Section 2-4 : Limit Properties

1. Given $\lim\limits_{x \to 8} f(x) = -9$, $\lim\limits_{x \to 8} g(x) = 2$ and $\lim\limits_{x \to 8} h(x) = 4$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim\limits_{x \to 8} \left[2f(x) - 12h(x)\right]$ **(b)** $\lim\limits_{x \to 8} \left[3h(x) - 6\right]$

(c) $\lim\limits_{x \to 8} \left[g(x)h(x) - f(x)\right]$ **(d)** $\lim\limits_{x \to 8} \left[f(x) - g(x) + h(x)\right]$

Hint : For each of these all we need to do is use the limit properties on the limit until the given limits appear and we can then compute the value.

(a) $\lim\limits_{x \to 8} \left[2f(x) - 12h(x)\right]$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\begin{aligned} \lim_{x \to 8} \left[2f(x) - 12h(x)\right] &= \lim_{x \to 8} \left[2f(x)\right] - \lim_{x \to 8} \left[12h(x)\right] && \text{Property 2} \\ &= 2\lim_{x \to 8} f(x) - 12\lim_{x \to 8} h(x) && \text{Property 1} \\ &= 2(-9) - 12(4) && \text{Plug in values of limits} \\ &= \boxed{-66} \end{aligned}$$

(b) $\lim\limits_{x \to 8} \left[3h(x) - 6\right]$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\begin{aligned} \lim_{x \to 8} \left[3h(x) - 6\right] &= \lim_{x \to 8} \left[3h(x)\right] - \lim_{x \to 8} 6 && \text{Property 2} \\ &= 3\lim_{x \to 8} h(x) - \lim_{x \to 8} 6 && \text{Property 1} \\ &= 3(4) - 6 && \text{Plug in value of limits \& Property 7} \\ &= \boxed{6} \end{aligned}$$

(c) $\lim\limits_{x \to 8} \left[g(x)h(x) - f(x)\right]$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\begin{aligned}\lim_{x\to 8}\left[g(x)h(x)-f(x)\right] &= \lim_{x\to 8}\left[g(x)h(x)\right]-\lim_{x\to 8} f(x) && \text{Property 2}\\ &= \left[\lim_{x\to 8} g(x)\right]\left[\lim_{x\to 8} h(x)\right]-\lim_{x\to 8} f(x) && \text{Property 3}\\ &= (2)(4)-(-9) && \text{Plug in values of limits}\\ &= \boxed{17}\end{aligned}$$

(d) $\lim_{x\to 8}\left[f(x)-g(x)+h(x)\right]$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\begin{aligned}\lim_{x\to 8}\left[f(x)-g(x)+h(x)\right] &= \lim_{x\to 8} f(x)-\lim_{x\to 8} g(x)+\lim_{x\to 8} h(x) && \text{Property 2}\\ &= -9-2+4 && \text{Plug in values of limits}\\ &= \boxed{-7}\end{aligned}$$

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2. Given $\lim_{x\to -4} f(x)=1$, $\lim_{x\to -4} g(x)=10$ and $\lim_{x\to -4} h(x)=-7$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim_{x\to -4}\left[\frac{f(x)}{g(x)}-\frac{h(x)}{f(x)}\right]$

(b) $\lim_{x\to -4}\left[f(x)g(x)h(x)\right]$

(c) $\lim_{x\to -4}\left[\frac{1}{h(x)}+\frac{3-f(x)}{g(x)+h(x)}\right]$

(d) $\lim_{x\to -4}\left[2h(x)-\frac{1}{h(x)+7f(x)}\right]$

Hint : For each of these all we need to do is use the limit properties on the limit until the given limits appear and we can then compute the value.

(a) $\lim_{x\to -4}\left[\frac{f(x)}{g(x)}-\frac{h(x)}{f(x)}\right]$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\lim_{x\to -4}\left[\frac{f(x)}{g(x)}-\frac{h(x)}{f(x)}\right]=\lim_{x\to -4}\frac{f(x)}{g(x)}-\lim_{x\to -4}\frac{h(x)}{f(x)} \qquad \text{Property 2}$$

$$=\frac{\lim\limits_{x\to -4} f(x)}{\lim\limits_{x\to -4} g(x)}-\frac{\lim\limits_{x\to -4} h(x)}{\lim\limits_{x\to -4} f(x)} \qquad \text{Property 4}$$

$$=\frac{1}{10}-\frac{-7}{1} \qquad \text{Plug in values of limits}$$

$$=\boxed{\frac{71}{10}}$$

Note that were able to use Property 4 in the second step only because after we evaluated the limit of the denominators (both of them) we found that the limits of the denominators were not zero.

(b) $\lim\limits_{x\to -4}\left[f(x)g(x)h(x)\right]$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\lim_{x\to -4}\left[f(x)g(x)h(x)\right]=\left[\lim_{x\to -4} f(x)\right]\left[\lim_{x\to -4} g(x)\right]\left[\lim_{x\to -4} h(x)\right] \qquad \text{Property 3}$$

$$=(1)(10)(-7) \qquad \text{Plug in value of limits}$$

$$=\boxed{-70}$$

Note that the properties 2 & 3 in this section were only given with two functions but they can easily be extended out to more than two functions as we did here for property 3.

(c) $\lim\limits_{x\to -4}\left[\frac{1}{h(x)}+\frac{3-f(x)}{g(x)+h(x)}\right]$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\lim_{x\to -4}\left[\frac{1}{h(x)}+\frac{3-f(x)}{g(x)+h(x)}\right]=\lim_{x\to -4}\frac{1}{h(x)}+\lim_{x\to -4}\frac{3-f(x)}{g(x)+h(x)} \qquad \text{Property 2}$$

$$=\frac{\lim\limits_{x\to -4}1}{\lim\limits_{x\to -4}h(x)}+\frac{\lim\limits_{x\to -4}\left[3-f(x)\right]}{\lim\limits_{x\to -4}\left[g(x)+h(x)\right]} \qquad \text{Property 4}$$

$$=\frac{\lim\limits_{x\to -4}1}{\lim\limits_{x\to -4}h(x)}+\frac{\lim\limits_{x\to -4}3-\lim\limits_{x\to -4}f(x)}{\lim\limits_{x\to -4}g(x)+\lim\limits_{x\to -4}h(x)} \qquad \text{Property 2}$$

$$=\frac{1}{-7}+\frac{3-1}{10-7} \qquad \text{Plug in values of limits \& Property 1}$$

$$=\boxed{\frac{11}{21}}$$

Note that were able to use Property 4 in the second step only because after we evaluated the limit of the denominators (both of them) we found that the limits of the denominators were not zero.

(d) $\lim\limits_{x\to -4}\left[2h(x)-\dfrac{1}{h(x)+7f(x)}\right]$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\lim_{x\to -4}\left[2h(x)-\frac{1}{h(x)+7f(x)}\right]=\lim_{x\to -4}2h(x)-\lim_{x\to -4}\frac{1}{h(x)+7f(x)} \qquad \text{Property 2}$$

$$=\lim_{x\to -4}2h(x)-\frac{\lim\limits_{x\to -4}1}{\lim\limits_{x\to -4}\left[h(x)+7f(x)\right]} \qquad \text{Property 4}$$

At this point let's step back a minute. In the previous parts we didn't worry about using property 4 on a rational expression. However, in this case let's be a little more careful. We can only use property 4 if the limit of the denominator is not zero. Let's check that limit and see what we get.

$$\lim_{x\to -4}\left[h(x)+7f(x)\right]=\lim_{x\to -4}h(x)+\lim_{x\to -4}\left[7f(x)\right] \qquad \text{Property 2}$$

$$=\lim_{x\to -4}h(x)+7\lim_{x\to -4}f(x) \qquad \text{Property 1}$$

$$=-7+7(1) \qquad \text{Plug in values of limits \& Property 1}$$

$$=0$$

Okay, we can see that the limit of the denominator in the second term will be zero so we cannot actually use property 4 on that term. This means that this limit cannot be done and note that the fact that we could determine a value for the limit of the first term will not change this fact. This limit cannot be done.

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3. Given $\lim\limits_{x \to 0} f(x) = 6$, $\lim\limits_{x \to 0} g(x) = -4$ and $\lim\limits_{x \to 0} h(x) = -1$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not.

(a) $\lim\limits_{x \to 0} \left[f(x) + h(x) \right]^3$

(b) $\lim\limits_{x \to 0} \sqrt{g(x)h(x)}$

(c) $\lim\limits_{x \to 0} \sqrt[3]{11 + \left[g(x) \right]^2}$

(d) $\lim\limits_{x \to 0} \sqrt{\dfrac{f(x)}{h(x) - g(x)}}$

Hint : For each of these all we need to do is use the limit properties on the limit until the given limits appear and we can then compute the value.

(a) $\lim\limits_{x \to 0} \left[f(x) + h(x) \right]^3$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\begin{aligned}
\lim_{x \to 0} \left[f(x) + h(x) \right]^3 &= \left[\lim_{x \to 0} \left(f(x) + h(x) \right) \right]^3 && \text{Property 5} \\
&= \left[\lim_{x \to 0} f(x) + \lim_{x \to 0} h(x) \right]^3 && \text{Property 2} \\
&= \left[6 - 1 \right]^3 && \text{Plug in values of limits} \\
&= \boxed{125}
\end{aligned}$$

(b) $\lim\limits_{x \to 0} \sqrt{g(x)h(x)}$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$\begin{aligned}
\lim_{x \to 0} \sqrt{g(x)h(x)} &= \sqrt{\lim_{x \to 0} g(x)h(x)} && \text{Property 6} \\
&= \sqrt{\left[\lim_{x \to 0} g(x) \right] \left[\lim_{x \to 0} h(x) \right]} && \text{Property 3} \\
&= \sqrt{(-4)(-1)} && \text{Plug in value of limits} \\
&= \boxed{2}
\end{aligned}$$

(c) $\lim\limits_{x \to 0} \sqrt[3]{11 + \left[g(x)\right]^2}$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$
\begin{aligned}
\lim_{x \to 0} \sqrt[3]{11 + \left[g(x)\right]^2} &= \sqrt[3]{\lim_{x \to 0}\left(11 + \left[g(x)\right]^2\right)} && \text{Property 6} \\
&= \sqrt[3]{\lim_{x \to 0} 11 + \lim_{x \to 0}\left[g(x)\right]^2} && \text{Property 2} \\
&= \sqrt[3]{\lim_{x \to 0} 11 + \left[\lim_{x \to 0} g(x)\right]^2} && \text{Property 5} \\
&= \sqrt[3]{11 + (-4)^2} && \text{Plug in values of limits \& Property 7} \\
&= \boxed{3}
\end{aligned}
$$

(d) $\lim\limits_{x \to 0} \sqrt{\dfrac{f(x)}{h(x) - g(x)}}$

Here is the work for this limit. At each step the property (or properties) used are listed and note that in some cases the properties may have been used more than once in the indicated step.

$$
\begin{aligned}
\lim_{x \to 0} \sqrt{\frac{f(x)}{h(x) - g(x)}} &= \sqrt{\lim_{x \to 0} \frac{f(x)}{h(x) - g(x)}} && \text{Property 6} \\
&= \sqrt{\frac{\lim\limits_{x \to 0} f(x)}{\lim\limits_{x \to 0}\left(h(x) - g(x)\right)}} && \text{Property 4} \\
&= \sqrt{\frac{\lim\limits_{x \to 0} f(x)}{\lim\limits_{x \to 0} h(x) - \lim\limits_{x \to 0} g(x)}} && \text{Property 2} \\
&= \sqrt{\frac{6}{-1 - (-4)}} && \text{Plug in values of limits} \\
&= \sqrt{2}
\end{aligned}
$$

Note that were able to use Property 4 in the second step only because after we evaluated the limit of the denominators (both of them) we found that the limits of the denominators were not zero.

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4. Use the limit properties given in this section to compute the following limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

$$\lim_{t \to -2} \left(14 - 6t + t^3\right)$$

Hint : All we need to do is use the limit properties on the limit until we can use Properties 7, 8 and/or 9 from this section to compute the limit.

$$\begin{aligned}
\lim_{t \to -2} \left(14 - 6t + t^3\right) &= \lim_{t \to -2} 14 - \lim_{t \to -2} 6t + \lim_{t \to -2} t^3 && \text{Property 2} \\
&= \lim_{t \to -2} 14 - 6 \lim_{t \to -2} t + \lim_{t \to -2} t^3 && \text{Property 1} \\
&= 14 - 6\left(-2\right) + \left(-2\right)^3 && \text{Properties 7, 8, \& 9} \\
&= \boxed{18}
\end{aligned}$$

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5. Use the limit properties given in this section to compute the following limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

$$\lim_{x \to 6} \left(3x^2 + 7x - 16\right)$$

Hint : All we need to do is use the limit properties on the limit until we can use Properties 7, 8 and/or 9 from this section to compute the limit.

$$\begin{aligned}
\lim_{x \to 6} \left(3x^2 + 7x - 16\right) &= \lim_{x \to 6} 3x^2 + \lim_{x \to 6} 7x - \lim_{x \to 6} 16 && \text{Property 2} \\
&= 3 \lim_{x \to 6} x^2 + 7 \lim_{x \to 6} x - \lim_{x \to 6} 16 && \text{Property 1} \\
&= 3\left(6^2\right) + 7\left(6\right) - 16 && \text{Properties 7, 8, \& 9} \\
&= \boxed{134}
\end{aligned}$$

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6. Use the limit properties given in this section to compute the following limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

$$\lim_{w \to 3} \frac{w^2 - 8w}{4 - 7w}$$

Hint : All we need to do is use the limit properties on the limit until we can use Properties 7, 8 and/or 9 from this section to compute the limit.

$$\lim_{w \to 3} \frac{w^2 - 8w}{4 - 7w} = \frac{\lim_{w \to 3}\left(w^2 - 8w\right)}{\lim_{w \to 3}\left(4 - 7w\right)} \qquad \text{Property 4}$$

$$= \frac{\lim_{w \to 3} w^2 - \lim_{w \to 3} 8w}{\lim_{w \to 3} 4 - \lim_{w \to 3} 7w} \qquad \text{Property 2}$$

$$= \frac{\lim_{w \to 3} w^2 - 8\lim_{w \to 3} w}{\lim_{w \to 3} 4 - 7\lim_{w \to 3} w} \qquad \text{Property 1}$$

$$= \frac{3^2 - 8(3)}{4 - 7(3)} \qquad \text{Properties 7, 8, \& 9}$$

$$= \boxed{\frac{15}{17}}$$

Note that we were able to use property 4 in the first step because after evaluating the limit in the denominator we found that it wasn't zero.

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7. Use the limit properties given in this section to compute the following limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

$$\lim_{x \to -5} \frac{x + 7}{x^2 + 3x - 10}$$

Hint : All we need to do is use the limit properties on the limit until we can use Properties 7, 8 and/or 9 from this section to compute the limit.

$$\lim_{x \to -5} \frac{x + 7}{x^2 + 3x - 10} = \frac{\lim_{x \to -5}\left(x + 7\right)}{\lim_{x \to -5}\left(x^2 + 3x - 10\right)} \qquad \text{Property 4}$$

Okay, at this point let's step back a minute. We used property 4 here and we know that we can only do that if the limit of the denominator is not zero. So, let's check that out and see what we get.

$$\lim_{x \to -5}\left(x^2 + 3x - 10\right) = \lim_{x \to -5} x^2 + \lim_{x \to -5} 3x - \lim_{x \to -5} 10 \qquad \text{Property 2}$$

$$= \lim_{x \to -5} x^2 + 3\lim_{x \to -5} x - \lim_{x \to -5} 10 \qquad \text{Property 1}$$

$$= (-5)^2 + 3(-5) - 10 \qquad \text{Properties 7, 8, \& 9}$$

$$= 0$$

So, the limit of the denominator is zero so we couldn't use property 4 in this case. Therefore, we cannot do this limit at this point (note that it will be possible to do this limit after the next section).

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8. Use the limit properties given in this section to compute the following limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

$$\lim_{z \to 0} \sqrt{z^2 + 6}$$

Hint : All we need to do is use the limit properties on the limit until we can use Properties 7, 8 and/or 9 from this section to compute the limit.

$$\begin{aligned} \lim_{z \to 0} \sqrt{z^2 + 6} &= \sqrt{\lim_{z \to 0} \left(z^2 + 6 \right)} && \text{Property 6} \\ &= \sqrt{\lim_{z \to 0} z^2 + \lim_{z \to 0} 6} && \text{Property 2} \\ &= \sqrt{0^2 + 6} && \text{Properties 7 \& 9} \\ &= \sqrt{6} \end{aligned}$$

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9. Use the limit properties given in this section to compute the following limit. At each step clearly indicate the property being used. If it is not possible to compute any of the limits clearly explain why not.

$$\lim_{x \to 10} \left(4x + \sqrt[3]{x - 2} \right)$$

Hint : All we need to do is use the limit properties on the limit until we can use Properties 7, 8 and/or 9 from this section to compute the limit.

$$\begin{aligned} \lim_{x \to 10} \left(4x + \sqrt[3]{x - 2} \right) &= \lim_{x \to 10} 4x + \lim_{x \to 10} \sqrt[3]{x - 2} && \text{Property 2} \\ &= \lim_{x \to 10} 4x + \sqrt[3]{\lim_{x \to 10} \left(x - 2 \right)} && \text{Property 6} \\ &= \lim_{x \to 10} 4x + \sqrt[3]{\lim_{x \to 10} x - \lim_{x \to 10} 2} && \text{Property 2} \\ &= 4 \lim_{x \to 10} x + \sqrt[3]{\lim_{x \to 10} x - \lim_{x \to 10} 2} && \text{Property 1} \\ &= 4\left(10\right) + \sqrt[3]{10 - 2} && \text{Properties 7 \& 8} \\ &= 42 \end{aligned}$$
