Paul's Online Notes

Section 2.1 : Tangent Lines And Rates Of Change - Practice Problems

1. For the function $f(x) = 3(x+2)^2$ and the point P given by $x = -3$ answer each of the following questions.

(a) For the points Q given by the following values of x compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points P and Q.

(i) -3.5 **(ii)** -3.1 **(iii)** -3.01 **(iv)** -3.001 **(v)** -3.0001

(vi) -2.5 **(vii)** -2.9 **(viii)** -2.99 **(ix)** -2.999 **(x)** -2.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $f(x)$ at $x = -3$ and write down the equation of the tangent line.

[Solution]

2. For the function $g(x) = \sqrt{4x+8}$ and the point P given by $x = 2$ answer each of the following questions.

(a) For the points Q given by the following values of x compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points P and Q.

(i) 2.5 **(ii)** 2.1 **(iii)** 2.01 **(iv)** 2.001 **(v)** 2.0001

(vi) 1.5 **(vii)** 1.9 **(viii)** 1.99 **(ix)** 1.999 **(x)** 1.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $g(x)$ at $x = 2$ and write down the equation of the tangent line.

[Solution]

3. For the function $W(x) = \ln\left(1 + x^4\right)$ and the point P given by $x = 1$ answer each of the following questions.

(a) For the points Q given by the following values of x compute (accurate to at least 8 decimal places) the slope, m_{PQ}, of the secant line through points P and Q.

(i) 1.5 **(ii)** 1.1 **(iii)** 1.01 **(iv)** 1.001 **(v)** 1.0001

(vi) 0.5 **(vii)** 0.9 **(viii)** 0.99 **(ix)** 0.999 **(x)** 0.9999

(b) Use the information from (a) to estimate the slope of the tangent line to $W(x)$ at $x = 1$ and write down the equation of the tangent line.

[Solution]

4. The volume of air in a balloon is given by $V(t) = \dfrac{6}{4t + 1}$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the volume of air in the balloon between $t = 0.25$ and the following values of t.

(i) 1 **(ii)** 0.5 **(iii)** 0.251 **(iv)** 0.2501 **(v)** 0.25001

(vi) 0 **(vii)** 0.1 **(viii)** 0.249 **(ix)** 0.2499 **(x)** 0.24999

(b) Use the information from (a) to estimate the instantaneous rate of change of the volume of air in the balloon at $t = 0.25$.

[Solution]

5. The population (in hundreds) of fish in a pond is given by $P(t) = 2t + \sin(2t - 10)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average rate of change of the population of fish between $t = 5$ and the following values of t. Make sure your calculator is set to radians for the computations.

(i) 5.5 **(ii)** 5.1 **(iii)** 5.01 **(iv)** 5.001 **(v)** 5.0001

(vi) 4.5 **(vii)** 4.9 **(viii)** 4.99 **(ix)** 4.999 **(x)** 4.9999

(b) Use the information from (a) to estimate the instantaneous rate of change of the population of the fish at $t = 5$.

[Solution]

6. The position of an object is given by $s(t) = \cos^2\left(\frac{3t - 6}{2}\right)$ answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t = 2$ and the following values of t. Make sure your calculator is set to radians for the computations.

(i) 2.5 **(ii)** 2.1 **(iii)** 2.01 **(iv)** 2.001 **(v)** 2.0001

(vi) 1.5 **(vii)** 1.9 **(viii)** 1.99 **(ix)** 1.999 **(x)** 1.9999

(b) Use the information from (a) to estimate the instantaneous velocity of the object at $t = 2$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

[Solution]

7. The position of an object is given by $s\left(t\right)=\left(8-t\right)\left(t+6\right)^{\frac{3}{2}}$. Note that a negative position here simply means that the position is to the left of the "zero position" and is perfectly acceptable. Answer each of the following questions.

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t=10$ and the following values of t.

(i) 10.5 **(ii)** 10.1 **(iii)** 10.01 **(iv)** 10.001 **(v)** 10.0001

(vi) 9.5 **(vii)** 9.9 **(viii)** 9.99 **(ix)** 9.999 **(x)** 9.9999

(b) Use the information from (a) to estimate the instantaneous velocity of the object at $t=10$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

[Solution]

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