3.4 Linear and Angular Speed

Linear speed is how fast the position of an object is changing.

$$v = \frac{s}{t}$$ v = distance / time

Angular speed is how fast an angle is changing.

$$\omega = \frac{\theta}{t}, \quad \theta \text{ in radians}$$ (How fast something is turning)

An object traveling in a cirular motion has BOTH linear and angluar speed. The linear speed is dependent on the radius of the circle and how fast the object is rotating (ω).

$$v = r\omega$$

Use the formula $\omega = \frac{\theta}{t}$ to find the value of the missing variable.

6. $\omega = \frac{\pi}{4}$ radian per min, $t = 5$ min

$\omega = \frac{\theta}{t}$

$(5)\frac{\pi}{4} = \frac{\theta}{5}(5)$

$\frac{5\pi}{4} = \theta$

10. $\theta = \frac{3\pi}{8}$ radians, $\omega = \frac{\pi}{24}$ radian per min

Find t

$\omega = \frac{\theta}{t}$

$t \cdot \frac{\pi}{24} = \frac{\frac{3\pi}{8}}{t} \cdot t$

$\left(\frac{24}{\pi}\right) t \cdot \frac{\pi}{24} = \frac{3\pi}{8}\left(\frac{24}{\pi}\right)$

$t = 9$ min

Use the formula $v = r\omega$ to find the value of the missing variable.

14. $r = 8$ cm, $\omega = \frac{9\pi}{5}$ radians per sec

$V = 8\left(\frac{9\pi}{5}\right)$

$V = 45.24 \frac{cm}{sec}$

16. $v = 18$ ft per sec, $r = 3$ ft

$V = r\omega$

$18 = 3\omega$

$6 \frac{rad}{sec} = \omega$

Use the formula $s = r\omega t$ to find the value of the missing variable.

22. $s = \frac{12\pi}{5}$ m, $r = \frac{3}{2}$ m, $\omega = \frac{2\pi}{5}$ radians per sec

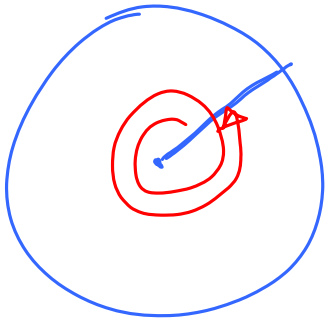
$\frac{12\pi}{5} = \frac{3}{2} \cdot \left(\frac{2\pi}{5}\right) t$

$12 = 3t$

$4 = t$

Find ω for the following:

26. A line from the center to the edge of a CD revolving 300 times per minute.



$\omega =$

$\omega = \frac{300 \text{ revolutions}}{1 \text{ min.}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}}$

$\omega = 600\pi \frac{\text{rad}}{\text{min}}$

28. the second hand of a clock Find ω

$\omega = \frac{1 \text{ rev}}{60 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}}$ $\omega = \frac{1 \text{ rev}}{1 \text{ min}}$

$\omega = \frac{\pi}{30} \frac{\text{rad}}{\text{sec}}$ or $\omega = 2\pi \frac{\text{rad}}{\text{min}}$

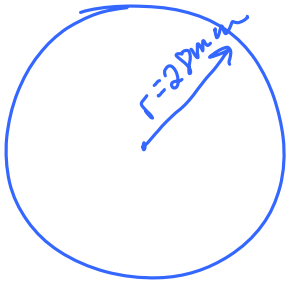
Find v for each of the following:

30. the tip of the second hand of a clock, if the hand is 28 mm long.

$V = r\omega$ – from previous prob.

$V = 28\left(\frac{\pi}{30}\frac{\text{rad}}{\text{sec}}\right)$

$V = 2.93\ \frac{\text{mm}}{\text{sec}}$



32. a point of the tread of a tire radius 18 cm, rotating 35 times per min.

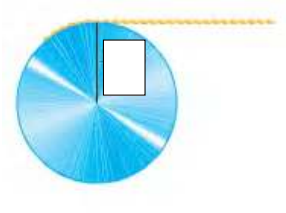
Find v

$\omega = \frac{35\ \text{rev}}{1\ \text{min}}$

$V = r\omega$

$V = 18\left(\frac{35\ \text{rev}}{1\ \text{min}} \cdot \frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)$

$V = 1260\pi\ \frac{\text{cm}}{\text{min}}$ or $3958.4\ \frac{\text{cm}}{\text{min}}$



like #39

A pulley has a radius of 15 cm. Suppose it takes 25 sec for 75 cm of belt to go around the pulley. Find the linear speed of the belt in centimeters per second. Find the angular speed of the pulley in radians per second.

$r = 15$ $\quad t = 25$ sec $\quad s = 75$ cm (distance)

v and ω

$v = \frac{s}{t}$

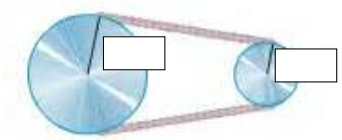
$v = \frac{75}{25} = \boxed{3 \frac{\text{cm}}{\text{sec}}}$

$v = r\omega$

$3 = (15)(\omega)$

$\frac{1}{5} = \omega$

$\boxed{.2 \frac{\text{rad}}{\text{sec}} = \omega}$



like #40

S = same

linear velocity same

The two pulleys have radii 20 cm and 6 cm, respectively. The smaller pulley rotates 30 times in 12 seconds. Find the angular speed of each pulley in radians per second.

BIG Pulley

$V = 30\pi \frac{cm}{sec}$

$r = 20 cm$

$\omega = ?$

$V = r\omega$

$30\pi = 20(\omega)$

$\frac{3}{2}\pi = \omega$ BIG pulley

ω for small pulley

$\omega = \frac{30 rev}{12 sec} \cdot \frac{2\pi}{1 rev}$

$\omega = 5\pi \frac{rad}{rev}$ small pulley

$V = r\omega$

$V = 6(5\pi) = 30\pi \frac{cm}{sec}$ BOTH

A thread is being pulled off a spool at the rate of 75 cm per sec. Find the radius of the spool if it makes 110 revolutions per min.

r ω v

$v = r\omega$

$v = \frac{75\text{ cm}}{\text{sec}}$

$\omega = \frac{110\text{ rev}}{\text{min}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} \cdot \frac{1\text{ min}}{60\text{ sec}}$

$\omega = \frac{11}{3}\pi \frac{\text{rad}}{\text{sec}}$

$\omega = 11.5 \frac{\text{rad}}{\text{sec}}$

$75 = r(11.5)$

$6.5\text{ cm} = r$