

Section 5-5 : Area Problem

1. Estimate the area of the region between $f(x) = x^3 - 2x^2 + 4$ the x -axis on $[1, 4]$ using $n = 6$ and using,

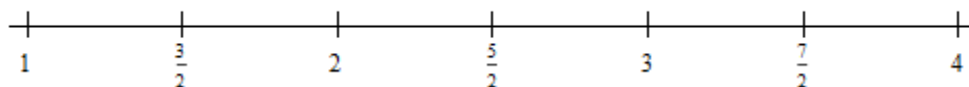
- (a) the right end points of the subintervals for the height of the rectangles,
- (b) the left end points of the subintervals for the height of the rectangles and,
- (c) the midpoints of the subintervals for the height of the rectangles.

(a) the right end points of the subintervals for the height of the rectangles,

The widths of each of the subintervals for this problem are,

$$\Delta x = \frac{4-1}{6} = \frac{1}{2}$$

We don't need to actually graph the function to do this problem. It would probably help to have a number line showing subintervals however. Here is that number line.



In this case we're going to be using right end points of each of these subintervals to determine the height of each of the rectangles.

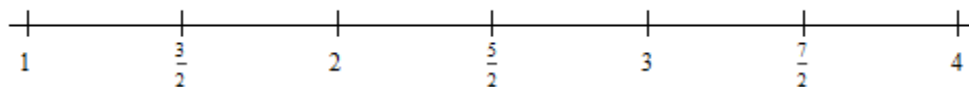
The area between the function and the x -axis is then approximately,

$$\begin{aligned} \text{Area} &\approx \frac{1}{2} f\left(\frac{3}{2}\right) + \frac{1}{2} f(2) + \frac{1}{2} f\left(\frac{5}{2}\right) + \frac{1}{2} f(3) + \frac{1}{2} f\left(\frac{7}{2}\right) + \frac{1}{2} f(4) \\ &= \frac{1}{2} \left(\frac{23}{8}\right) + \frac{1}{2} (4) + \frac{1}{2} \left(\frac{57}{8}\right) + \frac{1}{2} (13) + \frac{1}{2} \left(\frac{179}{8}\right) + \frac{1}{2} (36) = \boxed{\frac{683}{16} = 42.6875} \end{aligned}$$

(b) the left end points of the subintervals for the height of the rectangles and,

As we found in the previous part the widths of each of the subintervals are $\Delta x = \frac{1}{2}$.

Here is a copy of the number line showing the subintervals to help with the problem.



In this case we're going to be using left end points of each of these subintervals to determine the height of each of the rectangles.

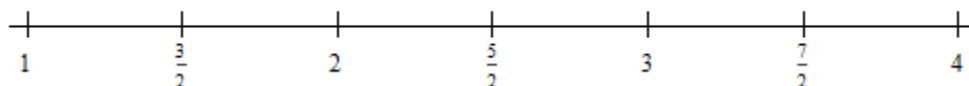
The area between the function and the x-axis is then approximately,

$$\begin{aligned}\text{Area} &\approx \frac{1}{2}f(1) + \frac{1}{2}f\left(\frac{3}{2}\right) + \frac{1}{2}f(2) + \frac{1}{2}f\left(\frac{5}{2}\right) + \frac{1}{2}f(3) + \frac{1}{2}f\left(\frac{7}{2}\right) \\ &= \frac{1}{2}(3) + \frac{1}{2}\left(\frac{23}{8}\right) + \frac{1}{2}(4) + \frac{1}{2}\left(\frac{57}{8}\right) + \frac{1}{2}(13) + \frac{1}{2}\left(\frac{179}{8}\right) = \boxed{\frac{419}{16} = 26.1875}\end{aligned}$$

(c) the midpoints of the subintervals for the height of the rectangles.

As we found in the first part the widths of each of the subintervals are $\Delta x = \frac{1}{2}$.

Here is a copy of the number line showing the subintervals to help with the problem.



In this case we're going to be using midpoints of each of these subintervals to determine the height of each of the rectangles.

The area between the function and the x-axis is then approximately,

$$\begin{aligned}\text{Area} &\approx \frac{1}{2}f\left(\frac{5}{4}\right) + \frac{1}{2}f\left(\frac{7}{4}\right) + \frac{1}{2}f\left(\frac{9}{4}\right) + \frac{1}{2}f\left(\frac{11}{4}\right) + \frac{1}{2}f\left(\frac{13}{4}\right) + \frac{1}{2}f\left(\frac{15}{4}\right) \\ &= \frac{1}{2}\left(\frac{181}{64}\right) + \frac{1}{2}\left(\frac{207}{64}\right) + \frac{1}{2}\left(\frac{337}{64}\right) + \frac{1}{2}\left(\frac{619}{64}\right) + \frac{1}{2}\left(\frac{1101}{64}\right) + \frac{1}{2}\left(\frac{1831}{64}\right) = \boxed{\frac{1069}{32} = 33.40625}\end{aligned}$$

2. Estimate the area of the region between $g(x) = 4 - \sqrt{x^2 + 2}$ the x-axis on $[-1, 3]$ using $n = 6$ and using,

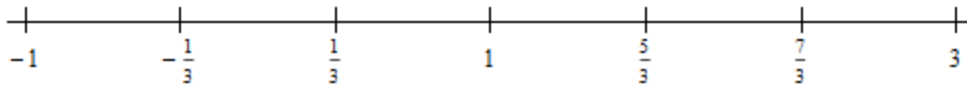
- (a)** the right end points of the subintervals for the height of the rectangles,
- (b)** the left end points of the subintervals for the height of the rectangles and,
- (c)** the midpoints of the subintervals for the height of the rectangles.

(a) the right end points of the subintervals for the height of the rectangles,

The widths of each of the subintervals for this problem are,

$$\Delta x = \frac{3 - (-1)}{6} = \frac{2}{3}$$

We don't need to actually graph the function to do this problem. It would probably help to have a number line showing subintervals however. Here is that number line.



In this case we're going to be using right end points of each of these subintervals to determine the height of each of the rectangles.

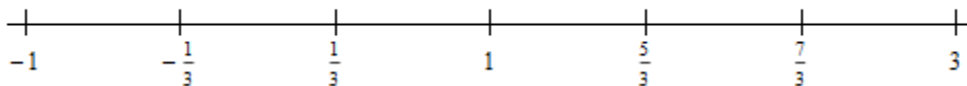
The area between the function and the x-axis is then approximately,

$$\begin{aligned} \text{Area} &\approx \frac{2}{3} f\left(-\frac{1}{3}\right) + \frac{2}{3} f\left(\frac{1}{3}\right) + \frac{2}{3} f(1) + \frac{2}{3} f\left(\frac{5}{3}\right) + \frac{2}{3} f\left(\frac{7}{3}\right) + \frac{2}{3} f(3) \\ &= \frac{2}{3} \left(4 - \frac{\sqrt{19}}{3}\right) + \frac{2}{3} \left(4 - \frac{\sqrt{19}}{3}\right) + \frac{2}{3} (4 - \sqrt{3}) + \frac{2}{3} \left(4 - \frac{\sqrt{43}}{3}\right) + \frac{2}{3} \left(4 - \frac{\sqrt{67}}{3}\right) + \frac{2}{3} (4 - \sqrt{11}) \\ &= \boxed{7.420752} \end{aligned}$$

(b) the left end points of the subintervals for the height of the rectangles and,

As we found in the previous part the widths of each of the subintervals are $\Delta x = \frac{2}{3}$.

Here is a copy of the number line showing the subintervals to help with the problem.



In this case we're going to be using left end points of each of these subintervals to determine the height of each of the rectangles.

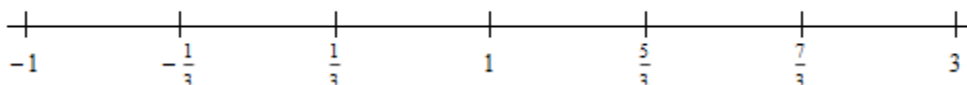
The area between the function and the x-axis is then approximately,

$$\begin{aligned} \text{Area} &\approx \frac{2}{3} f(-1) + \frac{2}{3} f\left(-\frac{1}{3}\right) + \frac{2}{3} f\left(\frac{1}{3}\right) + \frac{2}{3} f(1) + \frac{2}{3} f\left(\frac{5}{3}\right) + \frac{2}{3} f\left(\frac{7}{3}\right) \\ &= \frac{2}{3} (4 - \sqrt{3}) + \frac{2}{3} \left(4 - \frac{\sqrt{19}}{3}\right) + \frac{2}{3} \left(4 - \frac{\sqrt{19}}{3}\right) + \frac{2}{3} (4 - \sqrt{3}) + \frac{2}{3} \left(4 - \frac{\sqrt{43}}{3}\right) + \frac{2}{3} \left(4 - \frac{\sqrt{67}}{3}\right) \\ &= \boxed{8.477135} \end{aligned}$$

(c) the midpoints of the subintervals for the height of the rectangles.

As we found in the first part the widths of each of the subintervals are $\Delta x = \frac{2}{3}$.

Here is a copy of the number line showing the subintervals to help with the problem.



In this case we're going to be using midpoints of each of these subintervals to determine the height of each of the rectangles.

The area between the function and the x-axis is then approximately,

$$\begin{aligned} \text{Area} &\approx \frac{2}{3} f\left(-\frac{2}{3}\right) + \frac{2}{3} f(0) + \frac{2}{3} f\left(\frac{2}{3}\right) + \frac{2}{3} f\left(\frac{4}{3}\right) + \frac{2}{3} f(2) + \frac{2}{3} f\left(\frac{8}{3}\right) \\ &= \frac{2}{3} \left(4 - \frac{\sqrt{22}}{3}\right) + \frac{2}{3} (4 - \sqrt{2}) + \frac{2}{3} \left(4 - \frac{\sqrt{22}}{3}\right) + \frac{2}{3} \left(4 - \frac{\sqrt{34}}{3}\right) + \frac{2}{3} (4 - \sqrt{6}) + \frac{2}{3} \left(4 - \frac{\sqrt{82}}{3}\right) \\ &= \boxed{8.031494} \end{aligned}$$

3. Estimate the area of the region between $h(x) = -x \cos\left(\frac{x}{3}\right)$ the x-axis on $[0, 3]$ using $n = 6$ and using,

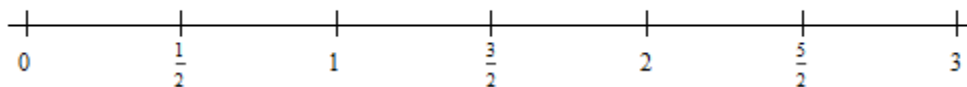
- (a) the right end points of the subintervals for the height of the rectangles,
- (b) the left end points of the subintervals for the height of the rectangles and,
- (c) the midpoints of the subintervals for the height of the rectangles.

(a) the right end points of the subintervals for the height of the rectangles,

The widths of each of the subintervals for this problem are,

$$\Delta x = \frac{3-0}{6} = \frac{1}{2}$$

We don't need to actually graph the function to do this problem. It would probably help to have a number line showing subintervals however. Here is that number line.



In this case we're going to be using right end points of each of these subintervals to determine the height of each of the rectangles.

The area between the function and the x-axis is then approximately,

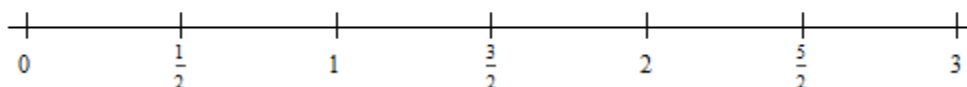
$$\begin{aligned} \text{Area} &\approx \frac{1}{2} f\left(\frac{1}{2}\right) + \frac{1}{2} f(1) + \frac{1}{2} f\left(\frac{3}{2}\right) + \frac{1}{2} f(2) + \frac{1}{2} f\left(\frac{5}{2}\right) + \frac{1}{2} f(3) \\ &= \frac{1}{2} \left(-\frac{1}{2} \cos\left(\frac{1}{6}\right)\right) + \frac{1}{2} \left(-\cos\left(\frac{1}{3}\right)\right) + \frac{1}{2} \left(-\frac{3}{2} \cos\left(\frac{1}{2}\right)\right) + \frac{1}{2} \left(-2 \cos\left(\frac{2}{3}\right)\right) \\ &\quad + \frac{1}{2} \left(-\frac{5}{2} \cos\left(\frac{5}{6}\right)\right) + \frac{1}{2} (-3 \cos(1)) \\ &= \boxed{-3.814057} \end{aligned}$$

Do not get excited about the negative area here. As we discussed in this section this just means that the graph, in this case, is below the x-axis as you could verify if you'd like to.

(b) the left end points of the subintervals for the height of the rectangles and,

As we found in the previous part the widths of each of the subintervals are $\Delta x = \frac{2}{3}$.

Here is a copy of the number line showing the subintervals to help with the problem.



In this case we're going to be using left end points of each of these subintervals to determine the height of each of the rectangles.

The area between the function and the x-axis is then approximately,

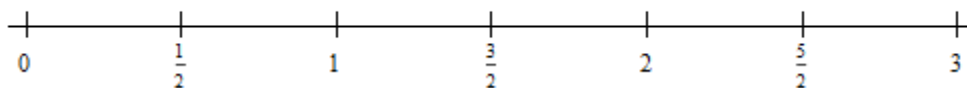
$$\begin{aligned} \text{Area} &\approx \frac{1}{2} f(0) + \frac{1}{2} f\left(\frac{1}{2}\right) + \frac{1}{2} f(1) + \frac{1}{2} f\left(\frac{3}{2}\right) + \frac{1}{2} f(2) + \frac{1}{2} f\left(\frac{5}{2}\right) \\ &= +\frac{1}{2}(0) + \frac{1}{2}\left(-\frac{1}{2}\cos\left(\frac{1}{6}\right)\right) + \frac{1}{2}\left(-\cos\left(\frac{1}{3}\right)\right) + \frac{1}{2}\left(-\frac{3}{2}\cos\left(\frac{1}{2}\right)\right) + \frac{1}{2}\left(-2\cos\left(\frac{2}{3}\right)\right) \\ &\quad + \frac{1}{2}\left(-\frac{5}{2}\cos\left(\frac{5}{6}\right)\right) \\ &= \boxed{-3.003604} \end{aligned}$$

Do not get excited about the negative area here. As we discussed in this section this just means that the graph, in this case, is below the x-axis as you could verify if you'd like to.

(c) the midpoints of the subintervals for the height of the rectangles.

As we found in the first part the widths of each of the subintervals are $\Delta x = \frac{2}{3}$.

Here is a copy of the number line showing the subintervals to help with the problem.



In this case we're going to be using midpoints of each of these subintervals to determine the height of each of the rectangles.

The area between the function and the x-axis is then approximately,

$$\begin{aligned}
\text{Area} &\approx \frac{1}{2} f\left(\frac{1}{4}\right) + \frac{1}{2} f\left(\frac{3}{4}\right) + \frac{1}{2} f\left(\frac{5}{4}\right) + \frac{1}{2} f\left(\frac{7}{4}\right) + \frac{1}{2} f\left(\frac{9}{4}\right) + \frac{1}{2} f\left(\frac{11}{4}\right) \\
&= \frac{1}{2} \left(-\frac{1}{4} \cos\left(\frac{1}{12}\right)\right) + \frac{1}{2} \left(-\frac{3}{4} \cos\left(\frac{1}{4}\right)\right) + \frac{1}{2} \left(-\frac{5}{4} \cos\left(\frac{5}{12}\right)\right) + \frac{1}{2} \left(-\frac{7}{4} \cos\left(\frac{7}{12}\right)\right) \\
&\quad + \frac{1}{2} \left(-\frac{9}{4} \cos\left(\frac{3}{4}\right)\right) + \frac{1}{2} \left(-\frac{11}{4} \cos\left(\frac{11}{12}\right)\right) \\
&= \boxed{-3.449532}
\end{aligned}$$

Do not get excited about the negative area here. As we discussed in this section this just means that the graph, in this case, is below the x -axis as you could verify if you'd like to.

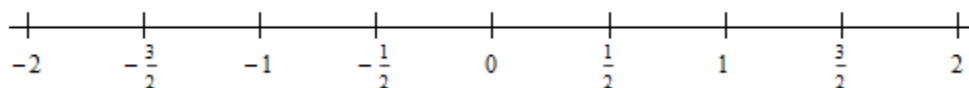
4. Estimate the net area between $f(x) = 8x^2 - x^5 - 12$ and the x -axis on $[-2, 2]$ using $n = 8$ and the midpoints of the subintervals for the height of the rectangles. Without looking at a graph of the function on the interval does it appear that more of the area is above or below the x -axis?

Step 1

First let's estimate the area between the function and the x -axis on the interval. The widths of each of the subintervals for this problem are,

$$\Delta x = \frac{2 - (-2)}{8} = \frac{1}{2}$$

We don't need to actually graph the function to do this problem. It would probably help to have a number line showing subintervals however. Here is that number line.



Now, we'll be using midpoints of each of these subintervals to determine the height of each of the rectangles.

The area between the function and the x -axis is then approximately,

$$\text{Area} \approx \frac{1}{2} f\left(-\frac{7}{4}\right) + \frac{1}{2} f\left(-\frac{5}{4}\right) + \frac{1}{2} f\left(-\frac{3}{4}\right) + \frac{1}{2} f\left(-\frac{1}{4}\right) + \frac{1}{2} f\left(\frac{1}{4}\right) + \frac{1}{2} f\left(\frac{3}{4}\right) + \frac{1}{2} f\left(\frac{5}{4}\right) + \frac{1}{2} f\left(\frac{7}{4}\right) = \boxed{-6}$$

We'll leave it to you to check all the function evaluations. They get a little messy, but after all the arithmetic is done we get a net area of -6.

Step 2

Now, as we (hopefully) recall from the discussion in this section area above the x -axis is positive and area below the x -axis is negative. In this case we have estimated that the net area is -6 and so, assuming that our estimate is accurate, it looks like we should have more area below the x -axis as above it.

Graph

For reference purposes here is the graph of the function with the area shaded in and as we can see it does appear that there is slightly more area below as above the x -axis.

