Section 5-4 : More Substitution Rule

1. Evaluate $\int 4\sqrt{5+9t} + 12(5+9t)^7\,dt$.

Hint : Each term seems to require the same substitution and recall that the same substitution can be used in multiple terms of an integral if we need to.

Step 1
Don't get too excited about the fact that there are two terms in this integrand. Each term requires the same substitution,

$$u = 5 + 9t$$

so we'll simply use that in both terms.

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 2
Here is the differential work for the substitution.

$$du = 9\,dt \qquad \to \qquad dt = \tfrac{1}{9}\,du$$

Doing the substitution and evaluating the integral gives,

$$\int \left[4u^{\frac{1}{2}} + 12u^7\right]\left(\tfrac{1}{9}\right)du = \tfrac{1}{9}\left[\tfrac{8}{3}u^{\frac{3}{2}} + \tfrac{3}{2}u^8\right] + c = \boxed{\tfrac{1}{9}\left[\tfrac{8}{3}(5+9t)^{\frac{3}{2}} + \tfrac{3}{2}(5+9t)^8\right] + c}$$

Be careful when dealing with the *dt* substitution here. Make sure that the $\frac{1}{9}$ gets multiplied times the whole integrand and not just one of the terms. You can do this either by using parenthesis (as we've done here) or pulling the $\frac{1}{9}$ completely out of the integral.

Do not forget to go back to the original variable after evaluating the integral!

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2. Evaluate $\int 7x^3\cos\left(2+x^4\right) - 8x^3\mathbf{e}^{2+x^4}\,dx$.

Hint : Each term seems to require the same substitution and recall that the same substitution can be used in multiple terms of an integral if we need to.

Step 1
Don't get too excited about the fact that there are two terms in this integrand. Each term requires the same substitution,

$$u = 2 + x^4$$

so we'll simply use that in both terms.

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 2

Here is the differential work for the substitution.

$$du = 4x^3\,dx \qquad \to \qquad x^3 dx = \tfrac{1}{4}\,du$$

Before doing the actual substitution it might be convenient to factor an x^3 out of the integrand as follows.

$$\int 7x^3\cos\left(2+x^4\right) - 8x^3\mathbf{e}^{2+x^4}\,dx = \int\left[7\cos\left(2+x^4\right) - 8\mathbf{e}^{2+x^4}\right]x^3 dx$$

Doing this should make the differential part (*i.e.* the *du* part) of the substitution clearer.

Now, doing the substitution and evaluating the integral gives,

$$\begin{aligned}\int 7x^3\cos\left(2+x^4\right) - 8x^3\mathbf{e}^{2+x^4}\,dx &= \tfrac{1}{4}\int 7\cos\left(u\right) - 8\mathbf{e}^{u}\,du \\ &= \tfrac{1}{4}\left[7\sin\left(u\right) - 8\mathbf{e}^{u}\right] + c = \boxed{\tfrac{1}{4}\left[7\sin\left(2+x^4\right) - 8\mathbf{e}^{2+x^4}\right] + c}\end{aligned}$$

Be careful when dealing with the *dx* substitution here. Make sure that the $\frac{1}{4}$ gets multiplied times the whole integrand and not just one of the terms. You can do this either by using parenthesis around the whole integrand or pulling the $\frac{1}{4}$ completely out of the integral (as we've done here).

Do not forget to go back to the original variable after evaluating the integral!

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3. Evaluate $\displaystyle\int \frac{6\mathbf{e}^{7w}}{\left(1-8\mathbf{e}^{7w}\right)^3} + \frac{14\mathbf{e}^{7w}}{1-8\mathbf{e}^{7w}}\,dw$.

Hint : Each term seems to require the same substitution and recall that the same substitution can be used in multiple terms of an integral if we need to.

Step 1

Don't get too excited about the fact that there are two terms in this integrand. Each term requires the same substitution,

$$u = 1 - 8\mathbf{e}^{7w}$$

so we'll simply use that in both terms.

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 2

Here is the differential work for the substitution.

$$du = -56\mathbf{e}^{7w}dw \qquad \to \qquad \mathbf{e}^{7w}dw = -\tfrac{1}{56}du$$

Before doing the actual substitution it might be convenient to factor an $\mathbf{e}^{7w}$ out of the integrand as follows.

$$\int \frac{6\mathbf{e}^{7w}}{\left(1-8\mathbf{e}^{7w}\right)^3} + \frac{14\mathbf{e}^{7w}}{1-8\mathbf{e}^{7w}}\,dw = \int \left[\frac{6}{\left(1-8\mathbf{e}^{7w}\right)^3} + \frac{14}{1-8\mathbf{e}^{7w}}\right]\mathbf{e}^{7w}\,dw$$

Doing this should make the differential part (*i.e.* the *du* part) of the substitution clearer.

Now, doing the substitution and evaluating the integral gives,

$$\int \frac{6\mathbf{e}^{7w}}{\left(1-8\mathbf{e}^{7w}\right)^3} + \frac{14\mathbf{e}^{7w}}{1-8\mathbf{e}^{7w}}\,dw = -\tfrac{1}{56}\int 6u^{-3} + \frac{14}{u}\,du = -\tfrac{1}{56}\left(-3u^{-2} + 14\ln\left|u\right|\right) + c$$

$$= \boxed{-\tfrac{1}{56}\left(-3\left(1-8\mathbf{e}^{7w}\right)^{-2} + 14\ln\left|1-8\mathbf{e}^{7w}\right|\right) + c}$$

Be careful when dealing with the *dw* substitution here. Make sure that the $-\frac{1}{56}$ gets multiplied times the whole integrand and not just one of the terms. You can do this either by using parenthesis around the whole integrand or pulling the $-\frac{1}{56}$ completely out of the integral (as we've done here).

Do not forget to go back to the original variable after evaluating the integral!

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4. Evaluate $\int x^4 - 7x^5\cos\left(2x^6 + 3\right)dx$.

Hint : Recall that terms that do not need substitutions should not be in the integral when the substitution is being done. At this point we should know how to "break" integrals up so that we can get the terms that require a substitution into a one integral and those that don't into another integral.

Step 1
Clearly the first term does not need a substitution while the second term does need a substitution. So, we'll first need to split up the integral as follows.

$$\int x^4 - 7x^5 \cos\left(2x^6 + 3\right) dx = \int x^4\,dx - \int 7x^5 \cos\left(2x^6 + 3\right) dx$$

Step 2
The substitution needed for the second integral is then,

$$u = 2x^6 + 3$$

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 3
Here is the differential work for the substitution.

$$du = 12x^5 dx \qquad \to \qquad x^5 dx = \tfrac{1}{12}\,du$$

Now, doing the substitution and evaluating the integrals gives,

$$\begin{aligned}\int x^4 - 7x^5 \cos\left(2x^6 + 3\right) dx &= \int x^4\,dx - \tfrac{7}{12}\int \cos\left(u\right) du = \tfrac{1}{5}x^5 - \tfrac{7}{12}\sin\left(u\right) + c \\ &= \boxed{\tfrac{1}{5}x^5 - \tfrac{7}{12}\sin\left(2x^6 + 3\right) + c}\end{aligned}$$

Do not forget to go back to the original variable after evaluating the integral!

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5. Evaluate $\displaystyle\int \mathbf{e}^z + \frac{4\sin\left(8z\right)}{1 + 9\cos\left(8z\right)}\,dz$.

Hint : Recall that terms that do not need substitutions should not be in the integral when the substitution is being done. At this point we should know how to "break" integrals up so that we can get the terms that require a substitution into a one integral and those that don't into another integral.

Step 1
Clearly the first term does not need a substitution while the second term does need a substitution. So, we'll first need to split up the integral as follows.

$$\int \mathbf{e}^z + \frac{4\sin\left(8z\right)}{1 + 9\cos\left(8z\right)}\,dz = \int \mathbf{e}^z\,dz + \int \frac{4\sin\left(8z\right)}{1 + 9\cos\left(8z\right)}\,dz$$

Step 2
The substitution needed for the second integral is then,

$$u = 1 + 9\cos(8z)$$

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 3
Here is the differential work for the substitution.

$$du = -72\sin(8z)\,dz \qquad \to \qquad \sin(8z)\,dz = -\tfrac{1}{72}\,du$$

Now, doing the substitution and evaluating the integrals gives,

$$\int \mathbf{e}^{z} + \frac{4\sin(8z)}{1+9\cos(8z)}\,dz = \int \mathbf{e}^{z}\,dz - \frac{4}{72}\int \frac{1}{u}\,du = \boxed{\mathbf{e}^{z} - \tfrac{1}{18}\ln\left|1+9\cos(8z)\right| + c}$$

Do not forget to go back to the original variable after evaluating the integral!

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6. Evaluate $\int 20\mathbf{e}^{2-8w}\sqrt{1+\mathbf{e}^{2-8w}} + 7w^{3} - 6\sqrt[3]{w}\,dw$.

Hint : Recall that terms that do not need substitutions should not be in the integral when the substitution is being done. At this point we should know how to "break" integrals up so that we can get the terms that require a substitution into a one integral and those that don't into another integral.

Step 1
Clearly the first term needs a substitution while the second and third terms don't. So, we'll first need to split up the integral as follows.

$$\int 20\mathbf{e}^{2-8w}\sqrt{1+\mathbf{e}^{2-8w}} + 7w^{3} - 6\sqrt[3]{w}\,dw = \int 20\mathbf{e}^{2-8w}\sqrt{1+\mathbf{e}^{2-8w}}\,dw + \int 7w^{3} - 6\sqrt[3]{w}\,dw$$

Step 2
The substitution needed for the first integral is then,

$$u = 1 + \mathbf{e}^{2-8w}$$

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 3
Here is the differential work for the substitution.

$$du = -8\mathbf{e}^{2-8w}dw \qquad \to \qquad \mathbf{e}^{2-8w}dw = -\tfrac{1}{8}du$$

Now, doing the substitutions and evaluating the integrals gives,

$$\begin{aligned}\int 20\mathbf{e}^{2-8w}\sqrt{1+\mathbf{e}^{2-8w}} + 7w^3 - 6\sqrt[3]{w}\,dw &= -\tfrac{20}{8}\int u^{\frac{1}{2}}\,du + \int 7w^3 - 6w^{\frac{1}{3}}\,dw \\ &= -\tfrac{5}{3}u^{\frac{3}{2}} + \tfrac{7}{4}w^4 - \tfrac{9}{2}w^{\frac{4}{3}} + c \\ &= \boxed{-\tfrac{5}{3}\left(1+\mathbf{e}^{2-8w}\right)^{\frac{3}{2}} + \tfrac{7}{4}w^4 - \tfrac{9}{2}w^{\frac{4}{3}} + c}\end{aligned}$$

Do not forget to go back to the original variable after evaluating the integral!

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7. Evaluate $\int \left(4+7t\right)^3 - 9t\sqrt[4]{5t^2+3}\,dt$.

Hint : You can only do one substitution per integral. At this point we should know how to "break" integrals up so that we can get the terms that require different substitutions into different integrals.

Step 1
Clearly each term needs a separate substitution. So, we'll first need to split up the integral as follows.

$$\int \left(4+7t\right)^3 - 9t\sqrt[4]{5t^2+3}\,dt = \int \left(4+7t\right)^3 dt - \int 9t\sqrt[4]{5t^2+3}\,dt$$

Step 2
The substitutions needed for each integral are then,

$$u = 4+7t \qquad\qquad v = 5t^2+3$$

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 3
Here is the differential work for each substitution.

$$du = 7dt \quad \to \quad dt = \tfrac{1}{7}du \qquad\qquad dv = 10t\,dt \quad \to \quad t\,dt = \tfrac{1}{10}dv$$

Now, doing the substitutions and evaluating the integrals gives,

$$\int (4+7t)^3\,dt - \int 9t\,\sqrt[4]{5t^2+3}\,dt = \tfrac{1}{7}\int u^3\,du - \tfrac{9}{10}\int v^{\frac{1}{4}}\,dv = \tfrac{1}{28}u^4 - \tfrac{18}{25}v^{\frac{5}{4}} + c$$
$$= \boxed{\tfrac{1}{28}(4+7t)^4 - \tfrac{18}{25}\left(5t^2+3\right)^{\frac{5}{4}} + c}$$

Do not forget to go back to the original variable after evaluating the integral!

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8. Evaluate $\displaystyle\int \frac{6x-x^2}{x^3-9x^2+8} - \csc^2\left(\frac{3x}{2}\right)dx$.

Hint : You can only do one substitution per integral. At this point we should know how to "break" integrals up so that we can get the terms that require different substitutions into different integrals.

Step 1
Clearly each term needs a separate substitution. So, we'll first need to split up the integral as follows.

$$\int \frac{6x-x^2}{x^3-9x^2+8} - \csc^2\left(\frac{3x}{2}\right)dx = \int \frac{6x-x^2}{x^3-9x^2+8}\,dx - \int \csc^2\left(\frac{3x}{2}\right)dx$$

Step 2
The substitutions needed for each integral are then,

$$u = x^3 - 9x^2 + 8 \qquad\qquad v = \frac{3x}{2}$$

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 3
Here is the differential work for each substitution.

$$du = \left(3x^2 - 18x\right)dx = -3\left(6x - x^2\right)dx \qquad \rightarrow \qquad \left(6x - x^2\right)dx = -\tfrac{1}{3}\,du$$
$$dv = \tfrac{3}{2}\,dx \qquad \rightarrow \qquad dx = \tfrac{2}{3}\,dv$$

Now, doing the substitutions and evaluating the integrals gives,

$$\int \frac{6x-x^2}{x^3-9x^2+8} - \csc^2\left(\frac{3x}{2}\right)dx = -\frac{1}{3}\int \frac{1}{u}\,du - \frac{2}{3}\int \csc^2(v)\,dv = -\frac{1}{3}\ln|u| + \frac{2}{3}\cot(v) + c$$
$$= \boxed{-\tfrac{1}{3}\ln\left|x^3 - 9x^2 + 8\right| + \tfrac{2}{3}\cot\left(\tfrac{3x}{2}\right) + c}$$

Do not forget to go back to the original variable after evaluating the integral!

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9. Evaluate $\int 7\left(3y+2\right)\left(4y+3y^2\right)^3+\sin\left(3+8y\right)dy$.

Hint : You can only do one substitution per integral. At this point we should know how to "break" integrals up so that we can get the terms that require different substitutions into different integrals.

Step 1
Clearly each term needs a separate substitution. So, we'll first need to split up the integral as follows.

$$\int 7\left(3y+2\right)\left(4y+3y^2\right)^3+\sin\left(3+8y\right)dy=\int 7\left(3y+2\right)\left(4y+3y^2\right)^3 dy+\int \sin\left(3+8y\right)dy$$

Step 2
The substitutions needed for each integral are then,

$$u=4y+3y^2 \qquad\qquad v=3+8y$$

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 3
Here is the differential work for each substitution.

$$\begin{aligned} du&=\left(4+6y\right)dy=2\left(3y+2\right)dy &&\to && \left(3y+2\right)dy=\tfrac{1}{2}du \\ dv&=8\,dy &&\to && dy=\tfrac{1}{8}dv \end{aligned}$$

Now, doing the substitutions and evaluating the integrals gives,

$$\begin{aligned} \int 7\left(3y+2\right)\left(4y+3y^2\right)^3+\sin\left(3+8y\right)dy &= \tfrac{7}{2}\int u^3du+\tfrac{1}{8}\int\sin\left(v\right)dv=\tfrac{7}{8}u^4-\tfrac{1}{8}\cos\left(v\right)+c \\ &= \boxed{\tfrac{7}{8}\left(4y+3y^2\right)^4-\tfrac{1}{8}\cos\left(3+8y\right)+c} \end{aligned}$$

Do not forget to go back to the original variable after evaluating the integral!

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10. Evaluate $\int \sec^2\left(2t\right)\left[9+7\tan\left(2t\right)-\tan^2\left(2t\right)\right]dt$.

Hint : Don't let this one fool you. This is simply an integral that requires you to use the same substitution more than once.

Step 1
This integral can be a little daunting at first glance. To do it all we need to notice is that the derivative of $\tan\left(x\right)$ is $\sec^2\left(x\right)$ and we can notice that there is a $\sec^2\left(2t\right)$ times the remaining portion of the integrand and that portion only contains constants and tangents.

So, it looks like the substitution is then,

$$u = \tan\left(2t\right)$$

If you aren't comfortable with the basic substitution mechanics you should work some problems in the previous section as we'll not be putting in as much detail with regards to the basics in this section. The problems in this section are intended for those that are fairly comfortable with the basic mechanics of substitutions and will involve some more "advanced" substitutions.

Step 2
Here is the differential work for the substitution.

$$du = 2\sec^2\left(2t\right)dt \qquad \to \qquad \sec^2\left(2t\right)dt = \tfrac{1}{2}du$$

Now, doing the substitution and evaluating the integrals gives,

$$\begin{aligned}\int \sec^2\left(2t\right)\left[9 + 7\tan\left(2t\right) - \tan^2\left(2t\right)\right]dt &= \tfrac{1}{2}\int 9 + 7u - u^2 du = \tfrac{1}{2}\left(9u + \tfrac{7}{2}u^2 - \tfrac{1}{3}u^3\right) + c \\ &= \boxed{\tfrac{1}{2}\left(9\tan\left(2t\right) + \tfrac{7}{2}\tan^2\left(2t\right) - \tfrac{1}{3}\tan^3\left(2t\right)\right) + c}\end{aligned}$$

Do not forget to go back to the original variable after evaluating the integral!

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11. Evaluate $\displaystyle\int \frac{8-w}{4w^2+9}\,dw$.

Hint : With the integrand written as it is here this problem can't be done.

Step 1
As written we can't do this problem. In order to do this integral we'll need to rewrite the integral as follows.

$$\int \frac{8-w}{4w^2+9}\,dw = \int \frac{8}{4w^2+9}\,dw - \int \frac{w}{4w^2+9}\,dw$$

Step 2
Now, the first integral looks like it might be an inverse tangent (although we'll need to do a rewrite of that integral) and the second looks like it's a logarithm (with a quick substitution).

So, here is the rewrite on the first integral.

$$\int \frac{8-w}{4w^2+9}\,dw = \frac{8}{9}\int \frac{1}{\frac{4}{9}w^2+1}\,dw - \int \frac{w}{4w^2+9}\,dw$$

Step 3

Now we'll need a substitution for each integral. Here are the substitutions we'll need for each integral.

$$u = \tfrac{2}{3}w \quad \left(\text{so } u^2 = \tfrac{4}{9}w^2\right) \qquad\qquad v = 4w^2+9$$

Step 4

Here is the differential work for the substitution.

$$du = \tfrac{2}{3}dw \quad \to \quad dw = \tfrac{3}{2}du \qquad\qquad dv = 8w\,dw \quad \to \quad w\,dw = \tfrac{1}{8}dv$$

Now, doing the substitutions and evaluating the integrals gives,

$$\begin{aligned}\int \frac{8-w}{4w^2+9}\,dw &= \frac{8}{9}\left(\frac{3}{2}\right)\int \frac{1}{u^2+1}\,du - \frac{1}{8}\int \frac{1}{v}\,dv = \tfrac{4}{3}\tan^{-1}(u) - \tfrac{1}{8}\ln|v| + c \\ &= \boxed{\tfrac{4}{3}\tan^{-1}\left(\tfrac{2}{3}w\right) - \tfrac{1}{8}\ln\left|4w^2+9\right| + c}\end{aligned}$$

Do not forget to go back to the original variable after evaluating the integral!

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12. Evaluate $\displaystyle\int \frac{7x+2}{\sqrt{1-25x^2}}\,dx$.

Hint : With the integrand written as it is here this problem can't be done.

Step 1

As written we can't do this problem. In order to do this integral we'll need to rewrite the integral as follows.

$$\int \frac{7x+2}{\sqrt{1-25x^2}}\,dx = \int \frac{7x}{\sqrt{1-25x^2}}\,dx + \int \frac{2}{\sqrt{1-25x^2}}\,dx$$

Step 2

Now, the second integral looks like it might be an inverse sine (although we'll need to do a rewrite of that integral) and the first looks like a simple substitution will work for us.

So, here is the rewrite on the second integral.

$$\int \frac{7x+2}{\sqrt{1-25x^2}}\,dx = \int \frac{7x}{\sqrt{1-25x^2}}\,dx + 2\int \frac{1}{\sqrt{1-(5x)^2}}\,dx$$

Step 3
Now we'll need a substitution for each integral. Here are the substitutions we'll need for each integral.

$$u = 1-25x^2 \qquad\qquad v = 5x \quad \left(\text{so } v^2 = 25x^2\right)$$

Step 4
Here is the differential work for the substitution.

$$du = -50x\,dx \quad \rightarrow \quad x\,dx = -\tfrac{1}{50}\,du \qquad\qquad dv = 5dx \quad \rightarrow \quad dx = \tfrac{1}{5}\,dv$$

Now, doing the substitutions and evaluating the integrals gives,

$$\begin{aligned}\int \frac{7x+2}{\sqrt{1-25x^2}}\,dx &= -\frac{7}{50}\int u^{-\frac{1}{2}}\,du + \frac{2}{5}\int \frac{1}{\sqrt{1-v^2}}\,dv = -\frac{7}{25}u^{\frac{1}{2}} + \frac{2}{5}\sin^{-1}(v) + c \\ &= \boxed{-\tfrac{7}{25}\left(1-25x^2\right)^{\frac{1}{2}} + \tfrac{2}{5}\sin^{-1}(5x) + c}\end{aligned}$$

Do not forget to go back to the original variable after evaluating the integral!

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13. Evaluate $\int z^7 \left(8+3z^4\right)^8 dz$.

Hint : Use the "obvious" substitution and don't forget that the substitution can be used more than once and in different ways.

Step 1
Okay, the "obvious" substitution here is probably,

$$u = 8+3z^4 \quad \rightarrow \quad du = 12z^3dz \quad \rightarrow \quad z^3dz = \tfrac{1}{12}\,du$$

however, that doesn't look like it might work because of the z^7.

Step 2
Let's do a quick rewrite of the integrand.

$$\int z^7\left(8+3z^4\right)^8 dz = \int z^4 z^3\left(8+3z^4\right)^8 dz = \int z^4\left(8+3z^4\right)^8 z^3dz$$

Step 3

Now, notice that we can convert all of the z's in the integrand except apparently for the z^4 that is in the front. However, notice from the substitution that we can solve for z^4 to get,

$$z^4 = \tfrac{1}{3}\left(u-8\right)$$

Step 4

With this we can now do the substitution and evaluate the integral.

$$\begin{aligned}\int z^7\left(8+3z^4\right)^8 dz &= \tfrac{1}{12}\int \tfrac{1}{3}\left(u-8\right)u^8du = \tfrac{1}{36}\int u^9 - 8u^8du = \tfrac{1}{36}\left(\tfrac{1}{10}u^{10} - \tfrac{8}{9}u^9\right)+c \\ &= \boxed{\tfrac{1}{36}\left(\tfrac{1}{10}\left(8+3z^4\right)^{10} - \tfrac{8}{9}\left(8+3z^4\right)^9\right)+c}\end{aligned}$$

Do not forget to go back to the original variable after evaluating the integral!