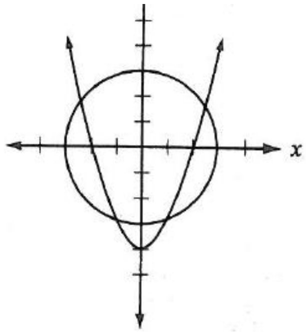
Name____Cumulative Mathematics Review 1

THIS IS ONE POINT IN CUMULATIVE GRADE

QUESTION 1



$$x^2 + y^2 = 9$$
$$y = x^2 - 4$$

A system of two equations and their graphs in the xy-plane are shown above. How many solutions does the system have?

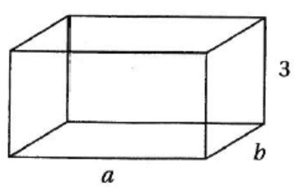
A) One

B) Two

C) Three

D) Four

QUESTION 2



Note: Figure not drawn to scale

A rectangular solid above has dimensions 3, a, and b, where a and b are integers. Which of the following CANNOT be the areas of three different faces of this solid?

A) 15, 18, and 30

B) 18, 24, and 48

C) 12, 15, and 24

D) 15, 24, and 40

QUESTION 3

If the function $m(x)$ satisfies the equation $\frac{m(x)}{x+3} - \frac{x+1}{x-1} = 1$ for all values of x greater than 1, then $m(x) =$

A) $\frac{2(x+3)}{x-1}$

B) $\frac{2(x^2+3x+3)}{x-1}$

C) $\frac{2(x+6)}{x-1}$

D) $\frac{2x(x+3)}{x-1}$

QUESTION 4

The graph of the equation $y = 2x^2 - 16x + 14$ intersects the y-axis at point A and the x-axis at points B and C. What is the area of triangle ABC?

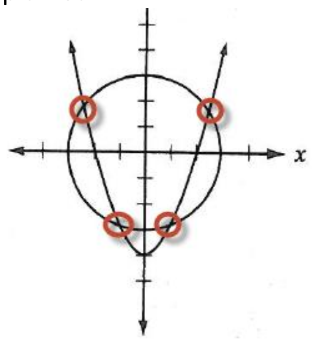
A) 42

B) 48

C) 54

D) 56

Question 1 explained



$$x^2 + y^2 = 9$$
$$y = x^2 - 4$$

A system of two equations and their graphs in the xy-plane are shown above. How many solutions does the system have?

A) One

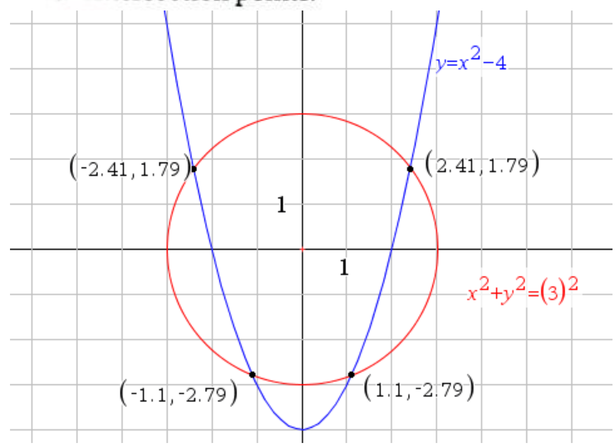
B) Two

C) Three

D) Four

D **Advanced Mathematics (nonlinear systems)**
EASY

The solutions to the system correspond to the points of intersection of the two graphs. The figure shows four such intersection points.



Actually solving this quadratic system involves WORK $y=x^2-4$

$x^2+y^2=9$

This leads to

$x^2-y=4$

$x^2+y^2=9$

This leads to

$-x^2+y=-4$

$x^2+y^2=9$

This leads to $y^2+y=5 \rightarrow y^2+y-5=0$

This leads to

$$y=\frac{-1\pm\sqrt{1^2-4\cdot -5\cdot 1}}{2\cdot 1}=\frac{-1\pm\sqrt{21}}{2}$$

Which gives us

$$x^2-\frac{-1\pm\sqrt{21}}{2}=4$$

$$\rightarrow x^2+\frac{1\pm\sqrt{21}}{2}=4$$

$$\rightarrow x^2=4-\frac{1\pm\sqrt{21}}{2} \quad x=\pm\sqrt{4-\frac{1\pm\sqrt{21}}{2.}}$$

$x=\frac{-\sqrt{2\cdot(\sqrt{21}+7)}}{2}$ and $y=\frac{\sqrt{21}-1}{2}$ or $x=\frac{-\sqrt{-2\cdot(\sqrt{21}-7)}}{2}$ and $y=\frac{-(\sqrt{21}+1)}{2}$ or

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$x=-2.40651$ and $y=1.79129$ or $x=-1.09941$ and $y=-2.79129$ or
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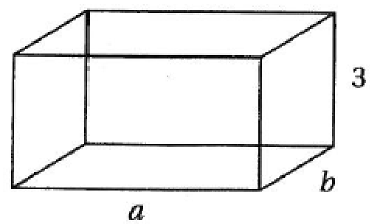
QUESTION 2



Note: Figure not drawn to scale

A rectangular solid above has dimensions 3, a, and b, where a and b are integers. Which of the following CANNOT be the areas of three different faces of this solid?

A) 15, 18, and 30

B) 18, 24, and 48

C) 12, 15, and 24

D) 15, 24, and 40

C Special Topics (three-dimensional geometry)
MEDIUM

On the drawing, we should first mark the areas of the three faces. The front and back faces both have an area of $3a$. The left and right faces both have an area of $3b$. The top and bottom faces both have an area of ab. We should now try to find integer values for a and b so that these areas match those given in the choices.

(A) 15, 18, and 30	This is possible if $a = 5$ and $b = 6$.
(B) 18, 24, and 48	This is possible if $a = 6$ and $b = 8$.
(C) 12, 15, and 24	This cannot work for any integer values of a and b.
(D) 15, 24, and 40	This is possible if $a = 5$ and $b = 8$.

A) 3(5) 3(6) (5)(6)
B) 3(6) 3(8) (6)(8)
C) 3(4) 3(5) (4)(6) X
D) 3(5) 3(8) (5)(8)

QUESTION 3

If the function $m(x)$ satisfies the equation $\frac{m(x)}{x+3}-\frac{x+1}{x-1}=1$ for all values of x greater than 1, then $m(x)=$

A) $\frac{2(x+3)}{x-1}$

B) $\frac{2(x^2+3x+3)}{x-1}$

C) $\frac{2(x+6)}{x-1}$

D) $\frac{2x(x+3)}{x-1}$

D Advanced Mathematics (rational equations)

MEDIUM

$$\frac{m(x)}{x+3}-\frac{x+1}{x-1}=1$$

Add $\frac{x+1}{x-1}$:

$$\frac{m(x)}{x+3}=\frac{x+1}{x-1}+1$$

Express right side in terms of a common denominator:

$$\frac{m(x)}{x+3}=\frac{x+1}{x-1}+\frac{x-1}{x-1}$$

Combine terms on right into one fraction:

$$\frac{m(x)}{x+3}=\frac{x+1+x-1}{x-1}$$

Combine terms:

$$\frac{m(x)}{x+3}=\frac{2x}{x-1}$$

Multiple by $x+3$:

$$m(x)=\frac{2x(x+3)}{x-1}$$

QUESTION 4

The graph of the equation $y = 2x^2 - 16x + 14$ intersects the y-axis at point A and the x-axis at points B and C. What is the area of triangle ABC?

A) 42

B) 48

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D) 56

A Advanced Mathematics (triangles/quadratics)
MEDIUM-HARD

Any point that intersects the y-axis has an x-value of 0. So, to find point A, plug in 0 for x and solve for y:

$$y = 2x^2 - 16x + 14$$

Plug in 0 for x: $y = 2(0)^2 - 16(0) + 14 = 14$

Any point that intersects the x-axis has a y-value of 0. So, to find points B and C, plug in 0 for y and solve for x:

$$y = 2x^2 - 16x + 14$$

Substitute 0 for y: $0 = 2x^2 - 16x + 14$

Divide by 2: $0 = x^2 - 8x + 7$

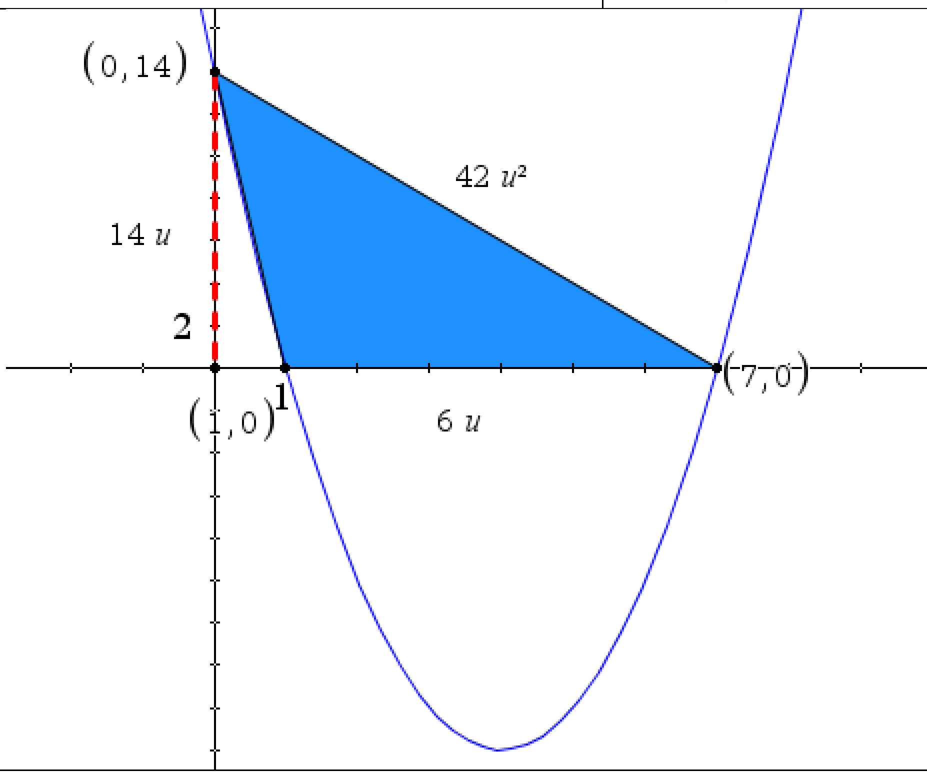
Factor: $0 = (x - 7)(x - 1)$

Use the Zero Product Property: $x = 7$ and $x = 1$

If we connect these three points, we get a triangle with a height of 14 (from $y = 0$ to $y = 14$) and a base of 6 (from $x = 1$ to $x = 7$).

Use the triangle area formula $A = \frac{1}{2}bh$:

$$A = \frac{1}{2}bh = \frac{1}{2}(14)(6) = 42$$



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BONUS
0.25</td></tr>
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