Chapter 5 : Integrals

Here are a set of practice problems for the Integrals chapter of the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

11. If you'd like a pdf document containing the solutions the download tab on the website contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems.

12. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Indefinite Integrals – In this section we will start off the chapter with the definition and properties of indefinite integrals. We will not be computing many indefinite integrals in this section. This section is devoted to simply defining what an indefinite integral is and to give many of the properties of the indefinite integral. Actually computing indefinite integrals will start in the next section.

Computing Indefinite Integrals – In this section we will compute some indefinite integrals. The integrals in this section will tend to be those that do not require a lot of manipulation of the function we are integrating in order to actually compute the integral. As we will see starting in the next section many integrals do require some manipulation of the function before we can actually do the integral. We will also take a quick look at an application of indefinite integrals.

Substitution Rule for Indefinite Integrals – In this section we will start using one of the more common and useful integration techniques – The Substitution Rule. With the substitution rule we will be able integrate a wider variety of functions. The integrals in this section will all require some manipulation of the function prior to integrating unlike most of the integrals from the previous section where all we really needed were the basic integration formulas.

More Substitution Rule – In this section we will continue to look at the substitution rule. The problems in this section will tend to be a little more involved than those in the previous section.

Area Problem – In this section we start off with the motivation for definite integrals and give one of the interpretations of definite integrals. We will be approximating the amount of area that lies between a function and the \(x\)-axis. As we will see in the next section this problem will lead us to the definition of the definite integral and will be one of the main interpretations of the definite integral that we'll be looking at in this material.

Definition of the Definite Integral – In this section we will formally define the definite integral, give many of its properties and discuss a couple of interpretations of the definite integral. We will also look at the first part of the Fundamental Theorem of Calculus which shows the very close relationship between derivatives and integrals.

Computing Definite Integrals – In this section we will take a look at the second part of the Fundamental Theorem of Calculus. This will show us how we compute definite integrals without using (the often very unpleasant) definition. The examples in this section can all be done with a basic knowledge of indefinite integrals and will not require the use of the substitution rule. Included in the examples in this section are computing definite integrals of piecewise and absolute value functions.

Substitution Rule for Definite Integrals – In this section we will revisit the substitution rule as it applies to definite integrals. The only real requirements to being able to do the examples in this section are being able to do the substitution rule for indefinite integrals and understanding how to compute definite integrals in general.

Section 5-1 : Indefinite Integrals

1. Evaluate each of the following indefinite integrals.
   (a) $\int 6x^5 - 18x^2 + 7\,dx$
   (b) $\int 6x^5\,dx - 18x^2 + 7$

2. Evaluate each of the following indefinite integrals.
   (a) $\int 40x^3 + 12x^2 - 9x + 14\,dx$
   (b) $\int 40x^3 + 12x^2 - 9x\,dx + 14$
   (c) $\int 40x^3 + 12x^2\,dx - 9x + 14$

For problems 3 – 5 evaluate the indefinite integral.

3. $\int 12t^7 - t^2 - t + 3\,dt$

4. $\int 10w^4 + 9w^3 + 7w\,dw$

5. $\int z^6 + 4z^4 - z^2\,dz$

6. Determine $f(x)$ given that $f'(x) = 6x^8 - 20x^4 + x^2 + 9$.

7. Determine $h(t)$ given that $h'(t) = t^4 - t^3 + t^2 + t - 1$.

Section 5-2 : Computing Indefinite Integrals

For problems 1 – 21 evaluate the given integral.

1. $\int 4x^6 - 2x^3 + 7x - 4\,dx$

2. $\int z^7 - 48z^{11} - 5z^{16}\,dz$

3. $\int 10t^{-3} + 12t^{-9} + 4t^3\,dt$

4. $\int w^{-2} + 10w^{-5} - 8\,dw$

5. $\int 12\,dy$

6. $\int \sqrt[3]{w} + 10\sqrt[5]{w^3}\,dw$

7. $\int \sqrt{x^7} - 7\sqrt[6]{x^5} + 17\sqrt[3]{x^{10}}\,dx$

8. $\int \frac{4}{x^2} + 2 - \frac{1}{8x^3}\,dx$

9. $\int \frac{7}{3y^6} + \frac{1}{y^{10}} - \frac{2}{\sqrt[3]{y^4}}\,dy$

10. $\int \left(t^2 - 1\right)\left(4 + 3t\right)dt$

11. $\int \sqrt{z}\left(z^2 - \frac{1}{4z}\right)dz$

12. $\int \frac{z^8 - 6z^5 + 4z^3 - 2}{z^4}\,dz$

13. $\int \frac{x^4 - \sqrt[3]{x}}{6\sqrt{x}}\,dx$

14. $\int \sin\left(x\right) + 10\csc^2\left(x\right)dx$

15. $\int 2\cos(w) - \sec(w)\tan(w)\,dw$

16. $\int 12 + \csc(\theta)\left[\sin(\theta) + \csc(\theta)\right]d\theta$

17. $\int 4\mathbf{e}^{z} + 15 - \frac{1}{6z}\,dz$

18. $\int t^{3} - \frac{\mathbf{e}^{-t} - 4}{\mathbf{e}^{-t}}\,dt$

19. $\int \frac{6}{w^{3}} - \frac{2}{w}\,dw$

20. $\int \frac{1}{1+x^{2}} + \frac{12}{\sqrt{1-x^{2}}}\,dx$

21. $\int 6\cos(z) + \frac{4}{\sqrt{1-z^{2}}}\,dz$

22. Determine $f(x)$ given that $f'(x) = 12x^{2} - 4x$ and $f(-3) = 17$.

23. Determine $g(z)$ given that $g'(z) = 3z^{3} + \frac{7}{2\sqrt{z}} - \mathbf{e}^{z}$ and $g(1) = 15 - \mathbf{e}$.

24. Determine $h(t)$ given that $h''(t) = 24t^{2} - 48t + 2$, $h(1) = -9$ and $h(-2) = -4$.

Section 5-3 : Substitution Rule for Indefinite Integrals

For problems 1 – 16 evaluate the given integral.

1. $\int (8x-12)\left(4x^2-12x\right)^4\,dx$

2. $\int 3t^{-4}\left(2+4t^{-3}\right)^{-7}\,dt$

3. $\int (3-4w)\left(4w^2-6w+7\right)^{10}\,dw$

4. $\int 5(z-4)\,\sqrt[3]{z^2-8z}\,dz$

5. $\int 90x^2\sin\left(2+6x^3\right)dx$

6. $\int \sec(1-z)\tan(1-z)\,dz$

7. $\int \left(15t^{-2}-5t\right)\cos\left(6t^{-1}+t^2\right)dt$

8. $\int \left(7y-2y^3\right)\mathbf{e}^{y^4-7y^2}\,dy$

9. $\int \frac{4w+3}{4w^2+6w-1}\,dw$

10. $\int \left(\cos(3t)-t^2\right)\left(\sin(3t)-t^3\right)^5\,dt$

11. $\int 4\left(\frac{1}{z}-\mathbf{e}^{-z}\right)\cos\left(\mathbf{e}^{-z}+\ln z\right)dz$

12. $\int \sec^2(v)\,\mathbf{e}^{1+\tan(v)}\,dv$

13. $\int 10\sin(2x)\cos(2x)\sqrt{\cos^2(2x)-5}\,dx$

14. $\int \frac{\csc(x)\cot(x)}{2-\csc(x)}\,dx$

15. $\int \frac{6}{7+y^2}\,dy$

16. $\displaystyle\int \frac{1}{\sqrt{4-9w^2}}\,dw$

17. Evaluate each of the following integrals.

(a) $\displaystyle\int \frac{3x}{1+9x^2}\,dx$

(b) $\displaystyle\int \frac{3x}{\left(1+9x^2\right)^4}\,dx$

(c) $\displaystyle\int \frac{3}{1+9x^2}\,dx$

Section 5-4 : More Substitution Rule

Evaluate each of the following integrals.

1. $\int 4\sqrt{5+9t}+12\left(5+9t\right)^{7}\,dt$

2. $\int 7x^{3}\cos\left(2+x^{4}\right)-8x^{3}\mathbf{e}^{2+x^{4}}\,dx$

3. $\int \frac{6\mathbf{e}^{7w}}{\left(1-8\mathbf{e}^{7w}\right)^{3}}+\frac{14\mathbf{e}^{7w}}{1-8\mathbf{e}^{7w}}\,dw$

4. $\int x^{4}-7x^{5}\cos\left(2x^{6}+3\right)dx$

5. $\int \mathbf{e}^{z}+\frac{4\sin\left(8z\right)}{1+9\cos\left(8z\right)}\,dz$

6. $\int 20\mathbf{e}^{2-8w}\sqrt{1+\mathbf{e}^{2-8w}}+7w^{3}-6\sqrt[3]{w}\,dw$

7. $\int \left(4+7t\right)^{3}-9t\sqrt[4]{5t^{2}+3}\,dt$

8. $\int \frac{6x-x^{2}}{x^{3}-9x^{2}+8}-\csc^{2}\left(\frac{3x}{2}\right)dx$

9. $\int 7\left(3y+2\right)\left(4y+3y^{2}\right)^{3}+\sin\left(3+8y\right)dy$

10. $\int \sec^{2}\left(2t\right)\left[9+7\tan\left(2t\right)-\tan^{2}\left(2t\right)\right]dt$

11. $\int \frac{8-w}{4w^{2}+9}\,dw$

12. $\int \frac{7x+2}{\sqrt{1-25x^{2}}}\,dx$

13. $\int z^{7}\left(8+3z^{4}\right)^{8}\,dz$

Section 5-5 : Area Problem

For problems 1 – 3 estimate the area of the region between the function and the *x*-axis on the given interval using $n = 6$ and using,

(a) the right end points of the subintervals for the height of the rectangles,
(b) the left end points of the subintervals for the height of the rectangles and,
(c) the midpoints of the subintervals for the height of the rectangles.

1. $f(x) = x^3 - 2x^2 + 4$ on $[1,4]$

2. $g(x) = 4 - \sqrt{x^2 + 2}$ on $[-1,3]$

3. $h(x) = -x\cos\left(\frac{x}{3}\right)$ on $[0,3]$

4. Estimate the net area between $f(x) = 8x^2 - x^5 - 12$ and the *x*-axis on $[-2,2]$ using $n = 8$ and the midpoints of the subintervals for the height of the rectangles. Without looking at a graph of the function on the interval does it appear that more of the area is above or below the *x*-axis?

Section 5-6 : Definition of the Definite Integral

For problems 1 & 2 use the definition of the definite integral to evaluate the integral. Use the right end point of each interval for x_i^*.

1. $\int_1^4 2x+3\,dx$

2. $\int_0^1 6x(x-1)\,dx$

3. Evaluate : $\displaystyle\int_4^4 \frac{\cos\left(\mathbf{e}^{3x}+x^2\right)}{x^4+1}\,dx$

For problems 4 & 5 determine the value of the given integral given that $\int_6^{11} f(x)\,dx=-7$ and $\int_6^{11} g(x)\,dx=24$.

4. $\int_{11}^6 9f(x)\,dx$

5. $\int_6^{11} 6g(x)-10f(x)\,dx$

6. Determine the value of $\int_2^9 f(x)\,dx$ given that $\int_5^2 f(x)\,dx=3$ and $\int_5^9 f(x)\,dx=8$.

7. Determine the value of $\int_{-4}^{20} f(x)\,dx$ given that $\int_{-4}^0 f(x)\,dx=-2$, $\int_{31}^0 f(x)\,dx=19$ and $\int_{20}^{31} f(x)\,dx=-21$.

For problems 8 & 9 sketch the graph of the integrand and use the area interpretation of the definite integral to determine the value of the integral.

8. $\int_1^4 3x-2\,dx$

9. $\int_0^5 -4x\,dx$

For problems 10 – 12 differentiate each of the following integrals with respect to *x*.

10. $\int_4^x 9\cos^2\left(t^2-6t+1\right)dt$

11. $\int_{7}^{\sin(6x)} \sqrt{t^2+4}\,dt$

12. $\int_{3x^2}^{-1} \frac{\mathbf{e}^t - 1}{t}\,dt$

Section 5-7 : Computing Definite Integrals

1. Evaluate each of the following integrals.

a. $\int \cos(x) - \frac{3}{x^5}\,dx$

b. $\int_{-3}^{4} \cos(x) - \frac{3}{x^5}\,dx$

c. $\int_{1}^{4} \cos(x) - \frac{3}{x^5}\,dx$

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

2. $\int_{1}^{6} 12x^3 - 9x^2 + 2\,dx$

3. $\int_{-2}^{1} 5z^2 - 7z + 3\,dz$

4. $\int_{3}^{0} 15w^4 - 13w^2 + w\,dw$

5. $\int_{1}^{4} \frac{8}{\sqrt{t}} - 12\sqrt{t^3}\,dt$

6. $\int_{1}^{2} \frac{1}{7z} + \frac{\sqrt[3]{z^2}}{4} - \frac{1}{2z^3}\,dz$

7. $\int_{-2}^{4} x^6 - x^4 + \frac{1}{x^2}\,dx$

8. $\int_{-4}^{-1} x^2(3-4x)\,dx$

9. $\int_{2}^{1} \frac{2y^3 - 6y^2}{y^2}\,dy$

10. $\int_{0}^{\frac{\pi}{2}} 7\sin(t) - 2\cos(t)\,dt$

11. $\int_{0}^{\pi} \sec(z)\tan(z) - 1\,dz$

12. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2(w) - 8\csc(w)\cot(w)\,dw$

13. $\int_0^2 \mathbf{e}^x + \frac{1}{x^2+1}\,dx$

14. $\int_{-5}^{-2} 7\mathbf{e}^y + \frac{2}{y}\,dy$

15. $\int_0^4 f(t)\,dt$ where $f(t) = \begin{cases} 2t & t > 1 \\ 1-3t^2 & t \le 1 \end{cases}$

16. $\int_{-6}^1 g(z)\,dz$ where $g(z) = \begin{cases} 2-z & z > -2 \\ 4\mathbf{e}^z & z \le -2 \end{cases}$

17. $\int_3^6 |2x-10|\,dx$

18. $\int_{-1}^0 |4w+3|\,dw$

Section 5-8 : Substitution Rule for Definite Integrals

Evaluate each of the following integrals, if possible. If it is not possible clearly explain why it is not possible to evaluate the integral.

1. $\int_0^1 3\left(4x+x^4\right)\left(10x^2+x^5-2\right)^6\,dx$

2. $\displaystyle\int_0^{\frac{\pi}{4}} \frac{8\cos\left(2t\right)}{\sqrt{9-5\sin\left(2t\right)}}\,dt$

3. $\int_{\pi}^{0} \sin\left(z\right)\cos^3\left(z\right)dz$

4. $\int_1^4 \sqrt{w}\,\mathbf{e}^{1-\sqrt{w^3}}\,dw$

5. $\displaystyle\int_{-4}^{-1} \sqrt[3]{5-2y}+\frac{7}{5-2y}\,dy$

6. $\int_{-1}^{2} x^3+\mathbf{e}^{\frac{1}{4}x}\,dx$

7. $\int_{\pi}^{\frac{3\pi}{2}} 6\sin\left(2w\right)-7\cos\left(w\right)dw$

8. $\displaystyle\int_1^5 \frac{2x^3+x}{x^4+x^2+1}-\frac{x}{x^2-4}\,dx$

9. $\displaystyle\int_{-2}^{0} t\sqrt{3+t^2}+\frac{3}{\left(6t-1\right)^2}\,dt$

10. $\int_{-2}^{1} \left(2-z\right)^3+\sin\left(\pi z\right)\left[3+2\cos\left(\pi z\right)\right]^3\,dz$