Chapter 6 : Applications of Integrals

Here are a set of practice problems for the Applications of Integrals chapter of the Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

13. If you'd like a pdf document containing the solutions the download tab on the website contains links to pdf's containing the solutions for the full book, chapter and section. At this time, I do not offer pdf's for solutions to individual problems.

14. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution to that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Here is a list of all the sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section.

Average Function Value – In this section we will look at using definite integrals to determine the average value of a function on an interval. We will also give the Mean Value Theorem for Integrals.

Area Between Curves – In this section we'll take a look at one of the main applications of definite integrals in this chapter. We will determine the area of the region bounded by two curves.

Volumes of Solids of Revolution / Method of Rings – In this section, the first of two sections devoted to finding the volume of a solid of revolution, we will look at the method of rings/disks to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the *x* or *y*-axis) around a vertical or horizontal axis of rotation.

Volumes of Solids of Revolution / Method of Cylinders – In this section, the second of two sections devoted to finding the volume of a solid of revolution, we will look at the method of cylinders/shells to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the *x* or *y*-axis) around a vertical or horizontal axis of rotation.

More Volume Problems – In the previous two sections we looked at solids that could be found by treating them as a solid of revolution. Not all solids can be thought of as solids of revolution and, in fact, not all solids of revolution can be easily dealt with using the methods from the previous two sections. So, in this section we'll take a look at finding the volume of some solids that are either not solids of revolutions or are not easy to do as a solid of revolution.

Work – In this section we will look at is determining the amount of work required to move an object subject to a force over a given distance.

Section 6-1 : Average Function Value

For problems 1 & 2 determine f_{avg} for the function on the given interval.

1. $f\left(x\right)=8x-3+5\mathbf{e}^{2-x}$ on $\left[0,2\right]$

2. $f\left(x\right)=\cos\left(2x\right)-\sin\left(\frac{x}{2}\right)$ on $\left[-\frac{\pi}{2},\pi\right]$

3. Find f_{avg} for $f\left(x\right)=4x^2-x+5$ on $\left[-2,3\right]$ and determine the value(s) of *c* in $\left[-2,3\right]$ for which $f\left(c\right)=f_{\text{avg}}$.

Section 6-2 : Area Between Curves

1. Determine the area below $f(x) = 3 + 2x - x^2$ and above the *x*-axis.

2. Determine the area to the left of $g(y) = 3 - y^2$ and to the right of $x = -1$.

For problems 3 – 11 determine the area of the region bounded by the given set of curves.

3. $y = x^2 + 2$, $y = \sin(x)$, $x = -1$ and $x = 2$

4. $y = \dfrac{8}{x}$, $y = 2x$ and $x = 4$

5. $x = 3 + y^2$, $x = 2 - y^2$, $y = 1$ and $y = -2$

6. $x = y^2 - y - 6$ and $x = 2y + 4$

7. $y = x\sqrt{x^2 + 1}$, $y = \mathbf{e}^{-\frac{1}{2}x}$, $x = -3$ and the *y*-axis

8. $y = 4x + 3$, $y = 6 - x - 2x^2$, $x = -4$ and $x = 2$

9. $y = \dfrac{1}{x+2}$, $y = (x+2)^2$, $x = -\dfrac{3}{2}$, $x = 1$

10. $x = y^2 + 1$, $x = 5$, $y = -3$ and $y = 3$

11. $x = \mathbf{e}^{1+2y}$, $x = \mathbf{e}^{1-y}$, $y = -2$ and $y = 1$

Section 6-3 : Volumes of Solids of Revolution / Method of Rings

For each of the following problems use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $y = \sqrt{x}$, $y = 3$ and the *y*-axis about the *y*-axis.

2. Rotate the region bounded by $y = 7 - x^2$, $x = -2$, $x = 2$ and the *x*-axis about the *x*-axis.

3. Rotate the region bounded by $x = y^2 - 6y + 10$ and $x = 5$ about the *y*-axis.

4. Rotate the region bounded by $y = 2x^2$ and $y = x^3$ about the *x*-axis.

5. Rotate the region bounded by $y = 6\mathbf{e}^{-2x}$ and $y = 6 + 4x - 2x^2$ between $x = 0$ and $x = 1$ about the line $y = -2$.

6. Rotate the region bounded by $y = 10 - 6x + x^2$, $y = -10 + 6x - x^2$, $x = 1$ and $x = 5$ about the line $y = 8$.

7. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $x = 24$.

8. Rotate the region bounded by $y = 2x + 1$, $x = 4$ and $y = 3$ about the line $x = -4$.

Section 6-4 : Volumes of Solids of Revolution / Method of Cylinders

For each of the following problems use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $x = \left(y-2\right)^2$, the *x*-axis and the *y*-axis about the *x*-axis.

2. Rotate the region bounded by $y = \dfrac{1}{x}$, $x = \dfrac{1}{2}$, $x = 4$ and the *x*-axis about the *y*-axis.

3. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the *y*-axis. For this problem assume that $x \ge 0$.

4. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the *x*-axis. For this problem assume that $x \ge 0$.

5. Rotate the region bounded by $y = 2x + 1$, $y = 3$ and $x = 4$ about the line $y = 10$.

6. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $y = -8$.

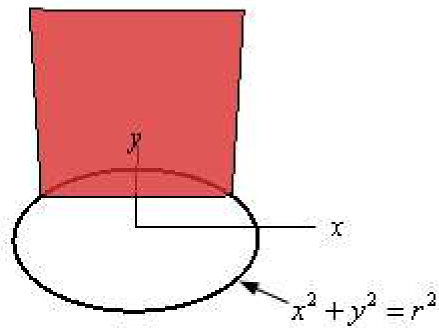
7. Rotate the region bounded by $y = x^2 - 6x + 9$ and $y = -x^2 + 6x - 1$ about the line $x = 8$.

8. Rotate the region bounded by $y = \dfrac{\mathbf{e}^{\frac{1}{2}x}}{x+2}$, $y = 5 - \frac{1}{4}x$, $x = -1$ and $x = 6$ about the line $x = -2$.

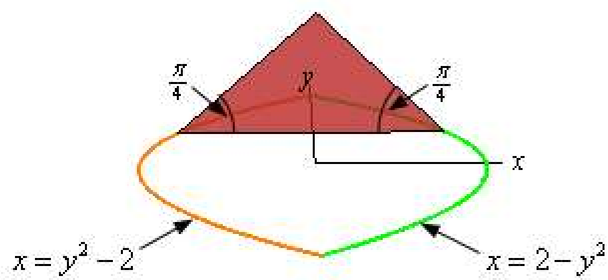
Section 6-5 : More Volume Problems

1. Find the volume of a pyramid of height *h* whose base is an equilateral triangle of length *L*.

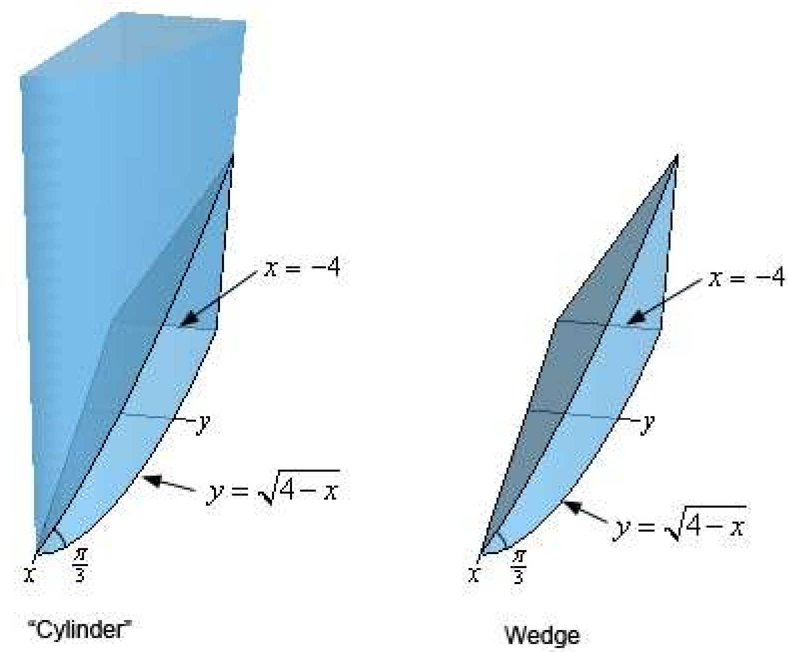
2. Find the volume of the solid whose base is a disk of radius *r* and whose cross-sections are squares. See figure below to see a sketch of the cross-sections.



3. Find the volume of the solid whose base is the region bounded by $x = 2 - y^2$ and $x = y^2 - 2$ and whose cross-sections are isosceles triangles with the base perpendicular to the *y*-axis and the angle between the base and the two sides of equal length is $\frac{\pi}{4}$. See figure below to see a sketch of the cross-sections.



4. Find the volume of a wedge cut out of a "cylinder" whose base is the region bounded by $y = \sqrt{4-x}$, $x = -4$ and the *x*-axis. The angle between the top and bottom of the wedge is $\frac{\pi}{3}$. See the figure below for a sketch of the "cylinder" and the wedge (the positive *x*-axis and positive *y*-axis are shown in the sketch – they are just in a different orientation).



Section 6-6 : Work

1. A force of $F(x) = x^2 - \cos(3x) + 2$, *x* is in meters, acts on an object. What is the work required to move the object from $x = 3$ to $x = 7$?

2. A spring has a natural length of 18 inches and a force of 20 lbs is required to stretch and hold the spring to a length of 24 inches. What is the work required to stretch the spring from a length of 21 inches to a length of 26 inches?

3. A cable with mass ½ kg/meter is lifting a load of 150 kg that is initially at the bottom of a 50 meter shaft. How much work is required to lift the load ¼ of the way up the shaft?

4. A tank of water is 15 feet long and has a cross section in the shape of an equilateral triangle with sides 2 feet long (point of the triangle points directly down). The tank is filled with water to a depth of 9 inches. Determine the amount of work needed to pump all of the water to the top of the tank. Assume that the density of water is 62 lb/ft^3.

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