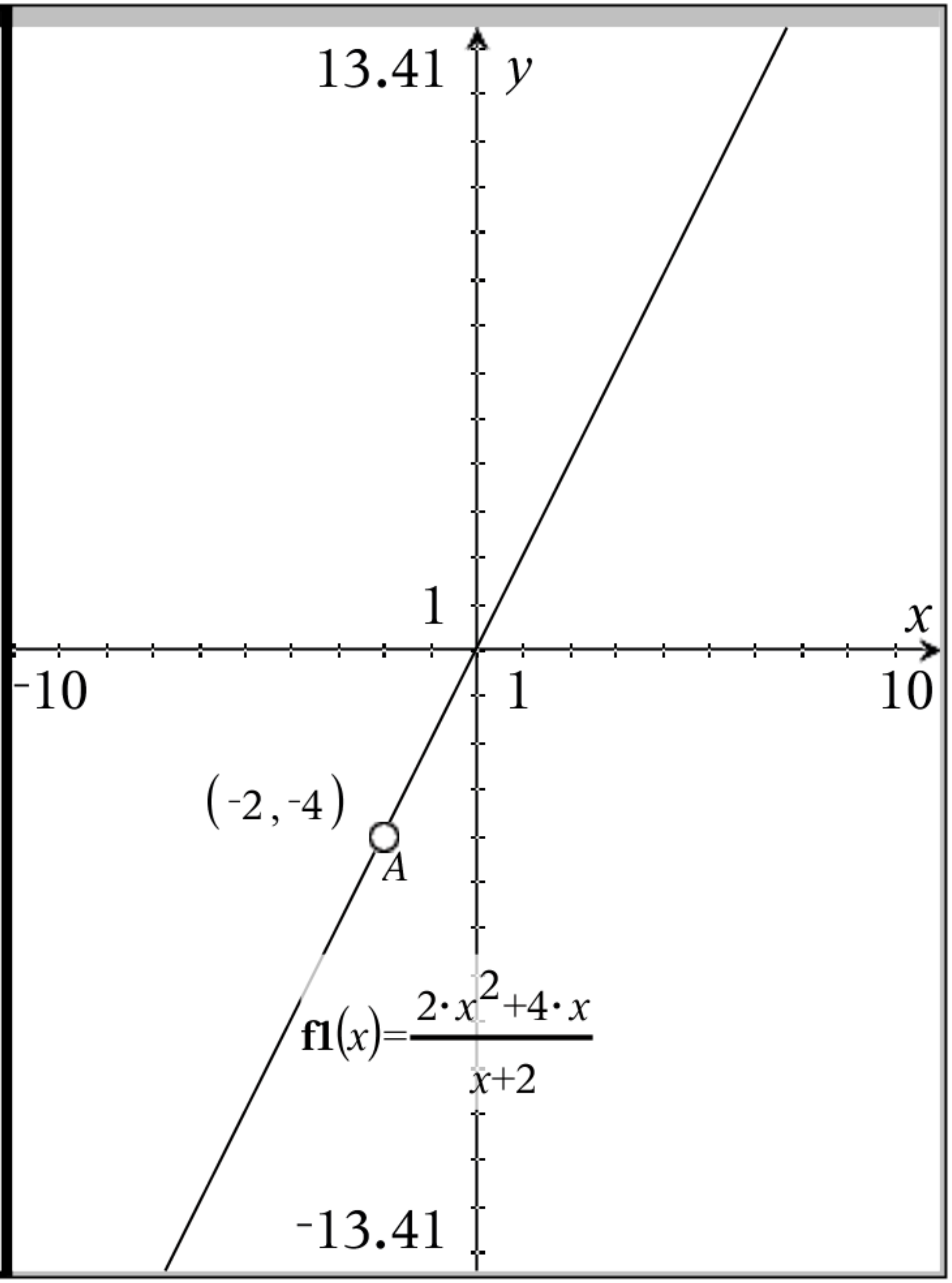
$\lim\limits_{x\to -2^-}(g(x))=-4$

&

$\lim\limits_{x\to -2^+}(g(x))=-4$

SO

$\lim\limits_{x\to -2}(g(x))=-4$



$h(x)=\dfrac{2x^2+4x}{x+3}$ Domain $x \neq -3$

Note: This is a rational function that can be reduced

$h(x)=\dfrac{2x^2+4x}{x+3}=\dfrac{2x(x+2)}{x+3}$ except at $x = -2$

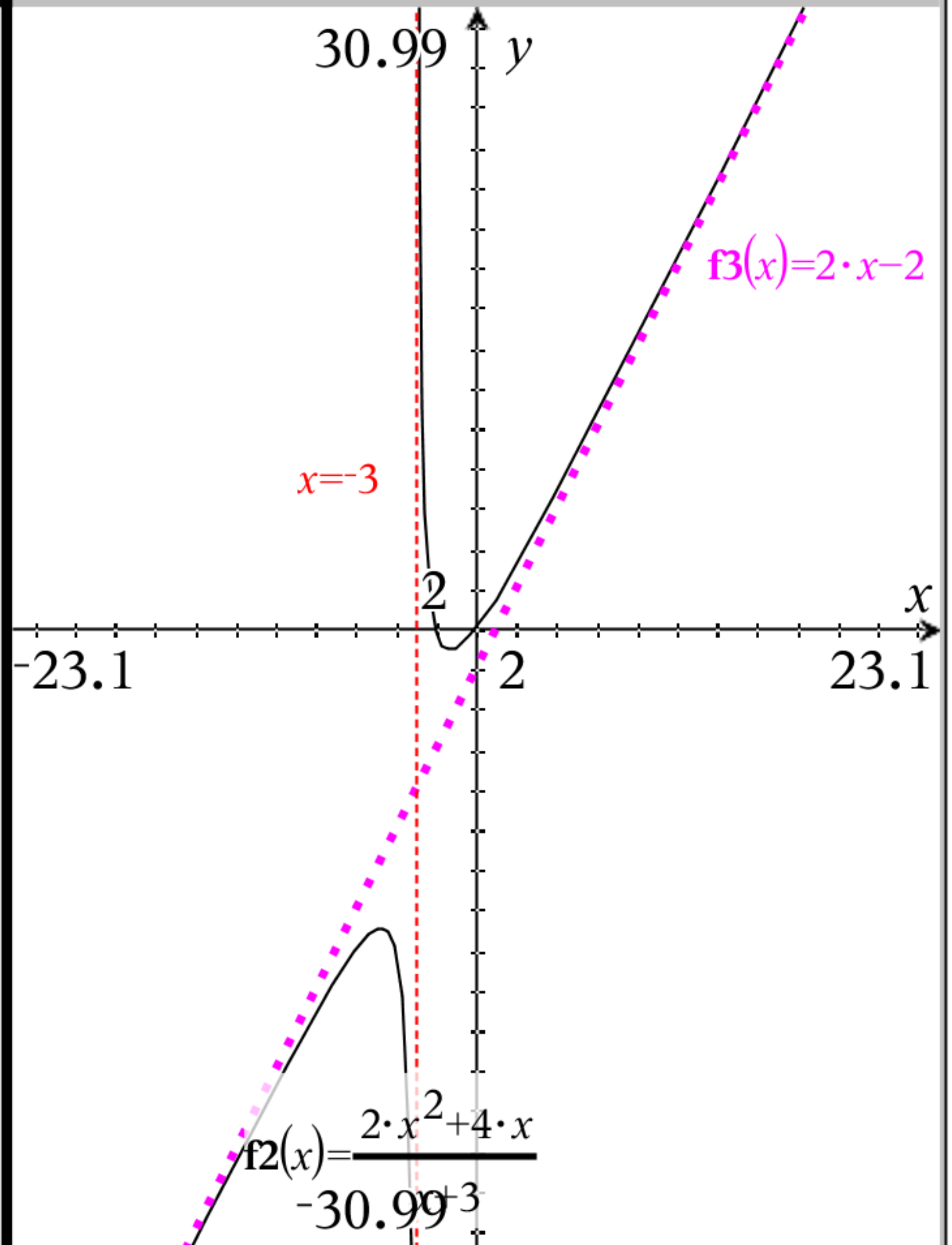
To find vertical asymptote

let $x = -3$ in "new" version of h(x)

$2(-3)(-3+4) = -6 \cdot (1) = -6$

$-3+3 = 0$

$h(-3) = \dfrac{-6}{0}$ which impossible and NOT indeterminate, so a vertical asynptote exists at $x = -3$



$h(x)=\frac{2x^2+4x}{x+3}$ Domain $x \neq -3$

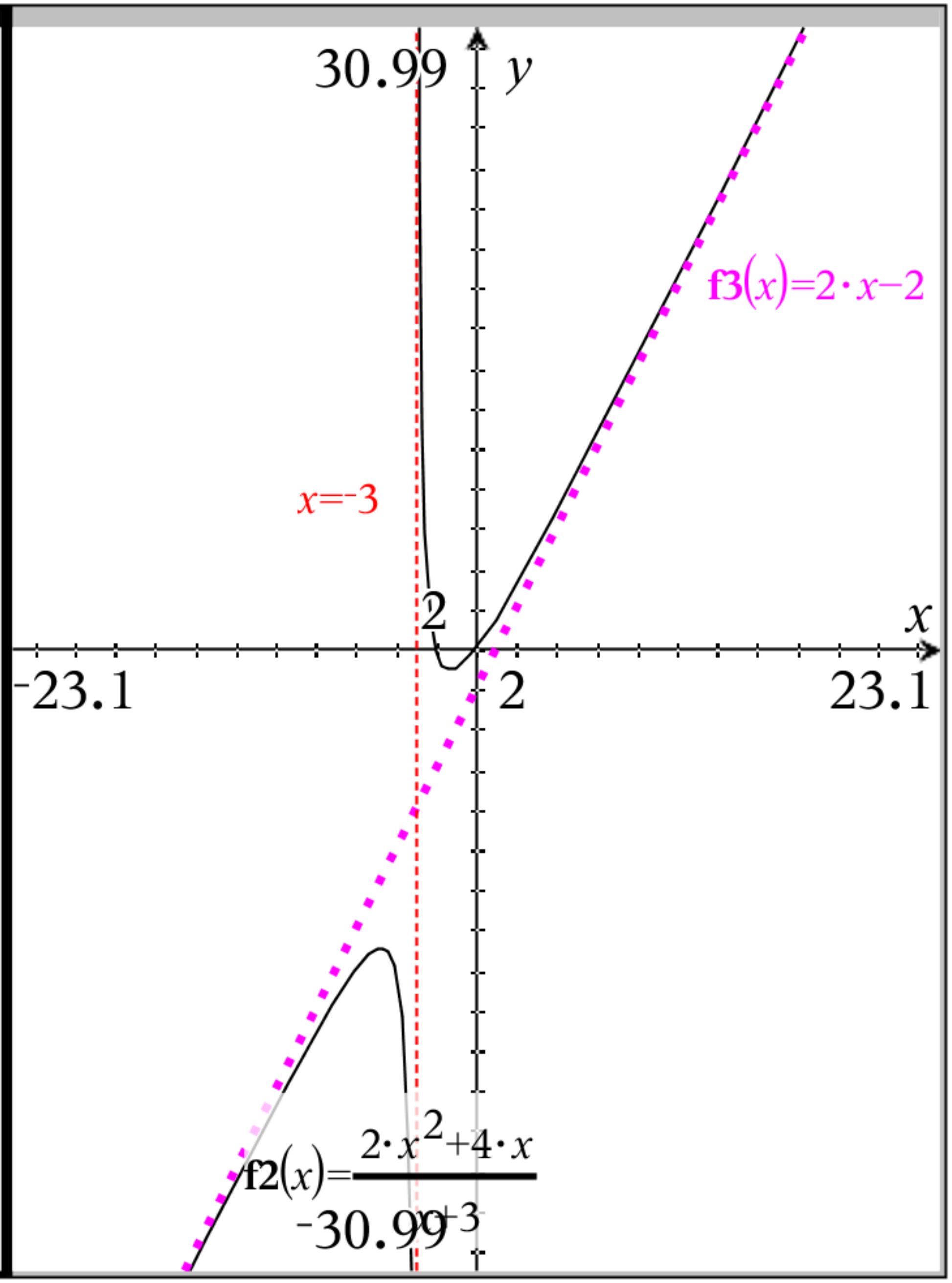
$\lim_{x\to -3^-} (h(x)) = -\infty$

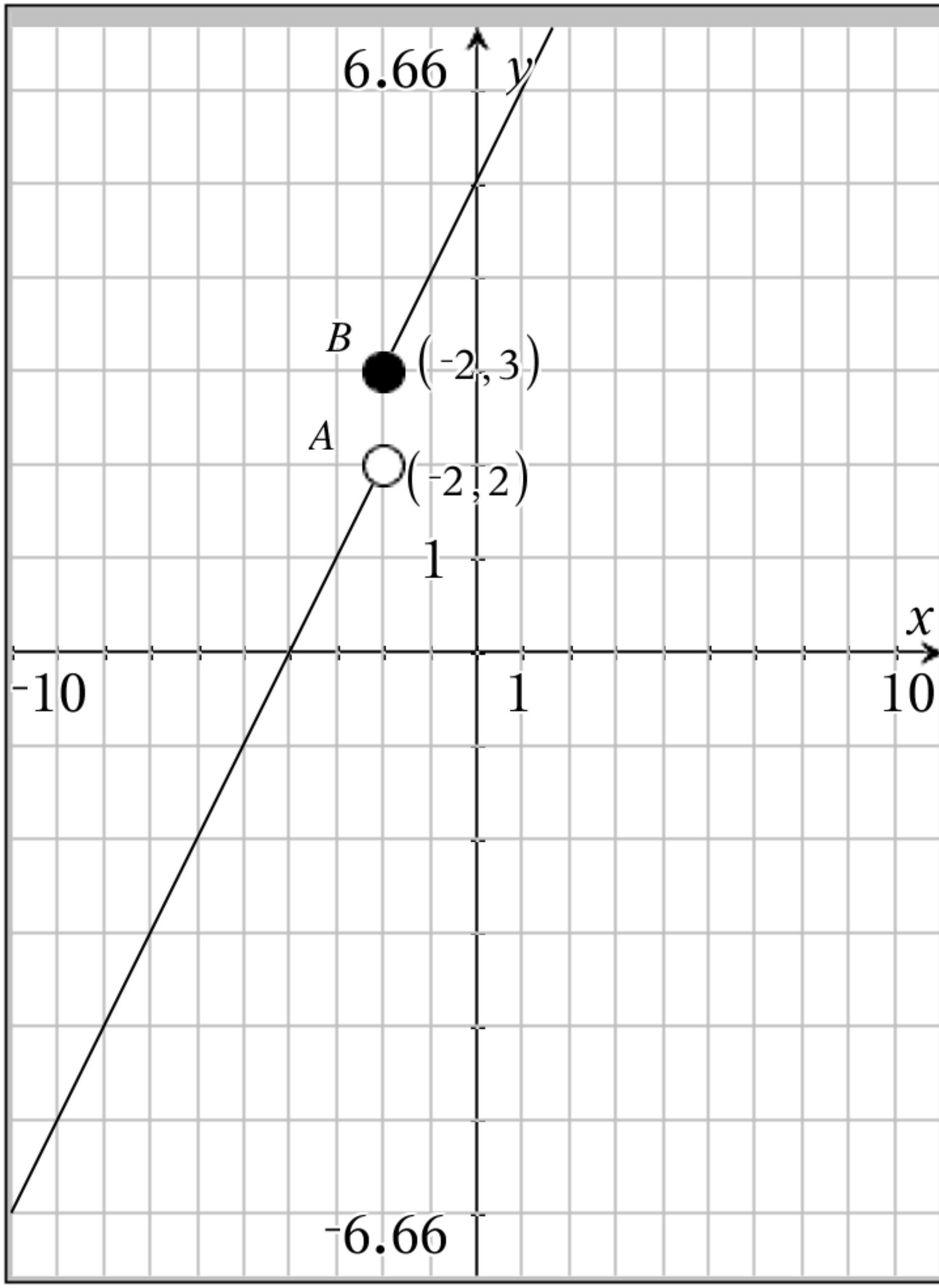
&

$\lim_{x\to -3^+} (h(x)) = +\infty$

SO

$\lim_{x\to -3} (h(x))$ =DNE because the left hand and right hand limit do not agree with each other





15F Solution

$$f(x) = \begin{cases} x+4, & x<-2 \\ x+3, & x\geq -2 \end{cases}$$

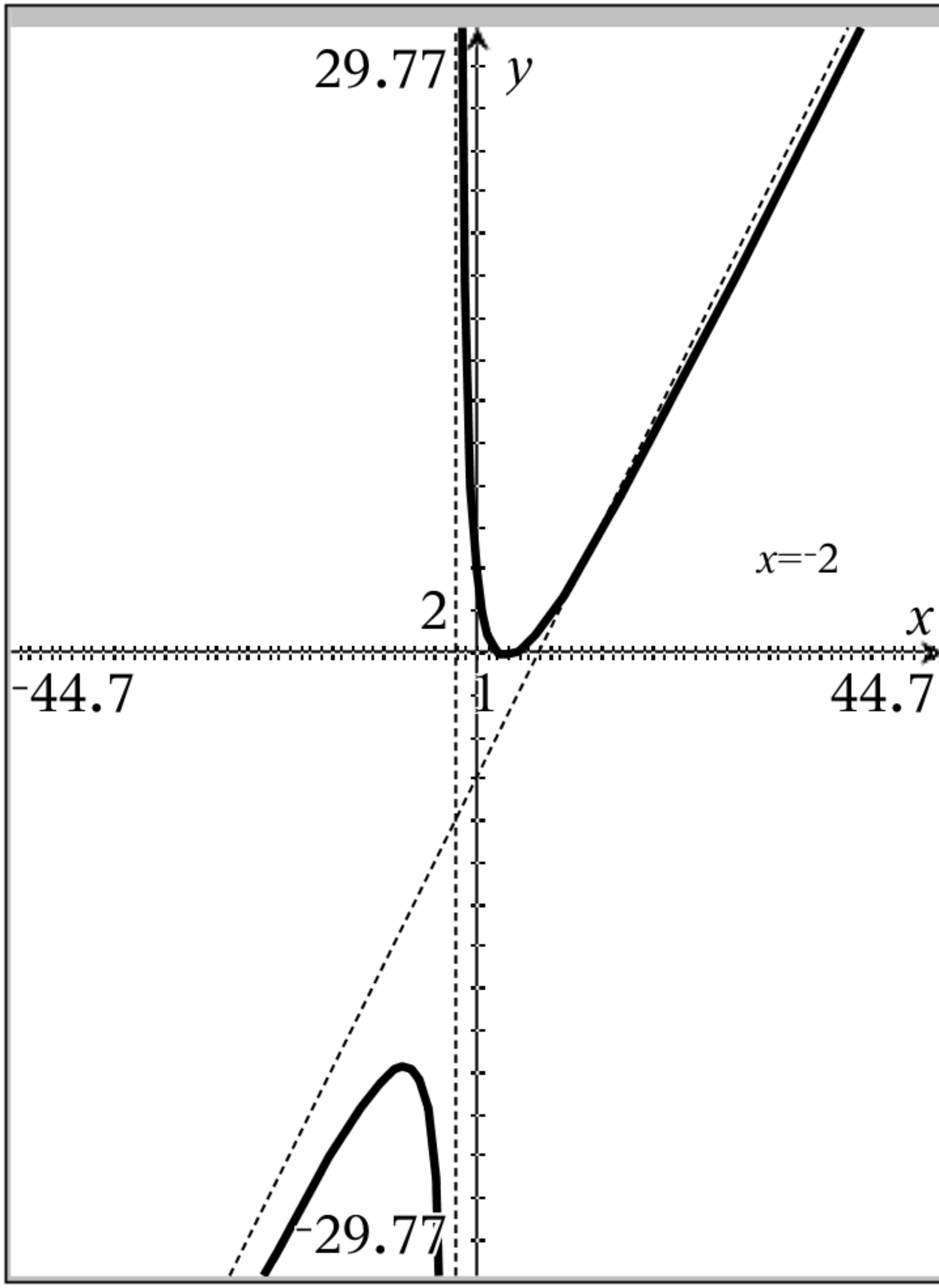
12a) $\lim_{x\to -2^+} (f(x)) = 3$

12b) $\lim_{x\to -2^-} (f(x)) = 2$

12c) $\lim_{x\to -2} (f(x)) =$ DNE

13F f(x) is undefinedat x = −2, but there exists a gap in y values at x = −2 and therefore despite the one sided limits existing, they do not agree so a limit at x = −2 does not existso g(x) has a limit for all values of x ∈(−5, 5) EXCEPT x = −2

14F f(x) is one of the two functions that has one sided limits that do not agree at x = −2



15G Solution

g(x) $= \frac{x^2-6\cdot x+8}{x+2} = \frac{(x-2)(x-4)}{x+2}$

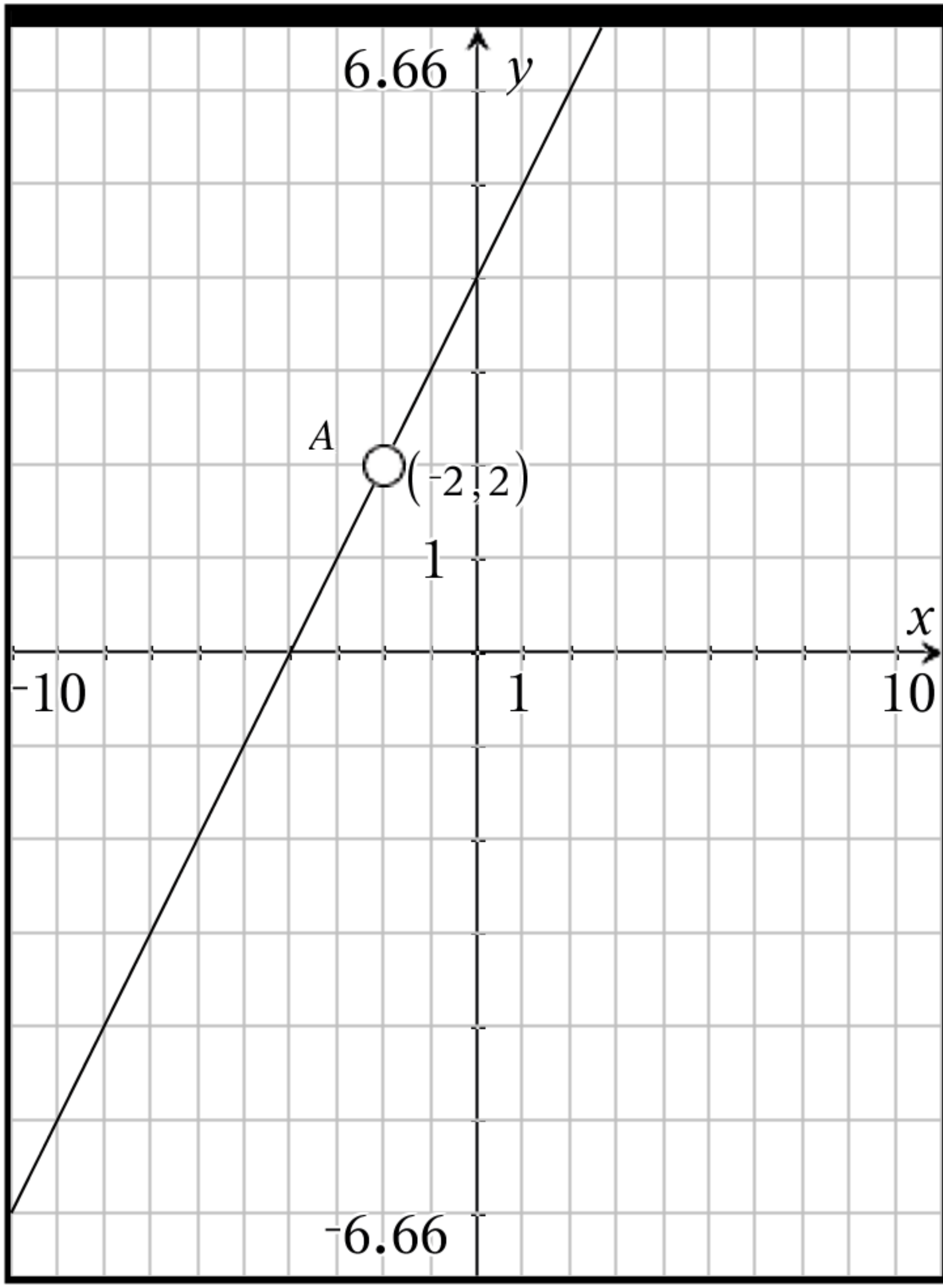
12d) $\lim\limits_{x\to -2^+} (g(x)) = +\infty$

12d) $\lim\limits_{x\to -2^-} (g(x)) = -\infty$

12e) $\lim\limits_{x\to -2} (g(x)) = \text{DNE}$

13G g(x) is undefinedat x = −2, but there exists a vertical asymptote at x = −2 and therefore despite the one sided limits existing, they do not agree so a limit at x = −2 does not existso g(x) has a limit for all values of x ∈(−5, 5) EXCEPT x = −2

14G g(x) is one of the two functions that has one sided limits that do not agree at x = −2



15H Solution

$$h(x) = \frac{x^2+6\cdot x+8}{x+2} = \frac{(x+2)(x+4)}{x+2}$$

f(x) = x+4 except at x = −2

f(−2)≠−2+4 ≠2

So the hole occurs at x = −2or the point (−2,2)

12g) $\lim_{x\to -2^+} (h(x)) = 2$

12h) $\lim_{x\to -2^-} (h(x)) = 2$

12i) $\lim_{x\to -2} (h(x)) = 2$

13H) h(x) has a limit for all x ∈(−5,5)

14H) h(x) has one sided limits that agree including at x = −2

$g(x)=\frac{2x^2+4x}{x+2}$ Domain $x \neq -2$

Note: This is a rational function that can be reduced

$g(x)=\frac{2x^2+4x}{x+2}=\frac{2x(x+2)}{x+2}=2x$ except at $x=-2$

To find hole let $x=-2$ in "new" version of g(x)

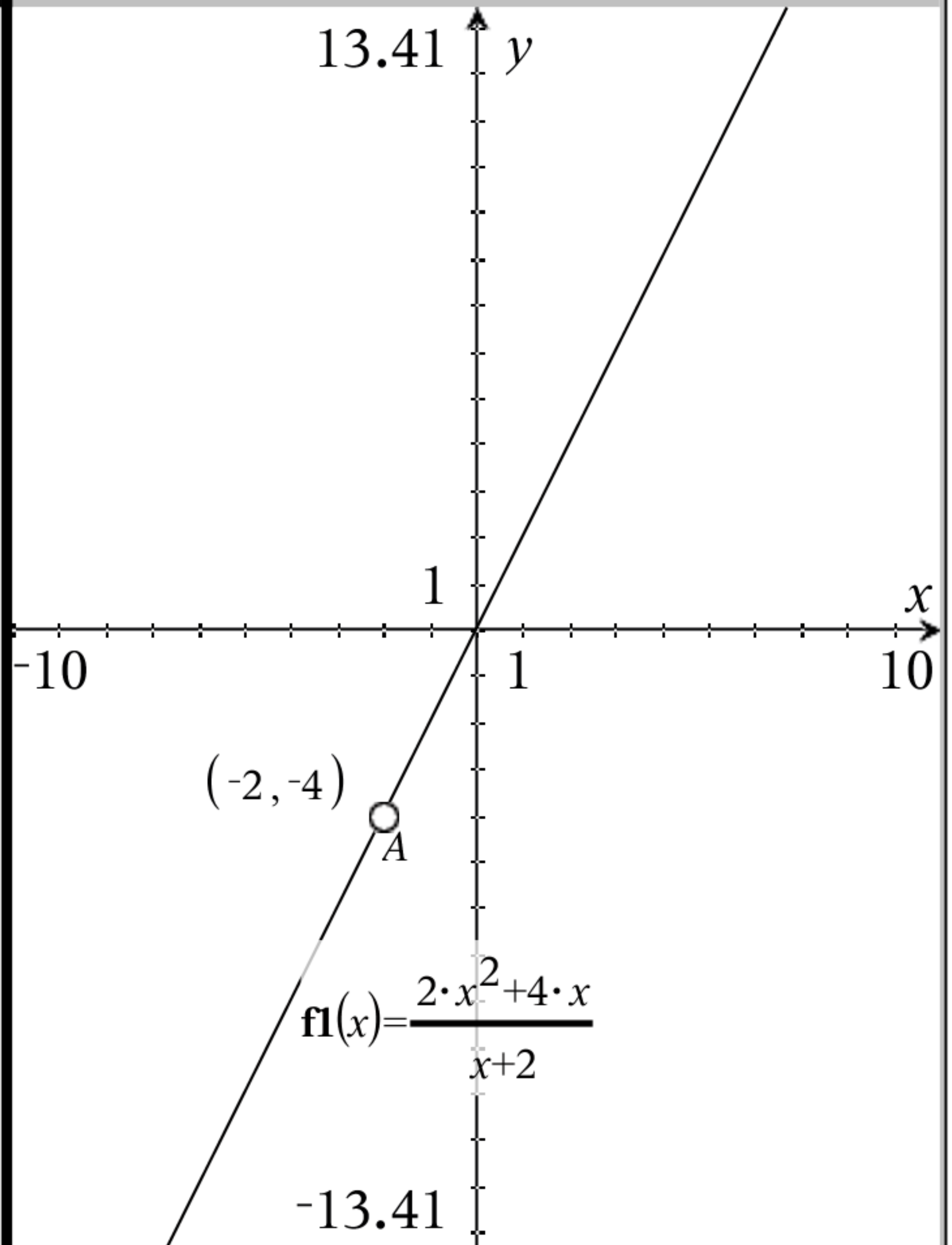
$g(-2)=2(-2)=-4$

So the hole in g(x) exists at $x=-2$

and it is $(-2,-4)$

Since a limit exists from both the left and right sides and it agrees as it occurs at $x=-2$ and the limit is -4

$$\lim_{x \to -2} (g(x)) = -4$$



$g(x)=\dfrac{2x^2+4x}{x+2}$

NOTE: $g(-2)=\dfrac{0}{0}=\text{undef}$

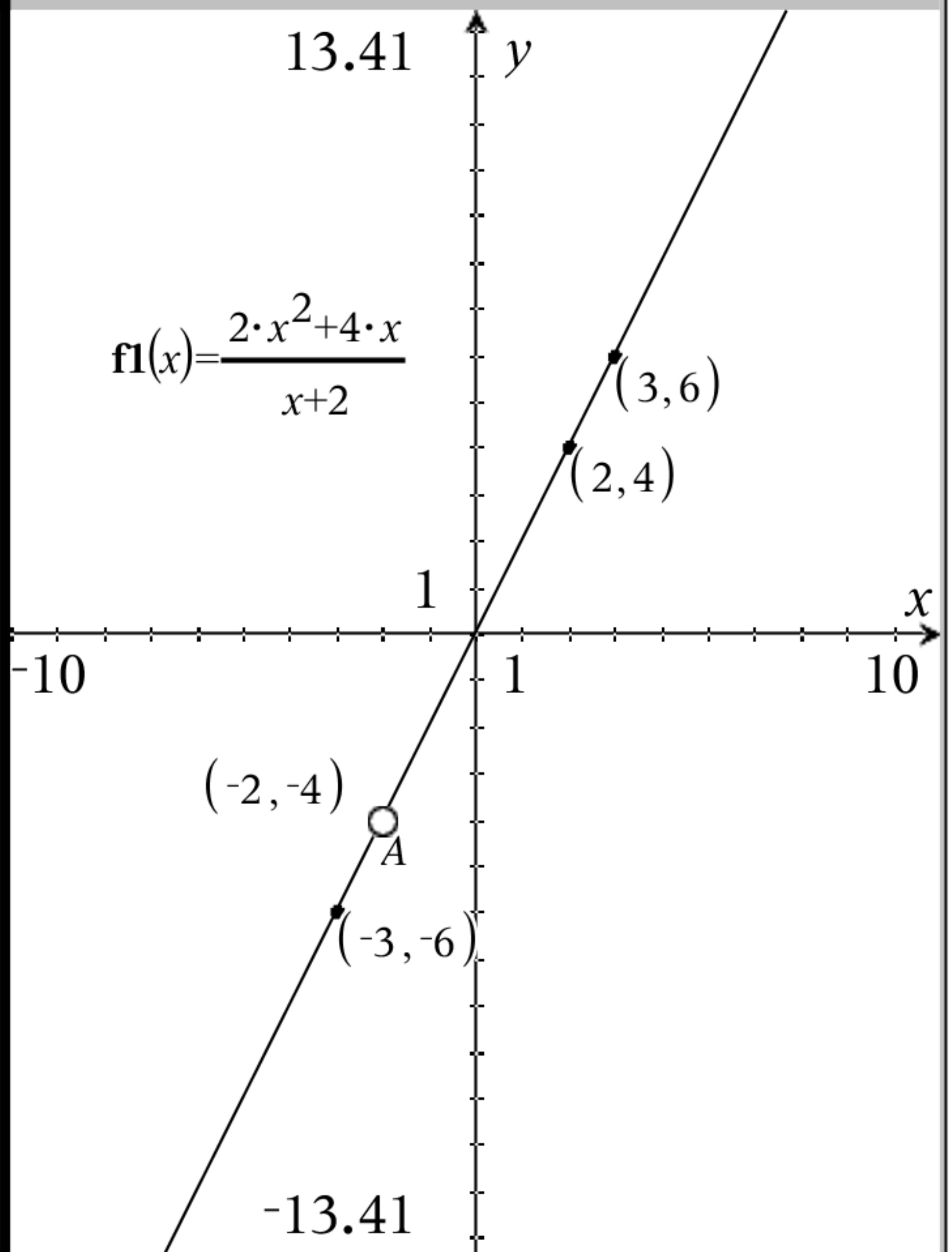
16a) $\displaystyle\lim_{x\to-2}(g(x))=-4$

(only value of xthat a limit that could not be found at is x = −2 through direct substitution or through g(−2))

16b) $\displaystyle\lim_{x\to 2}(g(x))=4$

16c) $\displaystyle\lim_{x\to 3}(g(x))=6$

16d) $\displaystyle\lim_{x\to-3}(g(x))=-6$



$h(x)=\dfrac{2x^2+4x}{x+3}$ Domain $x \neq -3$

Note: This is a rational function that can be reduced

$h(x)=\dfrac{2x^2+4x}{x+3}=\dfrac{2x(x+2)}{x+3}$ except at $x = -2$

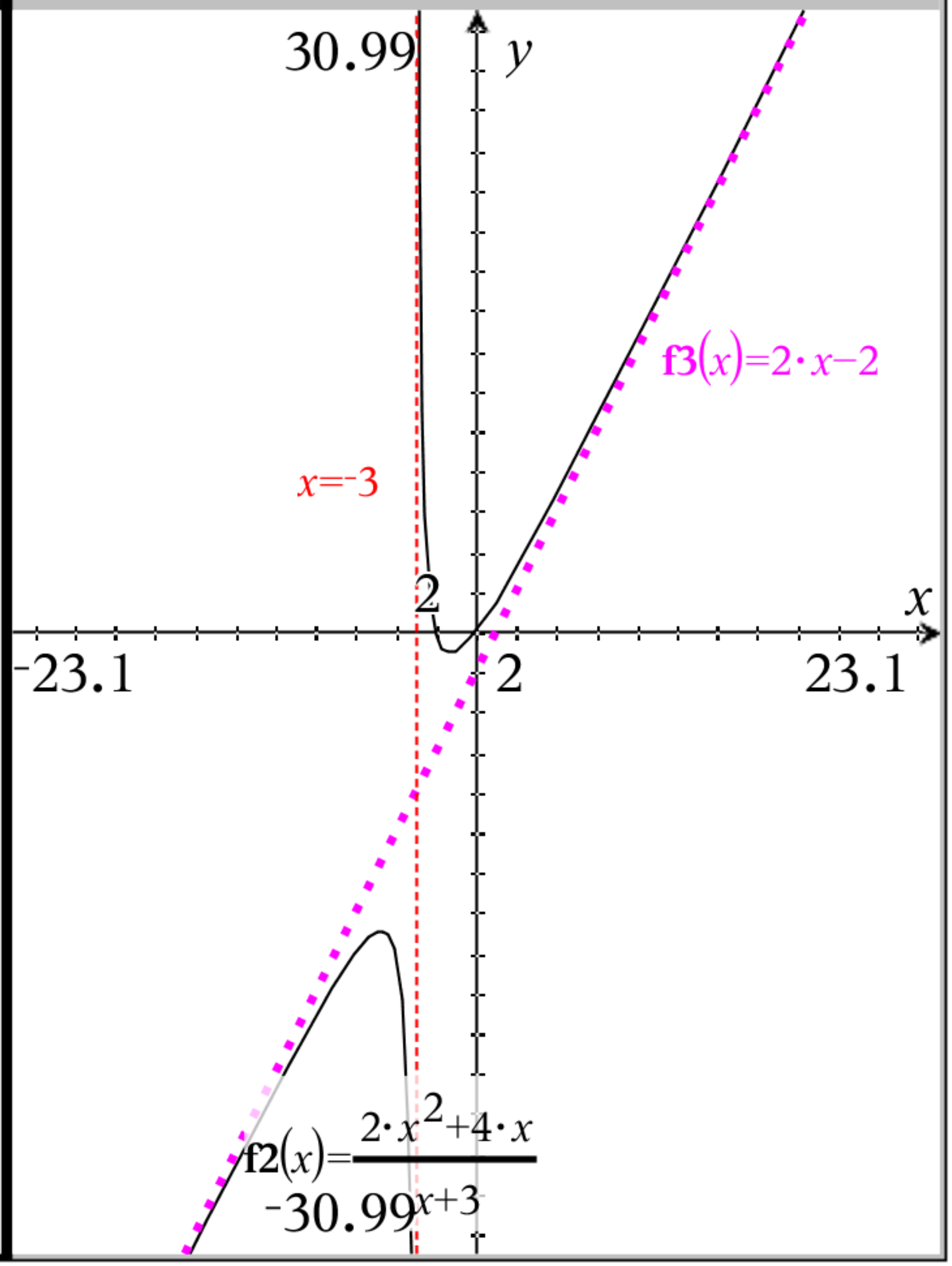
To find vertical asymptote

let $x = -3$ in "new" version of h(x)

$2(-3)(-3+4)=-6\cdot(1)=-6$

$-3+3 = 0$

$h(-3)=\dfrac{-6}{0}$ which impossible and NOT indeterminate, so a vertical asynptote exists at $x = -3$



$h(x)=\dfrac{2x^2+4x}{x+3}$ Domain $x \neq -3$

NOTE: $h(-3) = \dfrac{6}{0} = \text{undef}$

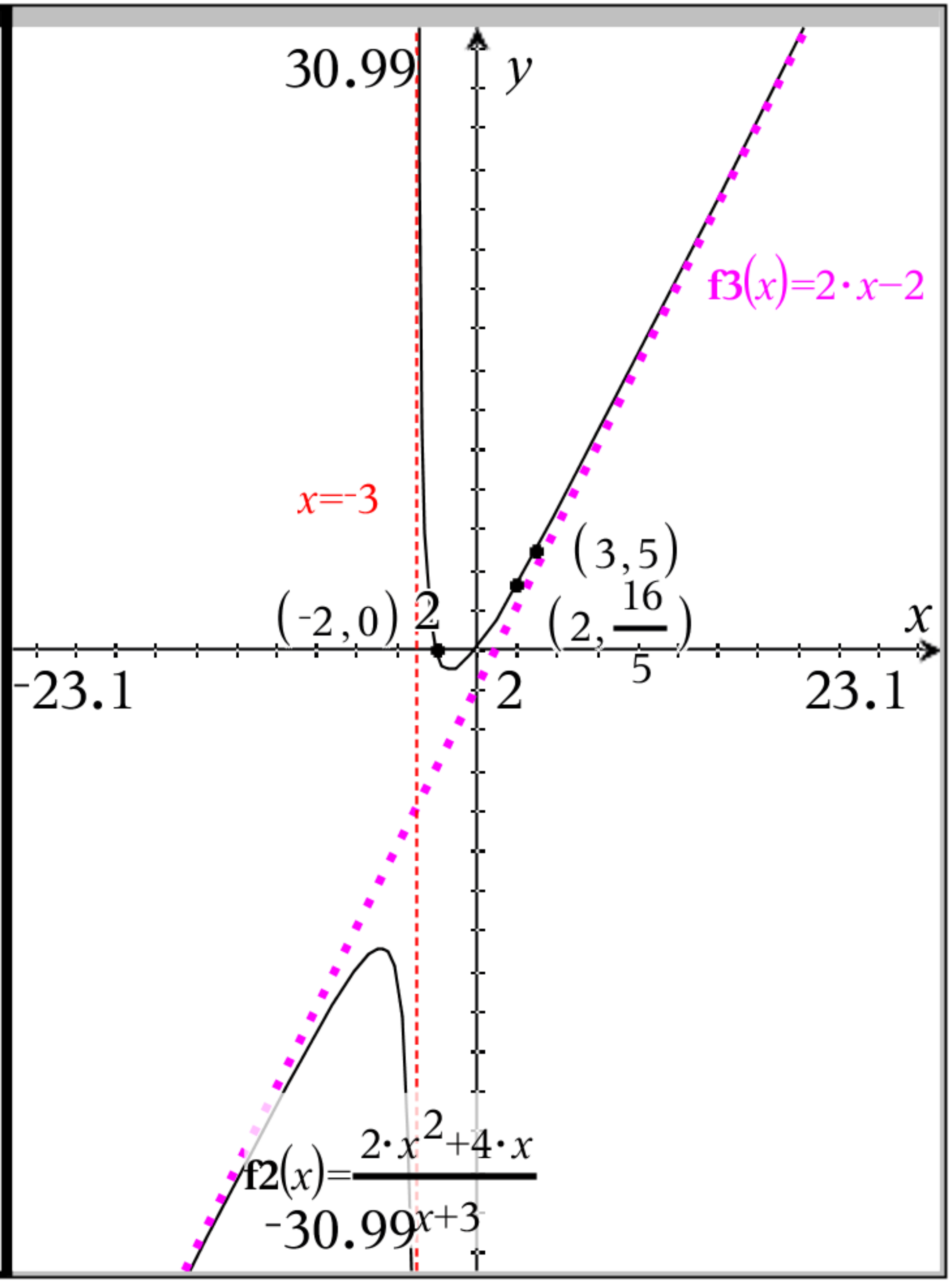
16e) $\lim\limits_{x \to -2} (h(x)) = 0$

16b) $\lim\limits_{x \to 2} (h(x)) = \dfrac{16}{5}$

16c) $\lim\limits_{x \to 3} (h(x)) = 6$

16d) $\lim\limits_{x \to -3} (h(x)) = \text{DNE}$

(only value of x that a limit that could not be found at is x = −3 through direct substitution or through h(−3))



17. What does a graph on a graphing calculator or a graphing program fail to directly show for the user of the graph? (mark all that apply)

a. Holes

b. Vertical asymptotes

c. Horizontal asymptotes

d. Slant asymptotes

e. Oblique Asymptotes

f. All of the above

F) ALL OF THE ABOVE

The user of the graphing technology still needs to know how to find the holes and asymptotes related to evry type of rational function.

TRUE or FALSE

18. TRUE 99.9% of the time instead of making a table of values to find a specific limit at $x = c$, you should try $f(c-0.0001)$ and $f(c+0.0001)$ for any real c and any function $f(x)$ to find this limit at $x = c$

19. FALSE A function must be defined at a value of x to have a limit at a value of x

(See Graoh C from #12)

20. FALSE limit of $f(x)$ at $x = c$ is ALWAYS $f(c)$

(See #11 $g(x)$ at $x = -2$)

21. FALSE is one of the example of a mathematic occurrence called the determinate form

The term is INDETERMINATE form not DETERMINATE form

22. TRUE A function can have a left side limit at $x = c$ and a right side limit at $x = c$, but not have a limit at $x = c$

(See Graph A from #12)

23. TRUE if a function's left hand limit at $x =c$ and right hand limit at $x = c$ are equal, then we can say that the function itself has a limit at $x = c$

(See Graph C from #12)