

## Section 2-1 : Tangent Lines and Rates of Change

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1. For the function  $f(x) = 3(x+2)^2$  and the point  $P$  given by  $x = -3$  answer each of the following questions.

(a) For the points  $Q$  given by the following values of  $x$  compute (accurate to at least 8 decimal places) the slope,  $m_{PQ}$ , of the secant line through points  $P$  and  $Q$ .

(i) -3.5      (ii) -3.1      (iii) -3.01      (iv) -3.001      (v) -3.0001  
(vi) -2.5      (vii) -2.9      (viii) -2.99      (ix) -2.999      (x) -2.9999

(b) Use the information from (a) to estimate the slope of the tangent line to  $f(x)$  at  $x = -3$  and write down the equation of the tangent line.

2. For the function  $g(x) = \sqrt{4x+8}$  and the point  $P$  given by  $x = 2$  answer each of the following questions.

(a) For the points  $Q$  given by the following values of  $x$  compute (accurate to at least 8 decimal places) the slope,  $m_{PQ}$ , of the secant line through points  $P$  and  $Q$ .

(i) 2.5      (ii) 2.1      (iii) 2.01      (iv) 2.001      (v) 2.0001  
(vi) 1.5      (vii) 1.9      (viii) 1.99      (ix) 1.999      (x) 1.9999

(b) Use the information from (a) to estimate the slope of the tangent line to  $g(x)$  at  $x = 2$  and write down the equation of the tangent line.

3. For the function  $W(x) = \ln(1+x^4)$  and the point  $P$  given by  $x = 1$  answer each of the following questions.

(a) For the points  $Q$  given by the following values of  $x$  compute (accurate to at least 8 decimal places) the slope,  $m_{PQ}$ , of the secant line through points  $P$  and  $Q$ .

(i) 1.5      (ii) 1.1      (iii) 1.01      (iv) 1.001      (v) 1.0001  
(vi) 0.5      (vii) 0.9      (viii) 0.99      (ix) 0.999      (x) 0.9999

(b) Use the information from (a) to estimate the slope of the tangent line to  $W(x)$  at  $x = 1$  and write down the equation of the tangent line.

4. The volume of air in a balloon is given by  $V(t) = \frac{6}{4t+1}$  answer each of the following questions.

**(a)** Compute (accurate to at least 8 decimal places) the average rate of change of the volume of air in the balloon between  $t = 0.25$  and the following values of  $t$ .

- |               |                  |                     |                    |                    |
|---------------|------------------|---------------------|--------------------|--------------------|
| <b>(i)</b> 1  | <b>(ii)</b> 0.5  | <b>(iii)</b> 0.251  | <b>(iv)</b> 0.2501 | <b>(v)</b> 0.25001 |
| <b>(vi)</b> 0 | <b>(vii)</b> 0.1 | <b>(viii)</b> 0.249 | <b>(ix)</b> 0.2499 | <b>(x)</b> 0.24999 |

**(b)** Use the information from **(a)** to estimate the instantaneous rate of change of the volume of air in the balloon at  $t = 0.25$ .

5. The population (in hundreds) of fish in a pond is given by  $P(t) = 2t + \sin(2t - 10)$  answer each of the following questions.

**(a)** Compute (accurate to at least 8 decimal places) the average rate of change of the population of fish between  $t = 5$  and the following values of  $t$ . Make sure your calculator is set to radians for the computations.

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|-----------------|------------------|--------------------|-------------------|-------------------|
| <b>(i)</b> 5.5  | <b>(ii)</b> 5.1  | <b>(iii)</b> 5.01  | <b>(iv)</b> 5.001 | <b>(v)</b> 5.0001 |
| <b>(vi)</b> 4.5 | <b>(vii)</b> 4.9 | <b>(viii)</b> 4.99 | <b>(ix)</b> 4.999 | <b>(x)</b> 4.9999 |

**(b)** Use the information from **(a)** to estimate the instantaneous rate of change of the population of the fish at  $t = 5$ .

6. The position of an object is given by  $s(t) = \cos^2\left(\frac{3t-6}{2}\right)$  answer each of the following questions.

**(a)** Compute (accurate to at least 8 decimal places) the average velocity of the object between  $t = 2$  and the following values of  $t$ . Make sure your calculator is set to radians for the computations.

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|-----------------|------------------|--------------------|-------------------|-------------------|
| <b>(i)</b> 2.5  | <b>(ii)</b> 2.1  | <b>(iii)</b> 2.01  | <b>(iv)</b> 2.001 | <b>(v)</b> 2.0001 |
| <b>(vi)</b> 1.5 | <b>(vii)</b> 1.9 | <b>(viii)</b> 1.99 | <b>(ix)</b> 1.999 | <b>(x)</b> 1.9999 |

**(b)** Use the information from **(a)** to estimate the instantaneous velocity of the object at  $t = 2$  and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

7. The position of an object is given by  $s(t) = (8-t)(t+6)^{\frac{3}{2}}$ . Note that a negative position here simply means that the position is to the left of the “zero position” and is perfectly acceptable. Answer each of the following questions.

**(a)** Compute (accurate to at least 8 decimal places) the average velocity of the object between  $t = 10$  and the following values of  $t$ .

- |                 |                  |                    |                    |                    |
|-----------------|------------------|--------------------|--------------------|--------------------|
| <b>(i)</b> 10.5 | <b>(ii)</b> 10.1 | <b>(iii)</b> 10.01 | <b>(iv)</b> 10.001 | <b>(v)</b> 10.0001 |
| <b>(vi)</b> 9.5 | <b>(vii)</b> 9.9 | <b>(viii)</b> 9.99 | <b>(ix)</b> 9.999  | <b>(x)</b> 9.9999  |

**(b)** Use the information from **(a)** to estimate the instantaneous velocity of the object at  $t = 10$  and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

