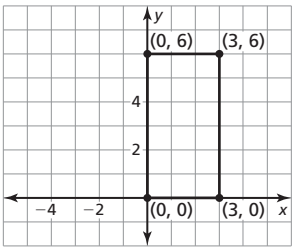
Selected Answers

Chapter 1

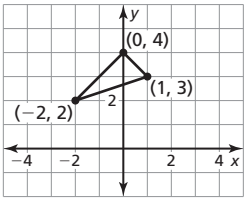
Chapter 1 Maintaining Mathematical Proficiency *(p. 1)*

1. 47 **2.** −46 **3.** $3\frac{3}{5}$ **4.** 4 **5.** 13 **6.** 0

7.



8.



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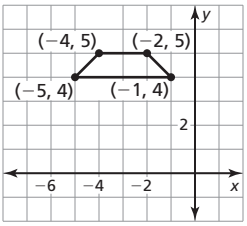


10. *Sample answer:* 12 + 18 ÷ 3 equals 18 when division is performed first and 10 when addition is performed first; yes; If the point (3, 2) is translated up 3 units then reflected across the x-axis, the new coordinate is (3, −5). If it is reflected across the x-axis first then translated up 3, the new coordinate is (3, 1).

1.1 Vocabulary and Core Concept Check *(p. 8)*

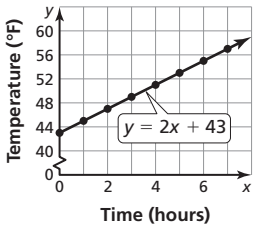
1. parent function

1.1 Monitoring Progress and Modeling with Mathematics *(pp. 8–10)*

3. absolute value; The graph is a vertical stretch with a translation 2 units left and 8 units down; The domain of each function is all real numbers, but the range of f is $y \geq -8$, and the range of the parent function is $y \geq 0$.

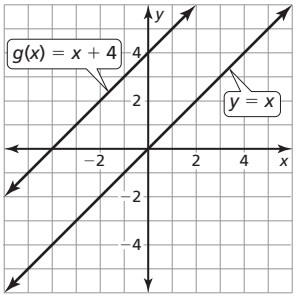
5. linear; The graph is a vertical stretch and a translation 2 units down; The domain and range of each function is all real numbers.

7.



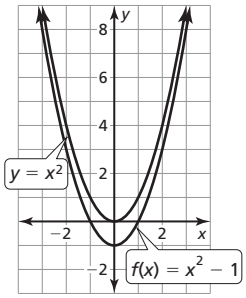
linear; The temperature is increasing by the same amount at each interval.

9.



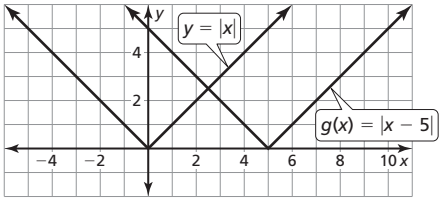
The graph of g is a vertical translation 4 units up of the parent linear function.

11.



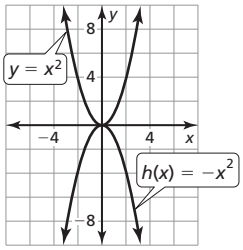
The graph of f is a vertical translation 1 unit down of the parent quadratic function.

13.



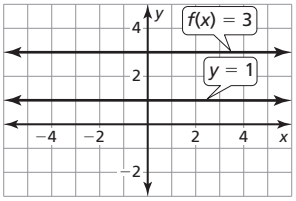
The graph of g is a horizontal translation 5 units right of the parent absolute value function.

15.



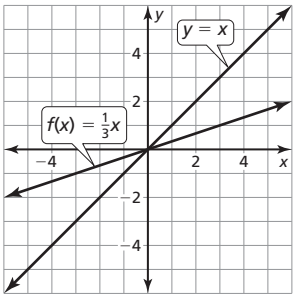
The graph of h is a reflection in the x-axis of the parent quadratic function.

17.



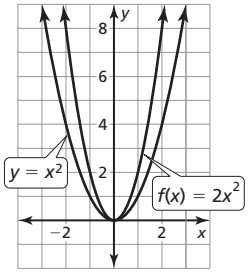
The graph of f is a vertical translation 2 units up of the parent constant function.

19.



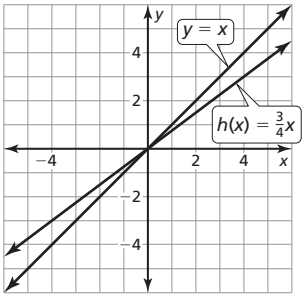
The graph of f is a vertical shrink of the parent linear function.

21.



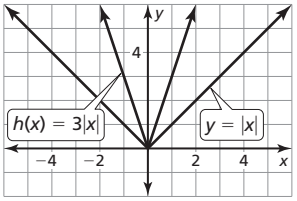
The graph of f is a vertical stretch of the parent quadratic function.

23.



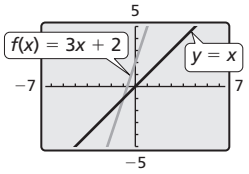
The graph of h is a vertical shrink of the parent linear function.

25.



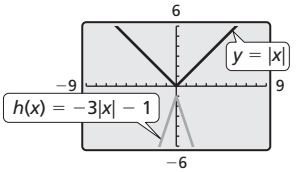
The graph of h is a vertical stretch of the parent absolute value function.

27.



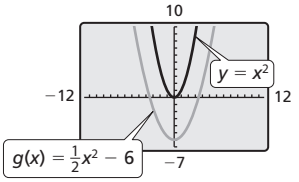
The graph of f is a vertical stretch followed by a translation 2 units up of the parent linear function.

29.



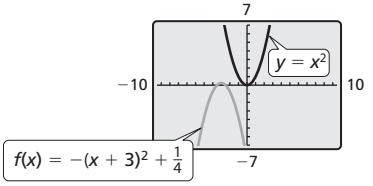
The graph of h is a vertical stretch and a reflection in the x-axis followed by a translation 1 unit down of the parent absolute value function.

31.



The graph of g is a vertical shrink followed by a translation 6 units down of the parent quadratic function.

33.



The graph of f is a reflection in the x-axis followed by a translation 3 units left and $\frac{1}{4}$ unit up of the parent quadratic function.

35. It is a vertical stretch, not shrink. The graph is a reflection in the x-axis followed by a vertical stretch of the parent quadratic function.

37. $(2, -1), (-1, -4), (2, -5)$

39. absolute value; domain is all real numbers; range is $y \geq -1$

41. linear; domain is all real numbers; range is all real numbers

43. quadratic; domain is all real numbers; range is $y \geq -2$

45. absolute value; 8 mi/h

47. no; f is shifted right and g is shifted down.

49. yes; Shifting the parent linear function down 2 units will create the same graph as shifting it 2 units right.

51. **a.** quadratic

b. 0; At the moment the ball is released, 0 seconds have passed.

c. 5.2; Because $f(t)$ represents the height of the ball, find $f(0)$.

53. **a.** vertical translation; The graph will have a vertical stretch and will be shifted 3 units down.

b. horizontal translation; The graph will be shifted 8 units right.

c. both; The graph will be shifted 2 units left and 4 units up.

d. neither; The graph will have a vertical stretch.

1.1 Maintaining Mathematical Proficiency *(p. 10)*

55. no **57.** yes **59.** x-intercept: 0; y-intercept: 0

61. x-intercept: $\frac{1}{3}$; y-intercept: 1

1.2 Vocabulary and Core Concept Check *(p. 16)*

1. shrink

1.2 Monitoring Progress and Modeling with Mathematics *(pp. 16–18)*

3. $g(x) = x - 1$ **5.** $g(x) = |4x + 3|$

7. $g(x) = 4 - |x - 2|$

9. f could be translated 3 units up or 3 units right.

11. $g(x) = 5x - 2$ **13.** $g(x) = |6x| - 2$

15. $g(x) = -3 + |x + 11|$ **17.** $g(x) = 5x + 10$

19. $g(x) = |4x| + 4$ **21.** $g(x) = -|x - 4| + 1$

23. C; The graph has been translated left.

25. D; The graph has been translated up. **27.** $g(x) = 2x + 1$

29. $g(x) = \left|\frac{1}{2}x - 1\right|$ **31.** $g(x) = -|x| - 8$

33. Translating a graph to the right requires subtraction, not addition; $g(x) = |x - 3| + 2$

35. no; Suppose a graph contains the point (3, 2) and is translated up 3 units then reflected in the x-axis. The new coordinate is (3, −5). If it is reflected in the x-axis first then translated up 3, the new coordinate is (3, 1).

37. The graph has been translated 6 units left; $A = 9$

39. The graph has been reflected in the x-axis; $A = 16$

41. **a.** $f(x) + (c - b)$ **b.** $f\left(x + \frac{c - b}{m}\right)$

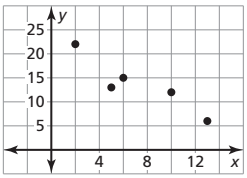
43. vertical stretch, translation, reflection; *Sample answer:* $-(4|x| - 2) = -4|x| + 2$

45. $a = -2$, $b = 1$, and $c = 0$, $g(x) = -2|x - 1|$ represents the transformation of $f(x)$.

1.2 Maintaining Mathematical Proficiency *(p. 18)*

47. −5 **49.** 0

51.



1.3 Vocabulary and Core Concept Check *(p. 26)*

1. slope-intercept

1.3 Monitoring Progress and Modeling with Mathematics *(pp. 26–28)*

3. $y = \frac{1}{5}x$; The tip increases \$0.20 for each dollar spent on the meal.

5. $y = 50x + 100$; The balance increases \$50 each week.

7. $y = 55x$; The number of words increases by 55 each minute.

9. Greenville Journal; 5 lines

11. The original balance of \$100 should have been included; After 7 years, the increase in balance will be \$70, resulting in a new balance of \$170.

13. yes; *Sample answer:* $y = 4.25x + 1.75$; $y = 65.5$; After 15 minutes, you have burned 65.5 calories.

15. yes; *Sample answer:* $y = -4.6x + 96$; $y = 27$; After 15 hours, the battery will have 27% of life remaining.

17. $y = 380.03x + 11{,}290$; \$16,990.45; The annual tuition increases about \$380 each year and the cost of tuition in 2005 is about \$11,290.

19. $y = 0.42x + 1.44$; $r = 0.61$; weak positive correlation

21. $y = -0.45x + 4.26$; $r = -0.67$; weak negative correlation

23. $y = 0.61x + 0.10$; $r = 0.95$; strong positive correlation

25. **a.** *Sample answer:* height and weight; temperature and ice cream sales; Correlation is positive because as the first goes up, so does the second.

b. *Sample answer:* miles driven and gas remaining; hours used and battery life remaining; Correlation is negative because as the first goes up, the second goes down.

c. *Sample answer:* age and length of hair; typing speed and shoe size; There is no relationship between the first and second.

27. no; Because r is close to 0, the points do not lie close to the line.

29. It is negative; As x increases, y increases, so z decreases.

31. about 2.2 mi

1.3 Maintaining Mathematical Proficiency *(p. 28)*

33. (16, −41) **35.** $\left(1, \frac{1}{2}\right)$ **37.** $\left(\frac{16}{17}, \frac{15}{17}\right)$

1.4 Vocabulary and Core Concept Check *(p. 34)*

1. ordered triple

1.4 Monitoring Progress and Modeling with Mathematics *(pp. 34–36)*

3. (1, 2, −1) **5.** (3, −1, −4) **7.** $\left(\frac{151}{64}, \frac{9}{8}, -\frac{51}{32}\right)$

9. The entire second equation should be multiplied by 4, not just the x-term.

$$\begin{array}{r} 4x - y + 2z = -18 \\ -4x + 8y + 4z = 44 \\ \hline 7y + 6z = 26 \end{array}$$

11. no solution **13.** $(z - 1, 1, z)$ **15.** no solution

17. A small pizza costs \$5, a liter of soda costs \$1, and a salad costs \$3.

19. (4, −3, 2) **21.** no solution **23.** (7, 3, 5)

25. (3, 2, 1) **27.** $\left(\frac{-3z + 3}{5}, \frac{-13z + 13}{5}, z\right)$ **29.** 1%

31. *Sample answer:* When one variable has the same coefficient or its opposite in each equation. The system

$3x + 2y - 4z = -5$

$2x + 2y + 3z = 8$

$5x - 2y - 7z = -9$

can be solved by eliminating y first.

33. $\ell + m + n = 65$, $n = \ell + m - 15$, $\ell = \frac{1}{3}m$; $\ell = 10$ ft, $m = 30$ ft, $n = 25$ ft

35. **a.** *Sample answer:* $a = -1$, $b = -1$, $c = -1$; Use elimination on equations 1 and 2.

b. *Sample answer:* $a = 4$, $b = 4$, $c = 5$; The solution is $\left(\frac{2}{3}, -\frac{2}{3}, 2\right)$.

c. *Sample answer*: $a = 5$, $b = 5$, $c = 5$; Use elimination on equations 1 and 2.

37. 350 ft^2

39. **a.** $r + \ell + i = 12$, $2.50r + 4l + 2i = 32$, $r = 2l + 2i$

b. 8 roses, 2 lilies, 2 irises

c. no; *Sample answer:* 8 roses, 4 lilies, 0 irises; 8 roses, 0 lilies, 4 irises; 8 roses, 3 lilies, 1 iris

41. $a = 12$, $b = -4$, $c = 10$; These values are the only ones which can satisfy the linear system at (−1, 2, −3).

43. $t + a = g$, $t + b = a$, $2g = 3b$; 5 tangerines

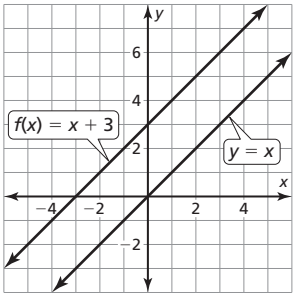
1.4 Maintaining Mathematical Proficiency *(p. 36)*

45. $9m^2 + 6m + 1$ **47.** $16 - 8y + y^2$

49. $g(x) = -|x| + 5$ **51.** $g(x) = 3|x| - 15$

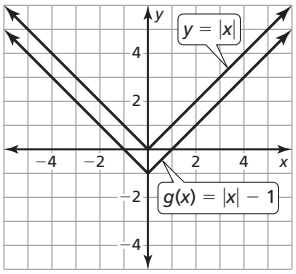
Chapter 1 Review *(pp. 38–40)*

1.



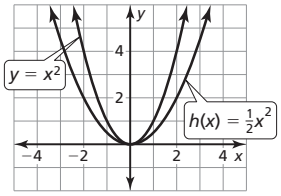
The graph of f is a translation 3 units up of the parent linear function.

2.



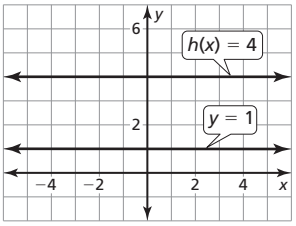
The graph of g is a translation 1 unit down of the parent absolute value function.

3.



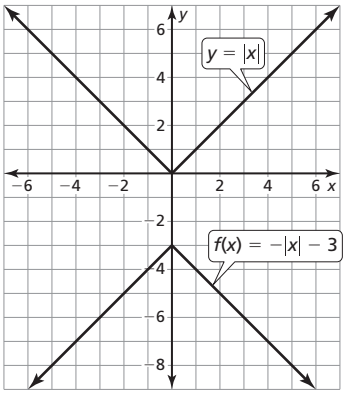
The graph of h is a vertical shrink by a factor of $\frac{1}{2}$ of the parent quadratic function.

4.



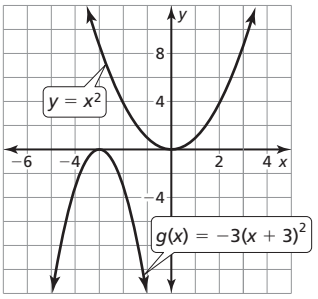
The graph of h is a translation 3 units up of the parent constant function.

5.



The graph of f is a reflection in the x-axis followed by a translation 3 units down of the parent absolute value function.

6.



The graph of g is a vertical stretch by a factor of 3 followed by a reflection in the x-axis and a translation 3 units left

7. $g(x) = -|x + 4|$ **8.** $g(x) = \frac{1}{2}|x| + 2$

9. $g(x) = -x - 3$ **10.** $y = 0.03x + 1.23$

11. $y = 0.35x$; 15.75 mi **12.** $(4, -2, 1)$

13. $\left(-\frac{4}{3}, -\frac{17}{3}, \frac{26}{3}\right)$ **14.** $(9 + 4y, y, -7 - 5y)$

15. no solution **16.** $(-11, -8, 3)$ **17.** $(-16, 12, 10)$

18. 200 student tickets, 350 adult tickets, and 50 children under 12 tickets

Chapter 2

Chapter 2 Maintaining Mathematical Proficiency *(p. 45)*

1. $-\frac{7}{2}$ **2.** $\frac{4}{3}$ **3.** -3.6 **4.** 5 **5.** -10

6. 8 **7.** about 6.32 **8.** about 8.06 **9.** about 2.24

10. about 12.65 **11.** 10 **12.** about 15.30

13. $d = |b - a|$; yes; Find the distance between the two y-coordinates by subtraction. Take the absolute value of the result, because distance is always positive. This is possible because when $x_1 = x_2$, the distance formula simplifies as shown.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(y_2 - y_1)^2}$$

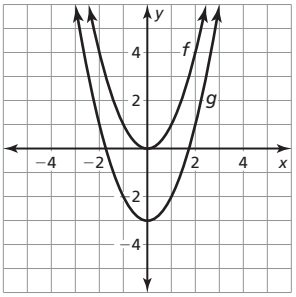
$$d = |y_2 - y_1|$$

2.1 Vocabulary and Core Concept Check *(p. 52)*

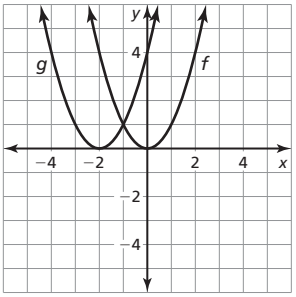
1. parabola

2.1 Monitoring Progress and Modeling with Mathematics *(pp. 52–54)*

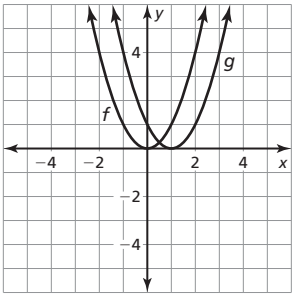
3. The graph of g is a translation 3 units down of the graph of f.



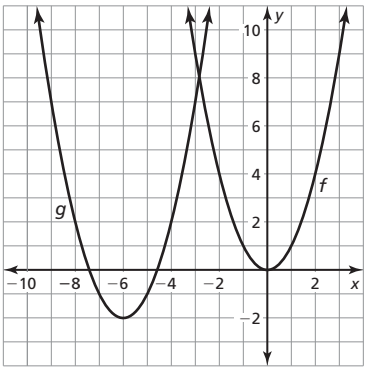
5. The graph of g is a translation 2 units left of the graph of f.



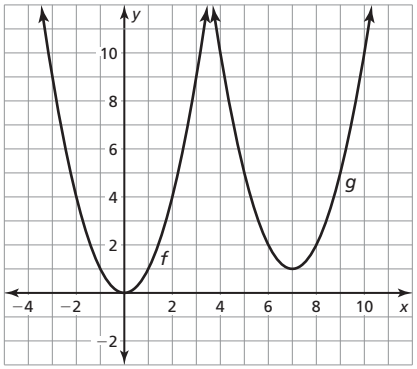
7. The graph of g is a translation 1 unit right of the graph of f.



9. The graph of g is a translation 6 units left and 2 units down of the graph of f.



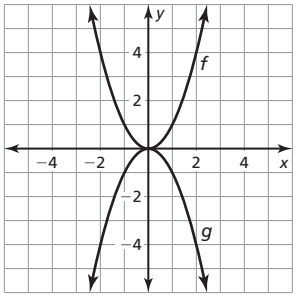
11. The graph of g is a translation 7 units right and 1 unit up of the graph of f.



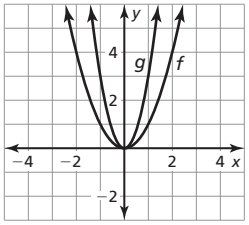
13. A; The graph has been translated 1 unit right.

15. C; The graph has been translated 1 unit right and 1 unit up.

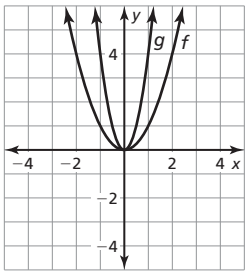
17. The graph of g is a reflection in the x-axis of the graph of f.



19. The graph of g is a vertical stretch by a factor of 3 of the graph of f.

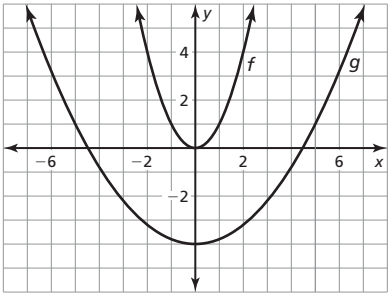


21. The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of f.



Selected Answers

23. The graph of g is a vertical shrink by a factor of $\frac{1}{5}$ followed by a translation 4 units down.



25. The graph is a reflection in the x-axis, not y-axis; The graph is a reflection in the x-axis and a vertical stretch by a factor of 6, followed by a translation 4 units up of the graph of the parent quadratic function.

27. The graph of f is a vertical stretch by a factor of 3 followed by a translation 2 units left and 1 unit up of the graph of the parent quadratic function; $(-2, 1)$

29. The graph of f is a vertical stretch by a factor of 2 followed by a reflection in the x-axis and a translation 5 units up of the graph of the parent quadratic function; $(0, 5)$

31. $g(x) = -4x^2 + 2$; $(0, 2)$

33. $g(x) = 8\left(\frac{1}{2}x\right)^2 - 4$; $(0, -4)$

35. C; The graph is a vertical stretch by a factor of 2 followed by a translation 1 unit right and 2 units down of the parent quadratic function.

37. D; The graph is a vertical stretch by a factor of 2 and a reflection in the x-axis, followed by a translation 1 unit right and 2 units up of the parent quadratic function.

39. F; The graph is a vertical stretch by a factor of 2 and a reflection in the x-axis followed by a translation 1 unit left and 2 units down of the parent quadratic function.

41. Subtract 6 from the output; Substitute $2x^2 + 6x$ for $f(x)$; Multiply the output by -1; Substitute $2x^2 + 6x - 6$ for $h(x)$; Simplify.

43. $h(x) = -0.03(x - 14)^2 + 10.99$

45. **a.** $y = \dfrac{-5}{1089}(x - 33)^2 + 5$

b. The domain is $0 \le x \le 66$ and the range is $0 \le y \le 5$; The domain represents the horizontal distance and the range represents the height of the fish.

c. yes; The value changes to $-\frac{1}{225}$; The vertex has changed but it still goes through the point $(0, 0)$, so there has been a horizontal stretch or shrink which changes the value of a.

47. **a.** $a = 2, h = 1, k = 6$; $g(x) = 2(x - 1)^2 + 6$

b. $g(x) = 2f(x - 1) + 6$; For each function, a, h, and k are the same but the second function does not indicate the type of function that is being translated.

c. $a = 2, h = 1, k = 3$; $g(x) = 2(x - 1)^2 + 3$; $g(x) = 2f(x - 1) + 3$; For each function, a, h, and k are the same, but the answer in part (b) does not indicate the type of function that is being translated.

d. *Sample answer:* vertex form; Writing a transformed function using function notation requires an extra step of substituting $f(x)$ into the newly transformed function.

49. a vertical shrink by a factor of $\frac{7}{16}$

2.1 Maintaining Mathematical Proficiency *(p. 54)*

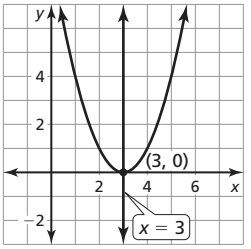
51. (4, 4)

2.2 Vocabulary and Core Concept Check *(p. 61)*

1. If a is positive, then the quadratic function will have a minimum. If a is negative, then the quadratic function will have a maximum.

2.2 Monitoring Progress and Modeling with Mathematics *(pp. 61–64)*

3.



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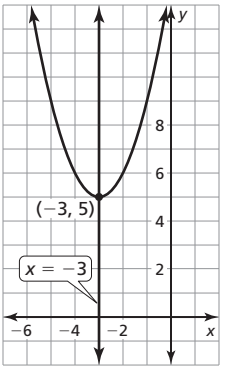

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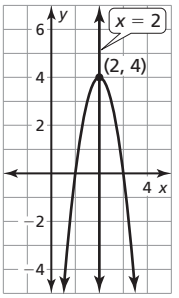

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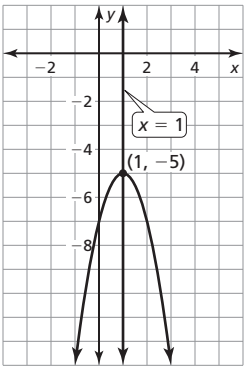

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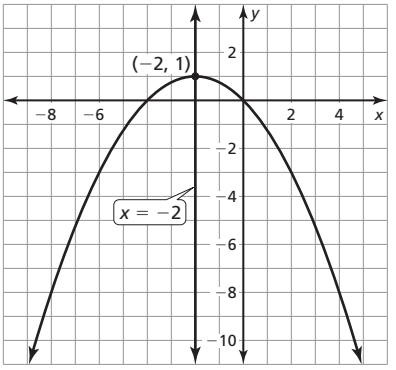

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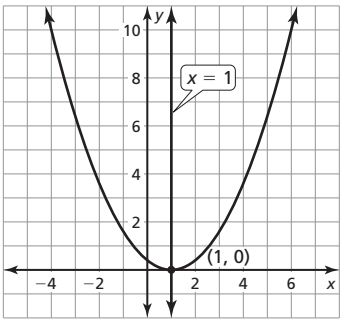
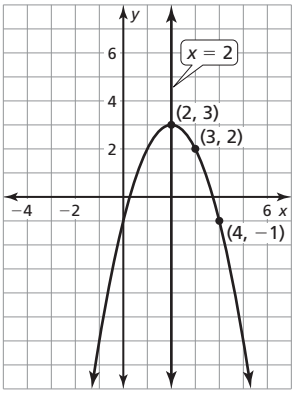


15. C **17.** B

19.



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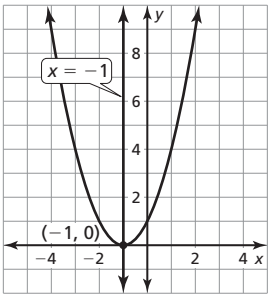

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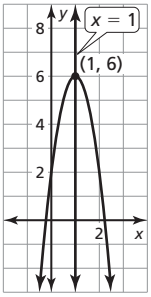

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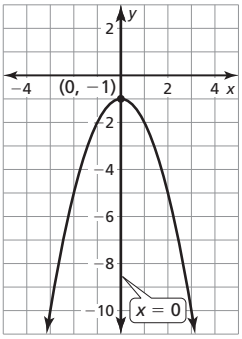

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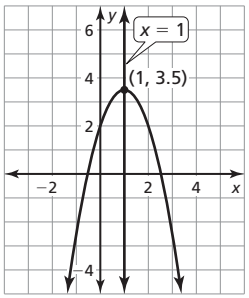

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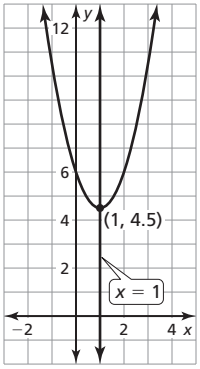


31. Both functions have an axis of symmetry of $x = 2$.

33. The formula is missing the negative sign; The x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{24}{2(4)} = -3.$$

35. (25, 18.5); When the basketball is at its highest point, it is 25 feet from its starting point and 18.5 feet off the ground.

37. B

39. The minimum value is -1. The domain is all real numbers and the range is $y \geq -1$. The function is decreasing to the left of $x = 0$ and increasing to the right of $x = 0$.

41. The maximum value is 2. The domain is all real numbers and the range is $y \leq 2$. The function is increasing to the left of $x = -2$ and decreasing to the right of $x = -2$.

43. The maximum value is 15. The domain is all real numbers and the range is $y \leq 15$. The function is increasing to the left of $x = 2$ and decreasing to the right of $x = 2$.

45. The minimum value is -18. The domain is all real numbers and the range is $y \geq -18$. The function is decreasing to the left of $x = 3$ and increasing to the right of $x = 3$.

47. The minimum value is -7. The domain is all real numbers and the range is $y \geq -7$. The function is decreasing to the left of $x = 6$ and increasing to the right of $x = 6$.

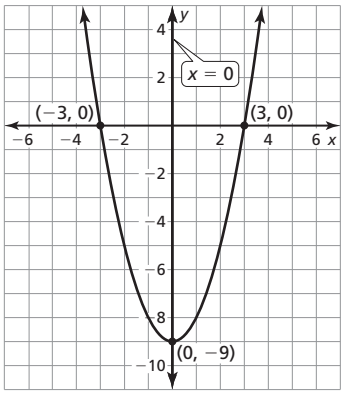
49. **a.** 1 m

b. 3.25 m

c. The diver is ascending from 0 meters to 0.5 meter and descending from 0.5 meter until hitting the water after approximately 1.1 meters.

51. $A = w(20 - w) = -w^2 + 20w$; The maximum area is 100 square units.

53.



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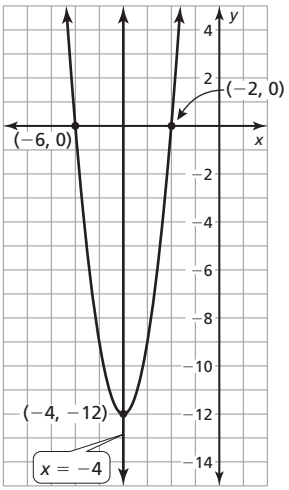

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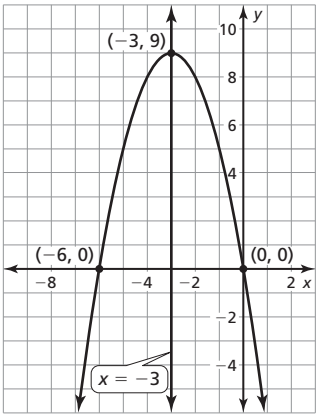

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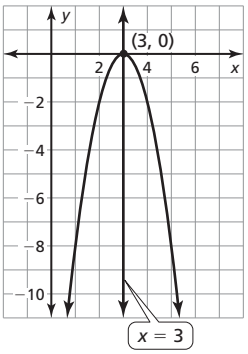


61. $p = 2$, $q = -6$; The graph is decreasing to the left of $x = -2$ and increasing to the right of $x = -2$.

63. $p = 4$, $q = 2$; The graph is increasing to the left of $x = 3$ and decreasing to the right of $x = 3$.

65. the second kick; the first kick

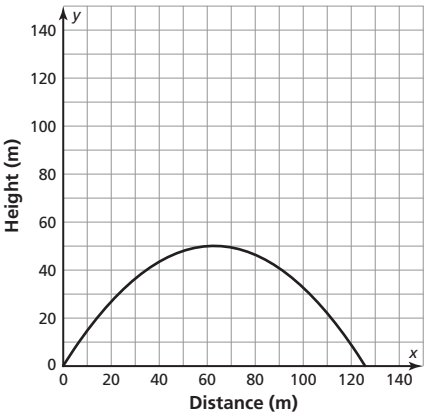
67. no; Either of the points could be the axis of symmetry, or neither of the points could be the axis of symmetry. You can only determine the axis of symmetry if the y-coordinates of the two points are the same, because the axis of symmetry would lie halfway between the two points.

69. \$1.75

71. All three graphs are the same; $f(x) = x^2 + 4x + 3$, $g(x) = x^2 + 4x + 3$

73. no; The vertex of the graph is (3.25, 2.1125), which means the mouse cannot jump over a fence that is higher than 2.1125 feet.

75.



The domain is $0 \le x \le 126$ and the range is $0 \le y \le 50$; The domain represents the distance from the start of the bridge on one side of the river, and the range represents the height of the bridge.

77. no; The vertex must lie on the axis of symmetry, and (0, 5) does not lie on $x = -1$.

79. **a.** about 14.1%; about 55.5 cm^3/g

b. about 13.6%; about 44.1 cm^3/g

c. The domain for hot-air popping is $5.52 \le x \le 22.6$, and the range is $0 \le y \le 55.5$. The domain for hot-oil popping is $5.35 \le x \le 21.8$, and the range is $0 \le y \le 44.1$. This means that the moisture content for the kernels can range from 5.52% to 22.6% and 5.35% to 21.8%, while the popping volume can range from 0 to 55.5 cubic centimeters per gram and 0 to 44.11 cubic centimeters per gram.

2.2 Maintaining Mathematical Proficiency *(p. 64)*

81. 4 **83.** no solution **85.** 2 **87.** −12

2.3 Vocabulary and Core Concept Check *(p. 72)*

1. focus; directrix

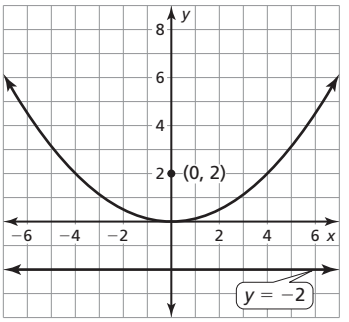
2.3 Monitoring Progress and Modeling with Mathematics *(pp. 72–74)*

3. $y = \frac{1}{4}x^2$ **5.** $y = -\frac{1}{8}x^2$ **7.** $y = \frac{1}{24}x^2$

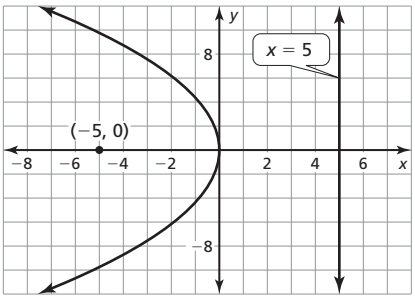
9. $y = -\frac{1}{40}x^2$

11. A, B and D; Each has a value for p that is negative. Substituting in a negative value for p in $y = \dfrac{1}{4p}x^2$ results in a parabola that has been reflected across the x-axis.

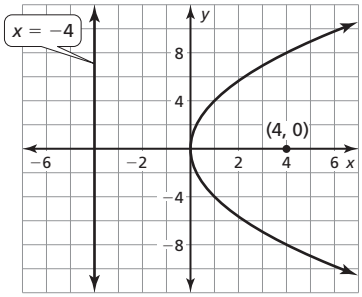
13. The focus is (0, 2). The directrix is $y = -2$. The axis of symmetry is the y-axis.



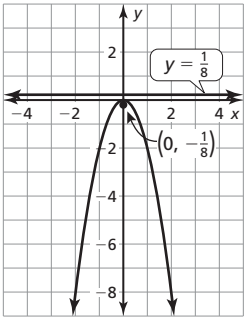
15. The focus is $(-5, 0)$. The directrix is $x = 5$. The axis of symmetry is the x-axis.



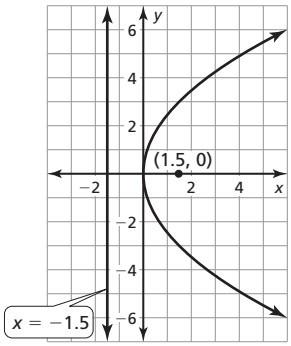
17. The focus is $(4, 0)$. The directrix is $x = -4$. The axis of symmetry is the x-axis.



19. The focus is $\left(0, -\frac{1}{8}\right)$. The directrix is $y = \frac{1}{8}$. The axis of symmetry is the y-axis.



21. Instead of a vertical axis of symmetry, the graph should have a horizontal axis of symmetry.



23. 9.5 in.; The receiver should be placed at the focus. The distance from the vertex to the focus is $p = \frac{38}{4} = 9.5$ in.

25. $y = \frac{1}{32}x^2$ **27.** $x = -\frac{1}{10}y^2$ **29.** $x = \frac{1}{12}y^2$

31. $x = \frac{1}{40}y^2$ **33.** $y = -\frac{3}{20}x^2$ **35.** $y = \frac{7}{24}x^2$

37. $x = -\frac{1}{16}y^2 - 4$ **39.** $y = \frac{1}{6}x^2 + 1$

41. The vertex is $(3, 2)$. The focus is $(3, 4)$. The directrix is $y = 0$. The axis of symmetry is $x = 3$. The graph is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 3 units right and 2 units up.

43. The vertex is $(1, 3)$. The focus is $(5, 3)$. The directrix is $x = -3$. The axis of symmetry is $y = 3$. The graph is a horizontal shrink by a factor of $\frac{1}{4}$ followed by a translation 1 unit right and 3 units up.

45. The vertex is $(2, -4)$. The focus is $\left(\frac{23}{12}, -4\right)$. The directrix is $x = \frac{25}{12}$. The axis of symmetry is $y = -4$. The graph is a horizontal stretch by a factor of 12 followed by a reflection in the y-axis and a translation 2 units right and 4 units down.

47. $x = \dfrac{1}{5.2}y^2$; about 3.08 in.

49. As $|p|$ increases, the graph gets wider; As $|p|$ increases, the constant in the function gets smaller which results in a vertical shrink, making the graph wider.

51. $y = \dfrac{1}{4}x^2$ **53.** $x = \dfrac{1}{4p}y^2$

2.3 Maintaining Mathematical Proficiency *(p. 74)*

55. $y = 3x - 7$ **57.** $y = -\frac{1}{2}x + \frac{5}{2}$

59. $y = 3.98x + 0.92$

2.4 Vocabulary and Core Concept Check *(p. 80)*

1. A quadratic model is appropriate when the second differences are constant.

2.4 Monitoring Progress and Modeling with Mathematics *(pp. 80–82)*

3. $y = -3(x + 2)^2 + 6$ **5.** $y = 0.06(x - 3)^2 + 2$

7. $y = -\frac{1}{3}(x + 6)^2 - 12$ **9.** $y = -4(x - 2)(x - 4)$

11. $y = \frac{1}{10}(x - 12)(x + 6)$ **13.** $y = 2.25(x + 16)(x + 2)$

15. If given the x-intercepts, it is easier to write the equation in intercept form. If given the vertex, it is easier to write the equation in vertex form.

17. $y = -16(x - 3)^2 + 150$ **19.** $y = -0.75x(x - 4)$

21. The x-intercepts were substituted incorrectly.

$y = a(x - p)(x - q)$

$4 = a(3 + 1)(3 - 2)$

$a = 1$

$y = (x + 1)(x - 2)$

23. $S(C) = 180C^2$; 18,000 lbs

25. intercept form; The three points can be substituted into the intercept form of a quadratic equation to solve for a, and then the equation can be written. This method is much shorter than writing and solving a system of three equations, although it can only be used when given the intercepts.

27. **a.** parabola; not a constant rate of change

b. $h = -16t^2 + 280$ **c.** about 4.18 sec

d. The domain is $0 \le t \le 4.18$ and represents the time the sponge was in the air. The range is $0 \le h \le 280$ and represents the height of the sponge.

29. quadratic; The second differences are constant; $y = -2x^2 + 42x + 470$

31. neither; The first and second differences are not constant.

33. **a.** The vertex indicates that on the 6th day, 19 people were absent, more than any other day.

b. $y = -0.5(10 - 6)^2 + 19$; 11 students

c. From 0 to 6 days, the average rate of change was 3 students per day. From 6 to 11 days, the average rate of change was -2.5 students per day. The rate at which students were missing school was changing more rapidly as more became ill, in comparison to when the students were becoming well.

35. $y = -16x^2 + 6x + 22$; after about 1.24 sec; 1.375 sec

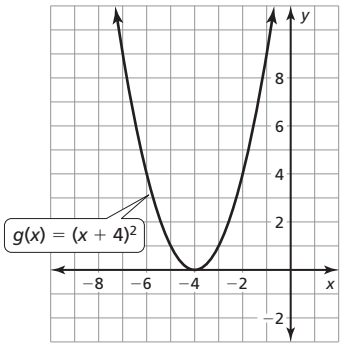
37. 155 tiles

2.4 Maintaining Mathematical Proficiency *(p. 82)*

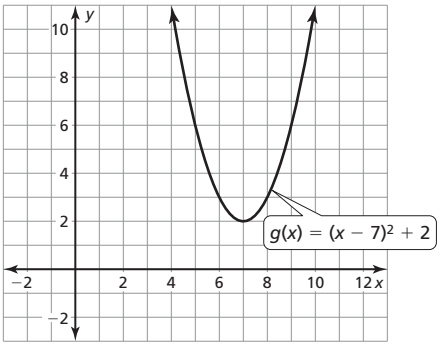
39. $(x - 2)(x - 1)$ **41.** $5(x + 3)(x - 2)$

Chapter 2 Review *(pp. 84–86)*

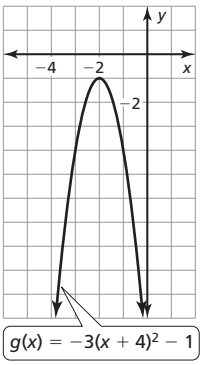
1. The graph is a translation 4 units left of the parent quadratic function.



2. The graph is a translation 7 units right and 2 units up of the parent quadratic function.



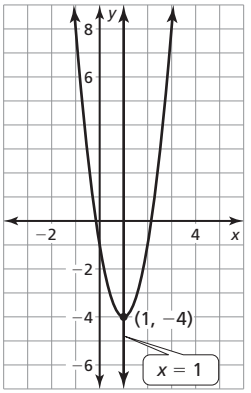
3. The graph is a vertical stretch by a factor of 3 followed by a reflection in the x-axis and a translation 2 units left and 1 unit down.



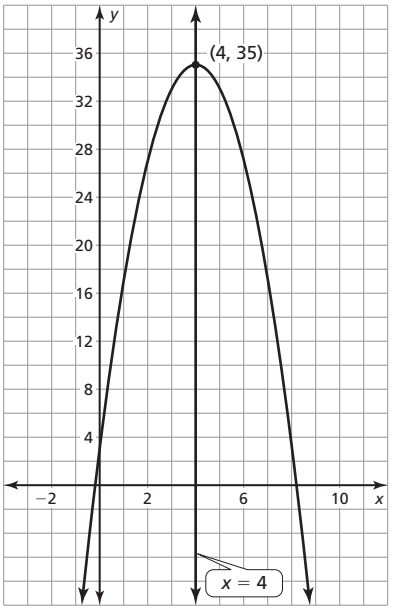
4. $g(x) = \frac{9}{4}(x + 5)^2 - 2$

5. $g(x) = (-x + 2)^2 - 2(-x + 2) + 3 = x^2 - 2x + 3$

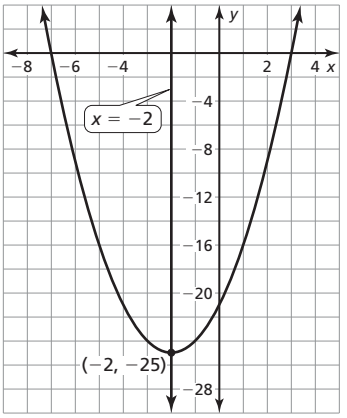
6. The minimum value is -4; The function is decreasing to the left of $x = 1$ and increasing to the right of $x = 1$.



7. The maximum value is 35; The function is increasing to the left of $x = 4$ and decreasing to the right of $x = 4$.

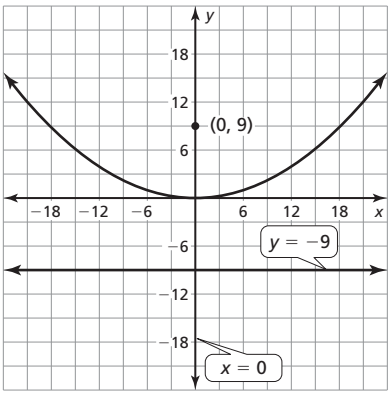


8. The minimum value is -25; The function is decreasing to the left of $x = -2$ and increasing to the right of $x = -2$.



9. 2.25 in.

10. The focus is (0, 9), the directrix is $y = -9$, and the axis of symmetry is $x = 0$.



11. $x = -\frac{1}{8}y^2$ **12.** $y = -\frac{1}{16}(x - 2)^2 + 6$

13. $y = \frac{16}{81}(x - 10)^2 - 4$ **14.** $y = -\frac{3}{5}(x + 1)(x - 5)$

15. $y = 4x^2 + 5x + 1$

16. $y = -16x^2 + 150$; about 3.06 sec

Chapter 3

Chapter 3 Maintaining Mathematical Proficiency *(p. 91)*

1. $3\sqrt{3}$ **2.** $-4\sqrt{7}$ **3.** $\frac{\sqrt{11}}{8}$ **4.** $\frac{7\sqrt{3}}{10}$ **5.** $\frac{3\sqrt{2}}{7}$

6. $-\frac{\sqrt{65}}{11}$ **7.** $-4\sqrt{5}$ **8.** $4\sqrt{2}$ **9.** $(x - 6)(x + 6)$

10. $(x - 3)(x + 3)$ **11.** $(2x - 5)(2x + 5)$ **12.** $(x - 11)^2$

13. $(x + 14)^2$ **14.** $(7x + 15)^2$

15. $a = 16$ and $c = 1$, $a = 4$ and $c = 4$, $a = 1$ and $c = 16$; $2\sqrt{ac} = 8$

3.1 Vocabulary and Core Concept Check *(p. 99)*

1. Use the graph to find the x-intercepts of the function.

3.1 Monitoring Progress and Modeling with Mathematics *(pp. 99–102)*

3. $x = -1$ and $x = -2$ **5.** $x = 3$ and $x = -3$

7. $x = -1$ **9.** no real solution **11.** no real solution

13. $s = \pm 12$ **15.** $z = 1$ and $z = 11$ **17.** $x = 1 \pm \sqrt{2}$

19. no real solution **21.** A, B, and E

23. The $\pm$ was not used when taking the square root; $2(x + 1)^2 + 3 = 21$; $2(x + 1)^2 = 18$; $(x + 1)^2 = 9$; $x + 1 = \pm 3$; $x = 2$ and $x = -4$

25. **a.** *Sample answer:* $x^2 = 16$ **b.** $x^2 = 0$
c. *Sample answer:* $x^2 = -9$

27. $x = -3$ **29.** $x = 6$ and $x = 2$ **31.** $n = 0$ and $n = 6$

33. $w = 12$ and $w = 2$ **35.** $x = 4$ **37.** $x = 3$

39. $u = 0$ and $u = -9$; *Sample answer:* factoring because the equation can be factored

41. no real solution; *Sample answer:* square roots because the equation can be written in the form $u^2 = d$

43. $x = 6$ and $x = 2$; *Sample answer:* square roots because the equation can be written in the form $u^2 = d$

45. $x = -0.5$ and $x = -2.5$; *Sample answer:* factoring because the equation can be factored

47. $x = -2$ and $x = -4$ **49.** $x = 3$ and $x = -10$

51. $x = 3$ and $x = -2$ **53.** $x = -11$

55. $f(x) = x^2 - 19x + 88$ **57.** \$5.75; \$1983.75

59. **a.** $h(t) = -16t^2 + 188$; about 3.4 sec
b. 80 ft; The log fell 80 feet between 2 and 3 seconds.

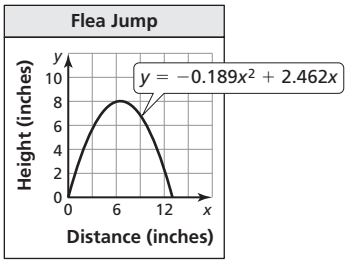
61. 0.5 ft or 6 in.

63. The 20-foot wave requires a wind speed twice as great as the wind speed required for a 5-foot wave.

65. $x \approx 34.64$; about 207.84 ft, 277.12 ft, 173.20 ft, and 300 ft

67. the rock on Jupiter; Because the first term is negative, the height of the falling object will decrease faster as g gets larger.

69.



The vertex (6.5, 8.0) indicates that the flea's maximum jump is 6.5 inches away from and 8.0 inches above the starting point. The zeros $x = 13$ and $x = 0$ indicate when the flea is on the ground.

71. you; The function does not cross the x-axis.

73. **a.** $mn = a^2$ and $m + n = 0$
b. $m = \sqrt{-a^2} = a\sqrt{-1}$,
$n = -\sqrt{-a^2} = -a\sqrt{-1}$;
m and n are not real numbers.

75. 60 ft

3.1 Maintaining Mathematical Proficiency *(p. 102)*

77. $x^3 + 4x^2 + 6$ **79.** $-3x^3 + 7x^2 - 15x + 9$

81. $10x^3 - 2x^2 + 6x$ **83.** $-44x^3 + 33x^2 + 88x$

3.2 Vocabulary and Core Concept Check *(p. 108)*

1. $i = \sqrt{-1}$ and is used to write the square root of any negative number.

3. Add the real parts and the imaginary parts separately.

3.2 Monitoring Progress and Modeling with Mathematics *(pp. 108–110)*

5. $6i$ **7.** $3i\sqrt{2}$ **9.** $8i$ **11.** $-16i\sqrt{2}$

13. $x = 2$ and $y = 2$ **15.** $x = -2$ and $y = 4$

17. $x = 7$ and $y = -12$ **19.** $x = 6$ and $y = 28$

21. $13 + 2i$ **23.** $9 + 11i$ **25.** 19 **27.** $4 + 2i$

29. $-4 - 14i$ **31.** **a.** $-4 + 5i$ **b.** $2\sqrt{2} + 10i$

33. $(12 + 2i)$ ohms **35.** $(8 + i)$ ohms **37.** $-3 - 15i$

39. $14 - 5i$ **41.** 20 **43.** $-27 - 36i$

45. Distributive Property; Simplify; Definition of complex addition; Write in standard form.

47. $(6 - 7i) - (4 - 3i) = 2 - 4i$ **49.** $x = \pm 3i$

51. $x = \pm i\sqrt{7}$ **53.** $x = \pm 2i\sqrt{5}$ **55.** $x = \pm i\sqrt{2}$

57. $x = \pm 6i$ **59.** $x = \pm 3i\sqrt{3}$ **61.** $x = \pm 4i\sqrt{3}$

63. i^2 can be simplified;
$15 - 3i + 10i - 2i^2 = 15 + 7i + 2 = 17 + 7i$

65. **a.** -8 **b.** $12 - 10i$ **c.** $21i$ **d.** $41 + 3i$ **e.** $-9i$ **f.** $-9 + 23i$ **g.** 14 **h.** $14i$

Real numbers	Imaginary numbers	Pure imaginary numbers
-8 14	$12 - 10i$ $41 + 3i$ $-9 + 23i$	$21i$ $-9i$ $14i$

67.

Powers of i	i^1	i^2	i^3	i^4	i^5	i^6	i^7	i^8	i^9	i^{10}	i^{11}	i^{12}
Simplified form	i	-1	$-i$	1	i	-1	$-i$	1	i	-1	$-i$	1

The results of i^n alternate in the pattern i, -1, $-i$, and 1.

69. $-28 + 27i$ **71.** $-15 - 25i$ **73.** $9 + 5i$

75. *Sample answer:* $3 + 2i$ and $3 - 2i$; The real parts are equal and the imaginary parts are opposites.

77. **a.** false; *Sample answer:* $(3 - 5i) + (4 + 5i) = 7$
b. true; *Sample answer:* $(3i)(2i) = 6i^2 = -6$
c. true; *Sample answer:* $3i = 0 + 3i$
d. false; *Sample answer:* $1 + 8i$

3.2 Maintaining Mathematical Proficiency *(p. 110)*

79. yes **81.** no **83.** $y = 2(x + 3)^2 - 3$

3.3 Vocabulary and Core Concept Check *(p. 116)*

1. $\left(\frac{b}{2}\right)^2$

3.3 Monitoring Progress and Modeling with Mathematics *(pp. 116–118)*

3. $x = 9$ and $x = -1$ **5.** $x = 9 \pm \sqrt{5}$

7. $y = 12 \pm 10i$ **9.** $w = \dfrac{-1 \pm 5\sqrt{3}}{2}$ **11.** 25; $(x + 5)^2$

13. 36; $(y - 6)^2$ **15.** 9; $(x - 3)^2$ **17.** $\frac{25}{4}$; $\left(z - \frac{5}{2}\right)^2$

19. $\frac{169}{4}$; $\left(w + \frac{13}{2}\right)^2$ **21.** 4; $x^2 + 4x + 4$

23. 36; $x^2 + 12x + 36$ **25.** $x = -3 \pm \sqrt{6}$

27. $x = -2 \pm \sqrt{6}$ **29.** $z = \dfrac{-9 \pm \sqrt{85}}{2}$

31. $t = -2 \pm 2i$ **33.** $x = -3 \pm i$ **35.** $x = 5 \pm 2\sqrt{7}$

37. 36 should have been added to the right side of the equation instead of 9; $4x^2 + 24x - 11 = 0$; $4(x^2 + 6x) = 11$; $4(x^2 + 6x + 9) = 11 + 36$; $4(x + 3)^2 = 47$; $(x + 3)^2 = \dfrac{47}{4}$; $x + 3 = \dfrac{\pm\sqrt{47}}{2}$; $x = -3 \pm \dfrac{\sqrt{47}}{2}$; $x = \dfrac{-6 \pm \sqrt{47}}{2}$

39. yes; All of the steps would be the same as with two real solutions, with the exception of the constant being negative when you take the square root.

41. factoring; The equation can be factored; $x = 7$ and $x = -3$

43. square roots; The equation can be written in the form $u^2 = d$; $x = -8$ and $x = 0$

45. factoring; The equation can be factored; $x = -6$

47. completing the square; The equation cannot be factored or written in the form $u^2 = d$; $x = -1 \pm \dfrac{\sqrt{10}}{2}$

49. square roots; The equation can be written in the form $u^2 = d$; $x = \pm 10$

51. $x = -5 + 5\sqrt{3}$ **53.** $x = -2 + 2\sqrt{21}$

55. $f(x) = (x - 4)^2 + 3$; $(4, 3)$

57. $g(x) = (x + 6)^2 + 1$; $(-6, 1)$

59. $h(x) = (x + 1)^2 - 49$; $(-1, -49)$

61. $f(x) = \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$; $\left(\frac{3}{2}, \frac{7}{4}\right)$

63. **a.** 22 ft **b.** about 2.1 sec

65. **a.** \$3600 **b.** $y = -(x - 10)^2 + 3600$
c. *Sample answer:* vertex form; The vertex of the graph gives the maximum value.

67. *Sample answer:* Complete the square to find the vertex. Factor it into intercept form to find the two roots, find their average to obtain the time when the water reaches its maximum height, and then substitute the time into the function. Use the coefficients of the original function $y = f(x)$ to find the maximum height, $f\left(-\dfrac{b}{2a}\right)$; 125.44 ft

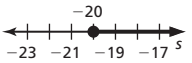
69. no; The problem cannot be solved by factoring because the answers are not rational.

71. $x = \dfrac{-b \pm \sqrt{b^2 - 4c}}{2}$ **73.** $x \approx 0.896$ cm

3.3 Maintaining Mathematical Proficiency *(p. 118)*

75. $y \le -1$

77. $s \ge -20$



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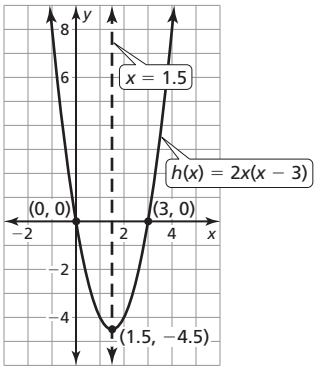
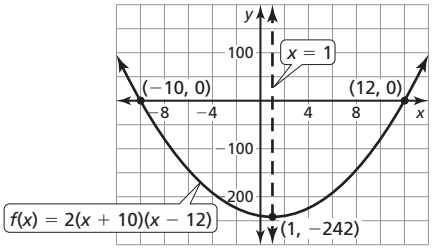


81.



3.4 Vocabulary and Core Concept Check *(p. 127)*

1. $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3. There will be two imaginary solutions.

3.4 Monitoring Progress and Modeling with Mathematics *(pp.127–130)*

5. $x = 3$ and $x = 1$ **7.** $x = -3 \pm i\sqrt{6}$ **9.** $x = 7$

11. $x = \dfrac{-1 \pm i\sqrt{14}}{3}$ **13.** $x = 5$ **15.** $x = \dfrac{3 \pm \sqrt{89}}{8}$

17. $z = 6 \pm \sqrt{30}$ **19.** 0; one real: $x = -6$

21. 400; two real: $n = 3$ and $n = -2$

23. -135; two imaginary: $x = \dfrac{5 \pm 3i\sqrt{15}}{8}$

25. 0; one real: $x = -4$ **27.** A

29. C; The discriminant is negative, so the graph has no x-intercepts.

31. A; The discriminant is positive, so the graph has two x-intercepts. The y-intercept is -9.

33. The i was left out after taking the square root;
$x = \dfrac{-10 \pm \sqrt{-196}}{2} = \dfrac{-10 \pm 14i}{2} = -5 \pm 7i$

35. *Sample answer:* $a = 1$ and $c = 5$; $x^2 + 4x + 5 = 0$

37. *Sample answer:* $a = 2$ and $c = 4$; $2x^2 - 8x + 4 = 0$

39. *Sample answer:* $a = 5$ and $c = -5$; $5x^2 + 10x + 5 = 0$

41. $-5x^2 + 8x - 12 = 0$ **43.** $-7x^2 + 4x - 5 = 0$

45. $3x^2 + 4x + 1 = 0$

47. $x = \pm 2\sqrt{2}$; *Sample answer:* square roots; The equation can be written in the form $u^2 = d$.

49. $x = 9$ and $x = -3$; *Sample answer:* factoring; The equation can be factored.

51. $x = 3$ and $x = 4$; *Sample answer:* factoring; The equation can be factored.

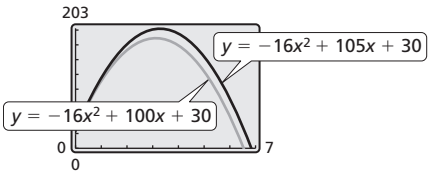
53. $x = 5 \pm i\sqrt{2}$; *Sample answer:* completing the square; Factor out 5, and $a = 1$ and b is an even number.

55. $x = \dfrac{-9 \pm \sqrt{33}}{8}$; *Sample answer:* Quadratic Formula; $a \neq 1$, b is not an even number, the equation cannot be factored, and it cannot be easily written in the form $u^2 = d$.

57. $x = \dfrac{-1 \pm \sqrt{5}}{2}$; *Sample answer:* Quadratic Formula; b is not an even number, the equation cannot be factored, and it cannot be easily written in the form $u^2 = d$.

59. $x = 6$ **61.** about 5.67 sec **63.** about 0.17 sec

65. **a.**



Both rockets start from the same height, but your friend's rocket does not go as high and lands about a half of a second earlier.

b. about 1 sec and 5.5625 sec; These are reasonable because $\dfrac{1 + 5.6}{2} = 3.3$ which is the axis of symmetry.

67. **a.** about 0.97 sec

b. the first bird; The second bird will reach the water after about 0.98 second.

69. 3.5 ft

71. **a.** $x = 6$, $x = -3$, $x = 5$, and $x = -2$ **b.** $x = \pm 3$

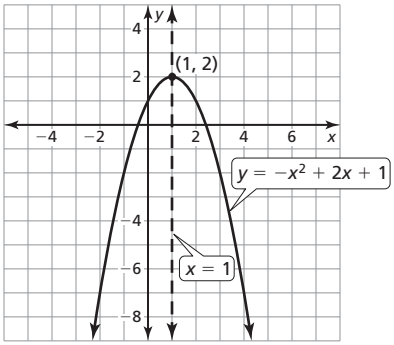
73. Add the solutions to get $\dfrac{-b}{a}$, then divide the result by 2 to get $-\dfrac{b}{2a}$; Because it is symmetric, the vertex of a parabola is in the middle of the two x-intercepts and the x-coordinate of the vertex is $-\dfrac{b}{2a}$.

75. If $x = 3i$ and $x = -2i$ are solutions, then the equation can be written as $a(x - 3i)(x + 2i) = ax^2 - aix + 6a$. a and ai cannot both be real numbers.

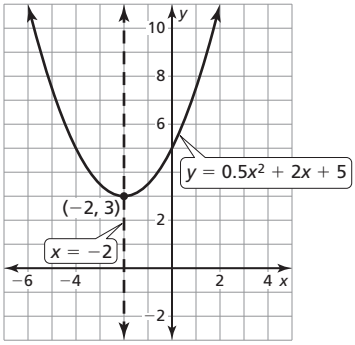
3.4 Maintaining Mathematical Proficiency *(p. 130)*

77. (4, 5) **79.** no solution

81.



83.



3.5 Vocabulary and Core Concept Check *(p. 136)*

1. There could be no solution, one solution, or two solutions.

3.5 Monitoring Progress and Modeling with Mathematics *(pp. 136–138)*

3. (0, 2) and (−2, 0) **5.** no solution **7.** (−4, 2)

9. (1, 1) and (3, 1) **11.** (−4, 1) **13.** (1, 4) and (9, 4)

15. (3, 8) and (−1, 4) **17.** (0, −8) **19.** no solution

21. (2, 3) and (−2, 3) **23.** no solution **25.** A and C

27. (2, 7) and (0, 5) **29.** no solution

31. about (−4.65, −4.71) and about (0.65, −15.29)

33. (−4, −4) and (−6, −4)

35. The terms that were added were not like terms; $0 = -2x^2 + 34x - 140$; $x = 7$ *or* $x = 10$

37. (0, −1); *Sample answer:* elimination because the equations are arranged with like terms in the same column

39. about (−11.31, 10) and about (5.31, 10); *Sample answer:* substitution because the second equation can be substituted into the first equation

41. (3, 3) and (5, 3); *Sample answer:* graphing because substitution and elimination would require more steps in this case

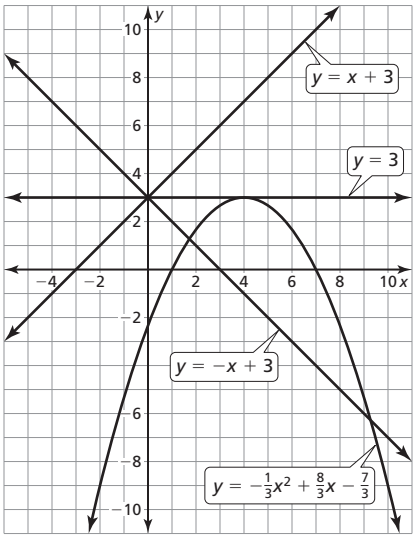
43. $x = 0$ **45.** $x \approx 0.63$ and $x \approx 2.37$

47. $x = 2$ and $x = 3$

49. The graphs intersect at the vertex of the quadratic function.

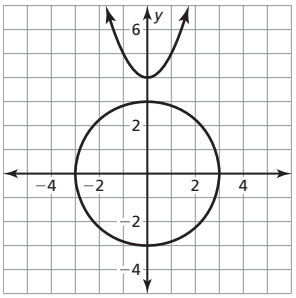
51. $d = 0.8t$; $d = 2.5t^2$; 0.32 min

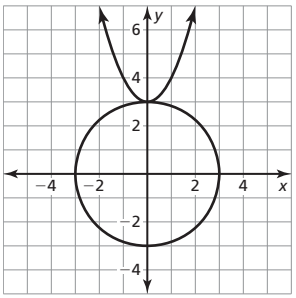
53. no solution: $m = 1$; one solution: $m = 0$; two solutions: $m = -1$

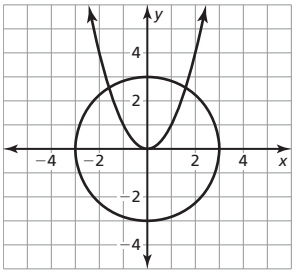


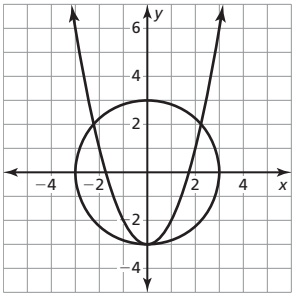
55. *Sample answer:* graphing and Quadratic Formula; graphing because it requires less time and steps than using the Quadratic Formula in this case

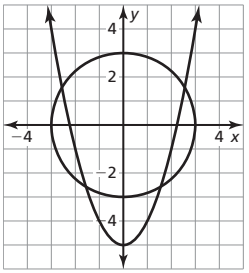
57. **a.** no solution, one solution, two solutions, three solutions, or four solutions



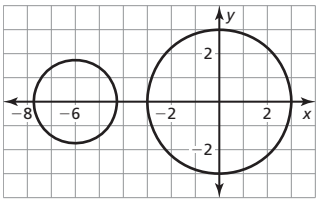


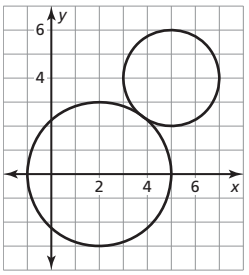


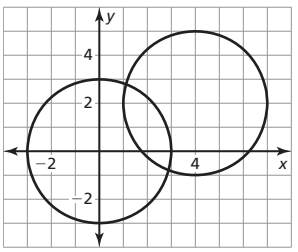


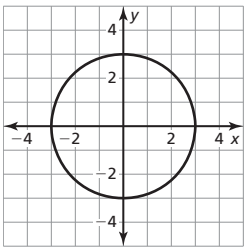


b. no solution, one solution, two solutions, or infinitely many solutions







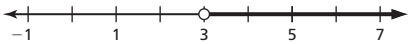


59. **a.** circle: $x^2 + y^2 = 1$, Oak Lane: $y = -\frac{1}{7}x + \frac{5}{7}$

b. $(-0.6, 0.8)$ and $(0.8, 0.6)$ **c.** about 1.41 mi

3.5 Maintaining Mathematical Proficiency *(p. 138)*

61. $x > 3$



63. $x \le -4$

65. $y < x - 2$

3.6 Vocabulary and Core Concept *(p. 144)*

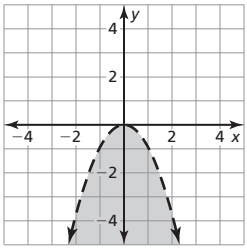
1. The graph of a quadratic inequality in one variable consists of a number line, but the graph of a quadratic inequality in two variables consists of both the x- and y-axis.

3.6 Monitoring Progress and Modeling with Mathematics *(pp. 144–146)*

3. C; The x-intercepts are $x = -1$ and $x = -3$. The test point $(-2, 5)$ does not satisfy the inequality.

5. B; The x-intercepts are $x = 1$ and $x = 3$. The test point $(2, 5)$ does not satisfy the inequality.

7.



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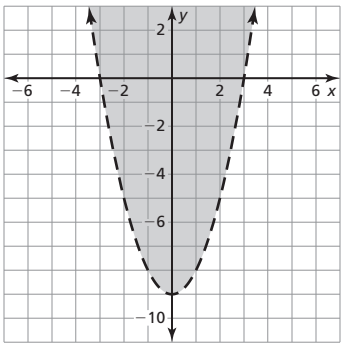

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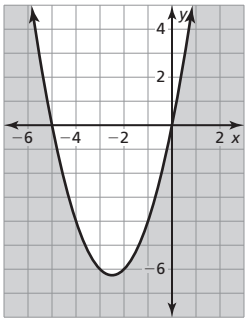

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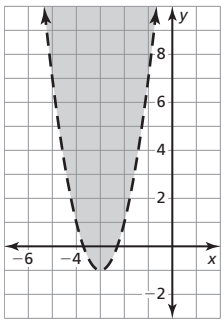
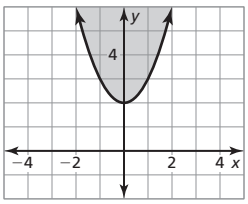
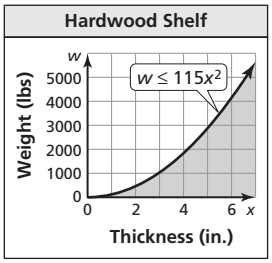


15. $y > f(x)$

17. The graph should be solid, not dashed.



19. The solution represents weights that can be supported by shelves with various thicknesses.



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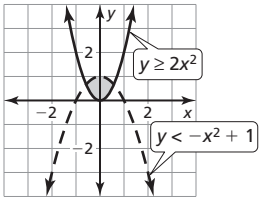

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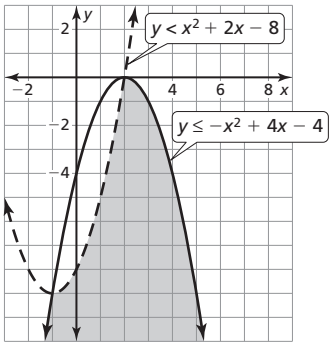

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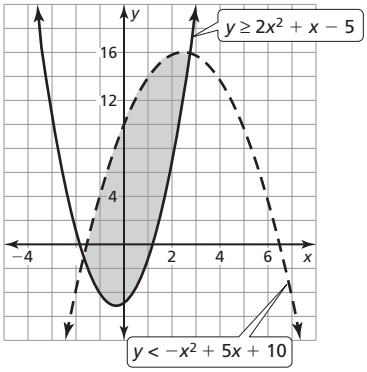


27. $-\frac{5}{2} < x < \frac{5}{2}$ **29.** $x \le 4$ or $x \ge 7$ **31.** $-0.5 \le x \le 3$

33. $x < -2$ or $x > 4$ **35.** about $0.38 < x < 2.62$

37. $x < -7$ or $x > -1$ **39.** $-2 \le x \le \frac{4}{3}$

41. about $x \le -6.87$ or $x \ge 0.87$

43. **a.** $x_1 < x < x_2$ **b.** $x < x_1$ or $x > x_2$ **c.** $x_1 < x < x_2$

45. about 55 m from the left pylon to about 447 m from the left pylon

47. **a.** $0.0051x^2 - 0.319x + 15 < 0.005x^2 - 0.23x + 22$, $16 \le x \le 70$

b. $A(x) < V(x)$ for $16 \le x \le 70$; Graph the inequalities only on $16 \le x \le 70$. $A(x)$ is always less than $V(x)$.

c. The driver would react more quickly to the siren of an approaching ambulance; The reaction time to audio stimuli is always less.

49. $0.00170x^2 + 0.145x + 2.35 > 10$, $0 \le x \le 40$; after about 37 days; Because $L(x)$ is a parabola, $L(x) = 10$ has two solutions. Because the x-value must be positive, the domain requires that the negative solution be rejected.

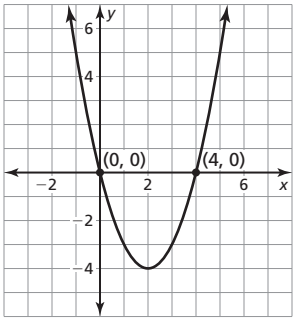
51. **a.** $\frac{32}{3} \approx 10.67$ square units **b.** $\frac{256}{3} \approx 85.33$ square units

53. **a.** yes; The points on the parabola that are exactly 11 feet high are (6, 11) and (14, 11). Because these points are 8 feet apart, there is enough room for a 7-foot wide truck.

b. 8 ft **c.** about 11.2 ft

3.6 Maintaining Mathematical Proficiency *(p. 146)*

55.



57. The maximum value is -1; The function is increasing to the left of $x = -3$ and decreasing to the right of $x = -3$.

59. The maximum value is 25; The function is increasing to the left of $x = -2$ and decreasing to the right of $x = -2$.

Chapter 3 Review *(pp. 148–150)*

1. $x = 4$ and $x = -2$ **2.** $x = \pm 2$

3. $x = 2$ and $x = -8$ **4.** $x = 6$ and $x = 2.5$

5. $(x + 18)(x + 35) = 1260$; $x = 10$; 28 ft by 45 ft

6. $x = 9$ and $y = -3$ **7.** $5 - 3i$ **8.** $11 + 10i$

9. $-62 + 11i$ **10.** $x = \pm i\sqrt{3}$ **11.** $x = \pm 4i$

12. 148 ft **13.** $x = -8 \pm \sqrt{47}$ **14.** $x = \dfrac{-4 \pm 3i}{2}$

15. $x = 3 \pm 3\sqrt{2}$ **16.** $y = (x - 1)^2 + 19$; (1, 19)

17. $x = \dfrac{5 \pm \sqrt{17}}{2}$ **18.** $x = 0.5$ and $x = -3$

19. $x = \dfrac{6 \pm i\sqrt{3}}{3}$ **20.** 0; one real solution: $x = -3$

21. 40; two real solutions: $x = 1 \pm \sqrt{10}$

22. 16; two real solutions: $x = -5$ and $x = -1$

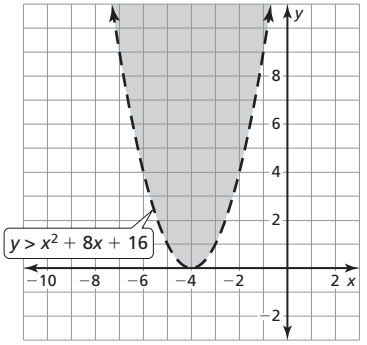
23. $(-2, 6)$ and $(1, 0)$; *Sample answer:* substitution because both equations are already solved for y

24. (4, 5); *Sample answer:* elimination because adding the like terms eliminates y

25. about $(-0.32, 1.97)$ and $(0.92, -1.77)$; substitution because elimination is not a possibility with no like terms

26. $x \approx -0.14$ and $x \approx 1.77$

27.



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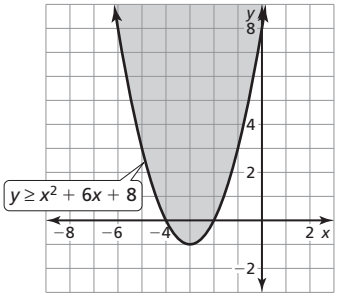

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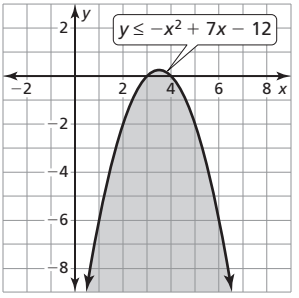

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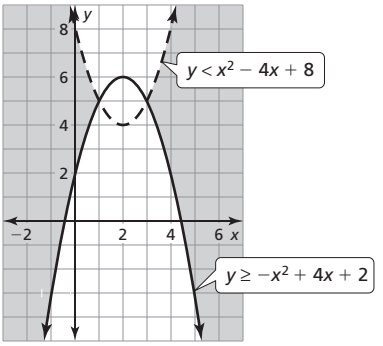

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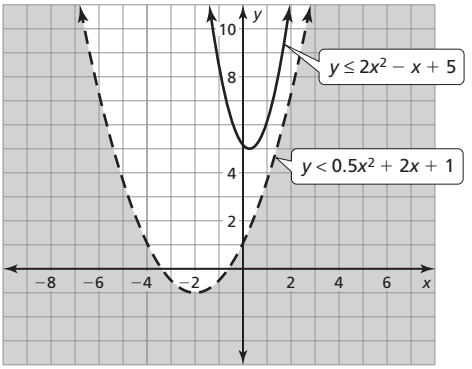
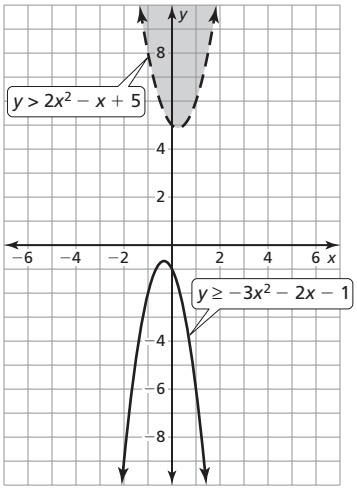


32.



33. $x \leq -5$ or $x \geq 4$ **34.** $x < -7$ or $x > -3$

35. $\frac{2}{3} \leq x \leq 1$

Chapter 4

Chapter 4 Maintaining Mathematical Proficiency *(p. 155)*

1. $2x$ **2.** $4m + 3$ **3.** $-y + 6$ **4.** $x + 4$

5. $z - 4$ **6.** $5x$ **7.** 64 in.^{3} **8.** $\frac{32\pi}{3}$ ft$^2 \approx 33.51$ ft^3

9. 48 ft^3 **10.** 45π cm$^3 \approx 141.37$ cm^3

11. no; If the volume of a cube is doubled, the side length is increased by a factor of $\sqrt[3]{2}$.

4.1 Vocabulary and Core Concept Check *(p. 162)*

1. The end behavior describes the behavior of a graph as x approaches positive infinity and negative infinity.

4.1 Monitoring Progress and Modeling with Mathematics *(pp. 162–164)*

3. polynomial function; $f(x) = 5x^3 - 6x^2 - 3x + 2$; degree: 3 (cubic), leading coefficient: 5

5. not a polynomial function

7. polynomial function; $h(x) = -\sqrt{7}x^4 + 8x^3 + \frac{5}{3}x^2 + x - \frac{1}{2}$; degree 4: (quartic), leading coefficient: $-\sqrt{7}$

9. The function is not in standard form so the wrong term was used to classify the function; f is a polynomial function. The degree is 4 and f is a quartic function. The leading coefficient is -7.

11. $h(-2) = -46$ **13.** $g(8) = -43$ **15.** $p\left(\frac{1}{2}\right) = \frac{45}{4}$

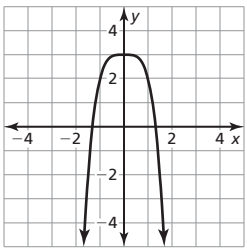
17. $h(x) \to -\infty$ as $x \to -\infty$ and $h(x) \to -\infty$ as $x \to \infty$

19. $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$

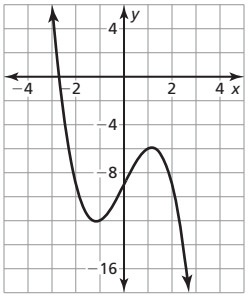
21. The degree of the function is odd and the leading coefficient is negative.

23. polynomial function; $f(x) = -4x^4 + \frac{5}{2}x^3 + \sqrt{2}x^2 + 4x - 6$; degree: 4 (quartic), leading coefficient: -4.

25.



27.



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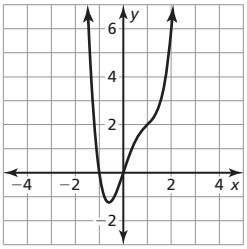
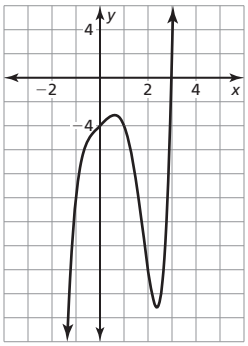


31.



33. **a.** The function is increasing when $x > 4$ and decreasing when $x < 4$.

b. $x < 3$ and $x > 5$

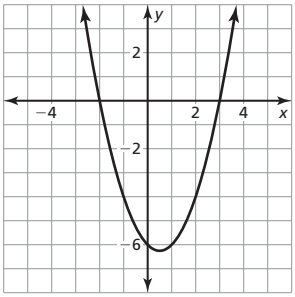
c. $3 < x < 5$

35. **a.** The function is increasing when $x < 0$ and $x > 2$ and decreasing when $0 < x < 2$.

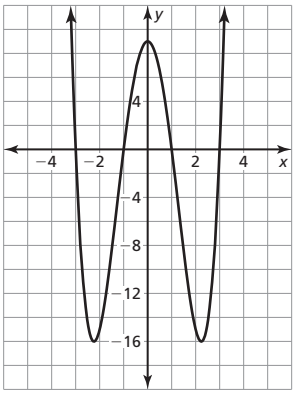
b. $-1 < x < 2$ and $x > 2$

c. $x < -1$

37. The degree is even and the leading coefficient is positive.

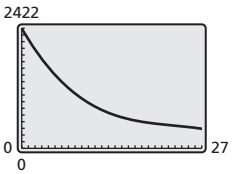


39. The degree is even and the leading coefficient is positive.



Selected Answers

41. **a.**



From 1980 to 2007 the number of open drive-in theaters decreased. Around the year 1995, the rate of decrease began to level off.

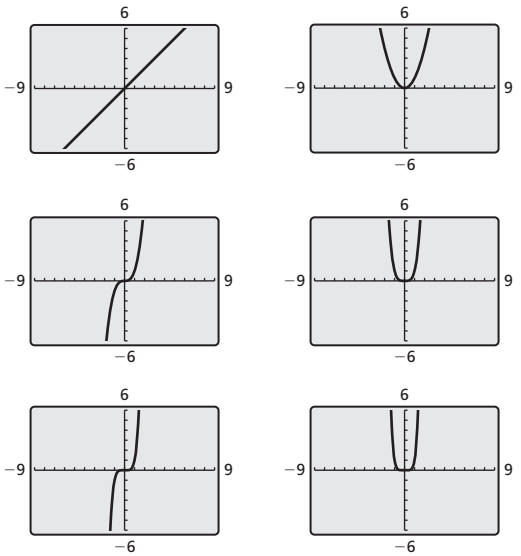
b. 1980 to 1995: about -119.6, 1995 to 2007: about -19.2; About 120 drive-in movie theaters closed each year on average from 1980 to 1995. From 1995 to 2007, drive-in movie theaters were closing at a much lower rate, with about 20 theaters closing each year.

c. Because the graph declines so sharply in the years leading up to 1980, it is most likely not accurate. The model may be valid for a few years before 1980, but in the long run, decline may not be reasonable. After 2007, the number of drive-in movie theaters declines sharply and soon becomes negative. Because negative values do not make sense given the context, the model cannot be used for years after 2007.

43. Because the graph of g is a reflection of the graph of f in the y-axis, the end behavior would be opposite; $g(x) \to -\infty$ as $x \to -\infty$ and $g(x) \to \infty$ as $x \to \infty$.

45. The viewing window is appropriate if it shows the end behavior of the graph as $x \to \infty$ and $x \to -\infty$.

47. **a.**



$y = x$, $y = x^3$, and $y = x^5$ are all symmetric with respect to the origin.

$y = x^2$, $y = x^4$, and $y = x^6$ are all symmetric with respect to the y-axis.

b. The graph of $y = x^{10}$ will be symmetric with respect to the y-axis. The graph of $y = x^{11}$ will be symmetric with respect to the origin; The exponent is even. The exponent is odd.

49. $f(-5) = -480$; Substituting the two given points into the function results in the system of equations $2 + b + c - 5 = 0$ and $16 + 4b + 2c - 5 = 3$. Solving for b and c gives $f(x) = 2x^3 - 7x^2 + 10x - 5$.

4.1 Maintaining Mathematical Proficiency *(p. 164)*

51. $-2x^2 + 3xy + y^2$ **53.** $12kz - 4kw$

55. $-x^3y^2 + 3x^2y + 13xy - 12x + 9$

4.2 Vocabulary and Core Concept Check *(p. 170)*

1. The binomials could be multiplied in a horizontal format or a vertical format. The patterns from Pascal's Triangle could also be used.

4.2 Monitoring Progress and Modeling with Mathematics *(pp. 170–172)*

3. $x^2 + x + 1$ **5.** $12x^5 + 5x^4 - 3x^3 + 6x - 4$

7. $7x^6 + 7x^5 + 8x^3 - 9x^2 + 11x - 5$

9. $-2x^3 - 14x^2 + 7x - 4$

11. $5x^6 - 7x^5 + 6x^4 + 9x^3 + 7$

13. $-x^5 + 7x^3 + 11x^2 + 10x - 4$

15. $P = 47.7t^2 + 678.5t + 17{,}667.4$; The constant term represents the total number of people attending degree-granting institutions at time $t = 0$.

17. $35x^5 + 21x^4 + 7x^3$ **19.** $-10x^3 + 23x^2 - 24x + 18$

21. $x^4 - 5x^3 - 3x^2 + 22x + 20$

23. $3x^5 - 6x^4 - 6x^3 + 25x^2 - 23x + 7$

25. The negative was not distributed through the entire second set of parenthesis;

$$(x^2 - 3x + 4) - (x^3 + 7x - 2) = x^2 - 3x + 4 - x^3 - 7x + 2$$
$$= -x^3 + x^2 - 10x + 6$$

27. $x^3 + 3x^2 - 10x - 24$ **29.** $12x^3 - 29x^2 + 7x + 6$

31. $-24x^3 + 86x^2 - 57x - 20$

33. $(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$;
Sample answer: $24 \cdot 16 = (20 + 4)(20 - 4)$
$= 20^2 - 4^2$
$= 400 - 16$
$= 384$

35. $x^2 - 81$ **37.** $9c^2 - 30c + 25$ **39.** $49h^2 + 56h + 16$

41. $8k^3 + 72k^2 + 216k + 216$ **43.** $8t^3 + 48t^2 + 96t + 64$

45. $16q^4 - 96q^3 + 216q^2 - 216q + 81$

47. $y^5z^5 + 5y^4z^4 + 10y^3z^3 + 10y^2z^2 + 5yz + 1$

49. $9a^8 + 66a^6b^2 + 97a^4b^4 - 88a^2b^6 + 16b^8$; *Sample answer:* Pascal's Triangle; Use Pascal's Triangle to expand the two binomials. Multiply the results vertically to find your final product.

51. $2x^3 + 10x^2 + 14x + 6$

53. **a.** $5000(1 + r)^3 + 1000(1 + r)^2 + 4000(1 + r)$

b. $7000r^3 + 25{,}000r^2 + 34{,}000r + 16{,}000$; 7000 is the total amount of money that gained interest for three years, 25,000 is the total amount of money that gained interest for two years, 34,000 is the total amount of money that gained interest for one year, and 16,000 is the total amount of money invested.

c. about \$17,763.38

55. no; The sum of $(x + 3)$ and $(x - 3)$ is $2x$, a monomial. The product of $(x + 3)$ and $(x - 3)$ is $x^2 - 9$, a binomial.

57. equivalent; They produce the same graph.

59. not equivalent; Although they appear to produce the same graph, the table of values shows they are off by a constant of 1.

61.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

$(x + 3)^7 = x^7 + 21x^6 + 189x^5 + 945x^4 + 2835x^3 + 5103x^2 + 5103x + 2187;$

$(x - 5)^9 = x^9 - 45x^8 + 900x^7 - 10{,}500x^6 + 78{,}750x^5 - 393{,}750x^4 + 1{,}312{,}500x^3 - 2{,}812{,}500x^2 + 3{,}515{,}625x - 1{,}953{,}125$

63. **a.** 5 **b.** 5 **c.** 9

d. $g(x) + h(x)$ has degree m. $g(x) - h(x)$ has degree m. $g(x) \cdot h(x)$ has degree $(m + n)$.

65. **a.** $(x^2 - y^2)^2 + (2xy)^2 = (x^2 + y^2)^2$

$(x^4 - 2x^2y^2 + y^4) + (4x^2y^2) = x^4 + 2x^2y^2 + y^4$

$x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4$

b. The Pythagorean triple is 11, 60, and 61.

c. $121 + 3600 = 3721$

$3721 = 3721$

4.2 Maintaining Mathematical Proficiency *(p. 172)*

67. $5 + 11i$ **69.** $9 - 2i$

4.3 Vocabulary and Core Concept Check *(p. 177)*

1. To evaluate the function $f(x) = x^3 - 2x + 4$ when $x = 3$, synthetic division can be used to divide $f(x)$ by the factor $x - 3$. The remainder is the value of $f(3)$. So, $f(3) = 25$.

Sample answer:

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -2 & 4 \\ & & 3 & 9 & 21 \\ \hline & 1 & 3 & 7 & 25 \end{array}$$

3. $(x^3 - 2x^2 - 9x + 18) \div (x + 3) = x^2 - 5x + 6$

4.3 Monitoring Progress and Modeling with Mathematics *(pp. 177–178)*

5. $x + 5 + \dfrac{3}{x - 4}$ **7.** $x + 1 + \dfrac{2x + 3}{x^2 - 1}$

9. $5x^2 - 12x + 37 + \dfrac{-122x + 109}{x^2 + 2x - 4}$ **11.** $x + 12 + \dfrac{49}{x - 4}$

13. $2x - 11 + \dfrac{62}{x + 5}$ **15.** $x + 3 + \dfrac{18}{x - 3}$

17. $x^3 + x^2 - 2x + 1 - \dfrac{6}{x - 6}$

19. D; $(2)^2 + (2) - 3 = 3$ so the remainder must be 3.

21. C; $(2)^2 - (2) + 3 = 5$ so the remainder must be 5.

23. The quotient should be one degree less than the dividend.

$\dfrac{x^3 - 5x + 3}{x - 2} = x^2 + 2x - 1 + \dfrac{1}{x - 2}$

25. $f(-1) = 37$ **27.** $f(2) = 11$

29. $f(6) = 181$ **31.** $f(3) = 115$

33. no; The Remainder Theorem states that $f(a) = 15$.

35. $\dfrac{A}{T} = \dfrac{-1.95x^3 + 70.1x^2 - 188x + 2150}{14.8x + 725}$

$= 0.13x^2 + 11.19x - 560.90 + \dfrac{408{,}563.25}{14.8x + 725}, 0 < x < 18$

37. A **39.** $2x + 5$

4.3 Maintaining Mathematical Proficiency *(p. 178)*

41. $x = 3$ **43.** $x = -7$

4.4 Vocabulary and Core Concept Check *(p. 184)*

1. quadratic; $3x^2$

3. It is written as a product of unfactorable polynomials with integer coefficients.

4.4 Monitoring Progress and Modeling with Mathematics *(pp. 184–186)*

5. $x(x - 6)(x + 4)$ **7.** $3p^3(p - 8)(p + 8)$

9. $q^2(2q - 3)(q + 6)$ **11.** $w^8(5w - 2)(2w - 3)$

13. $(x + 4)(x^2 - 4x + 16)$ **15.** $(g - 7)(g^2 + 7g + 49)$

17. $3h^6(h - 4)(h^2 + 4h + 16)$

19. $2t^4(2t + 5)(4t^2 - 10t + 25)$

21. $x^2 + 9$ is not a factorable binomial because it is not the difference of two squares; $3x^3 + 27x = 3x(x^2 + 9)$

23. $(y^2 + 6)(y - 5)$ **25.** $(3a^2 + 8)(a + 6)$

27. $(x - 2)(x + 2)(x - 8)$ **29.** $(2q + 3)(2q - 3)(q - 4)$

31. $(7k^2 + 3)(7k^2 - 3)$ **33.** $(c^2 + 5)(c^2 + 4)$

35. $(4z^2 + 9)(2z + 3)(2z - 3)$ **37.** $3r^2(r^3 + 5)(r^3 - 4)$

39. factor **41.** not a factor **43.** factor

45.

$$\begin{array}{r|rrrr} -4 & 1 & -1 & -20 & 0 \\ & & -4 & 20 & 0 \\ \hline & 1 & -5 & 0 & 0 \end{array}$$

$g(x) = x(x + 4)(x - 5)$

47.

$$\begin{array}{r|rrrrr} 6 & 1 & -6 & 0 & -8 & 48 \\ & & 6 & 0 & 0 & -48 \\ \hline & 1 & 0 & 0 & -8 & 0 \end{array}$$

$f(x) = (x - 6)(x - 2)(x^2 + 2x + 4)$

49.

$$\begin{array}{r|rrrr} -7 & 1 & 0 & -37 & 84 \\ & & -7 & 49 & -84 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

$r(x) = (x + 7)(x - 3)(x - 4)$

51. D; The x-intercepts of the graph are 2, 3, and -1.

53. A; The x-intercepts of the graph are -2, -3, and 1.

55. The model makes sense for $x > 6.5$; When factored completely, the volume is $V = x(2x - 13)(x - 3)$. For all three dimensions of the box to have positive lengths, the value of x must be greater than 6.5.

57. $a^4(a + 6)(a - 5)$; A common monomial can be factored out to obtain a factorable trinomial in quadratic form.

59. $(z - 3)(z + 3)(z - 7)$; Factoring by grouping can be used because the expression contains pairs of monomials that have a common factor. Difference of two squares can be used to factor one of the resulting binomials.

61. $(4r + 9)(16r^2 - 36r + 81)$; The sum of two cubes pattern can be used because the expression is of the form $a^3 + b^3$.

63. $(4n^2 + 1)(2n - 1)(2n + 1)$; The difference of two squares pattern can be used to factor the original expression and one of the resulting binomials.

65. **a.** no; $7z^4(2z + 3)(z - 2)$

b. no; $n(2 - n)(n + 6)(3n - 11)$ **c.** yes

67. 0.7 million

69. *Sample answer:* Factor Theorem and synthetic division; Calculations without a calculator are easier with this method because the values are lesser.

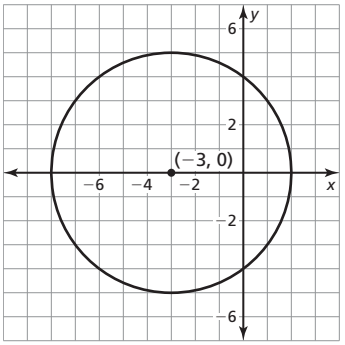
71. $k = 22$

7	2	−13	−22	105
		14	7	−105
	2	1	−15	0

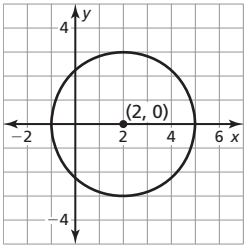
73. **a.** $(c - d)(c + d)(7a + b)$ **b.** $(x^n - 1)(x^n - 1)$

c. $(a^3 - b^2)(ab + 1)^2$

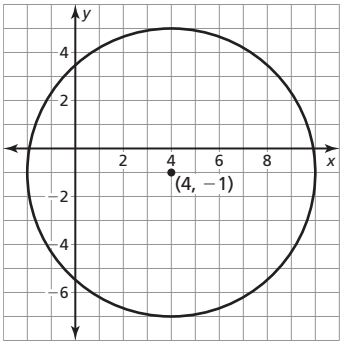
75. **a.** $(x + 3)^2 + y^2 = 5^2$; The center of the circle is $(-3, 0)$ and the radius is 5.



b. $(x - 2)^2 + y^2 = 3^2$; The center of the circle is $(2, 0)$ and the radius is 3.



c. $(x - 4)^2 + (y + 1)^2 = 6^2$; The center of the circle is $(4, -1)$ and the radius is 6.



4.4 Maintaining Mathematical Proficiency *(p. 186)*

77. $x = 6$ and $x = -5$ **79.** $x = \frac{5}{3}$ and $x = 2$

81. $x = 18$ and $x = -6$ **83.** $x = -3$ and $x = -7$

4.5 Vocabulary and Core Concept Check *(p. 194)*

1. constant term; leading coefficient

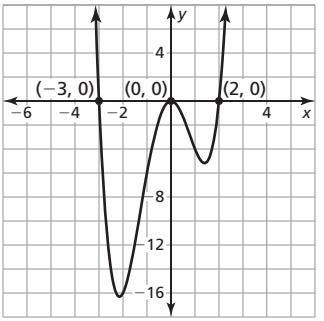
4.5 Monitoring Progress and Modeling with Mathematics *(pp. 194–196)*

3. $z = -3$, $z = 0$, and $z = 4$ **5.** $x = 0$ and $x = 1$

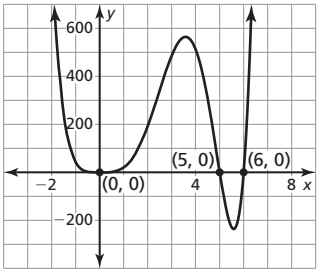
7. $w = 0$ and $w = \pm\sqrt{10} \approx \pm 3.16$

9. $c = 0$, $c = 3$, and $c = \pm\sqrt{6} \approx \pm 2.45$ **11.** $n = -4$

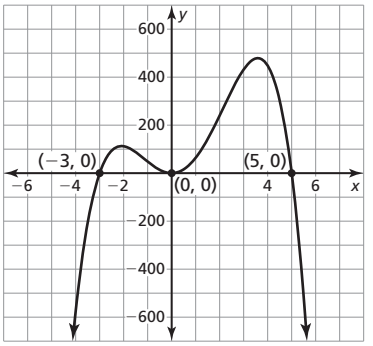
13. $x = -3$, $x = 0$, and $x = 2$



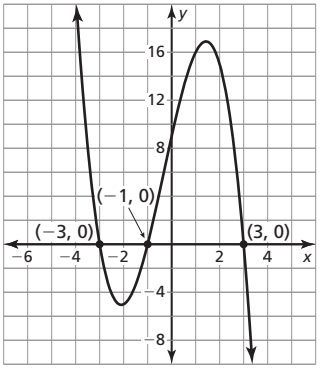
15. $x = 0$, $x = 5$, and $x = 6$



17. $x = -3$, $x = 0$, and $x = 5$



19. $x = -3$, $x = -1$, and $x = 3$



21. C

23. The $\pm$ was not included with each factor; ± 1, ± 3, ± 5, ± 9, ± 15, ± 45

25. $x = -5$, $x = 1$, and $x = 3$ **27.** $x = -1$, $x = 5$, and $x = 6$

29. $x = -3$, $x = 4$, and $x = 5$

31. $x = -4$, $x = -0.5$, and $x = 6$ **33.** -5, 3, and 4

35. -5, -3, and -2 **37.** -4, 1.5, and 3

39. $1, \dfrac{-1 + \sqrt{17}}{2} \approx 1.56$, and $\dfrac{-1 - \sqrt{17}}{2} \approx -2.56$

41. $f(x) = x^3 - 7x^2 + 36$ **43.** $f(x) = x^3 - 10x - 12$

45. $f(x) = x^4 - 32x^2 + 24x$

47. $x = -3$, $x = 3$, and $x = 4$; *Sample answer:* graphing; The equation has three real solutions, all which can be found by graphing to find the x-intercepts.

49. 4 cm by 4 cm by 7 cm

51. The block is 3 meters high, 21 meters long, and 15 meters wide.

53. **a.** $-20t^3 + 252t^2 - 280t - 2400 = 0$

b. 1, 2, 3, 4, 5, 6, 8, 10 **c.** $t = 5$ years and $t = 10$ years

55. The length should be 8 feet, the width should be 4 feet, and the height should be 4 feet.

57. **a.** $k = 60$ **b.** $k = 33$ **c.** $k = 6$ **59.** $x = 1$

61. $x = 2$

63. The height of each ramp is $\frac{5}{3}$ feet and the width of each ramp is 5 feet. The left ramp is to be 24 feet in length while the right ramp is to be 12 feet in length.

65. rs; Each factor of a_0 can be written as the numerator with each factor of a_n as the denominator, creating $r \times s$ factors.

4.5 Maintaining Mathematical Proficiency *(p. 196)*

67. not a polynomial function

69. not a polynomial function; The term $\sqrt[4]{x}$ has an exponent that is not a whole number.

71. $x = \pm 3i$ **73.** $x = \pm\dfrac{\sqrt{2}}{4}$

4.6 Vocabulary and Core Concept Check *(p. 202)*

1. complex conjugates

4.6 Monitoring Progress and Modeling with Mathematics *(pp. 202–204)*

3. 4 **5.** 6 **7.** 7 **9.** -1, 1, 2, and 4

11. -2, -2, 1, and 3 **13.** -3, -1, $2i$, and $-2i$

15. -4, -1, 2, $i\sqrt{2}$, and $-i\sqrt{2}$

17. 2; The graph shows 2 real zeros, so the remaining zeros must be imaginary.

19. 2; The graph shows no real zeros, so all of the zeros must be imaginary.

21. $f(x) = x^3 + 4x^2 - 7x - 10$

23. $f(x) = x^3 - 11x^2 + 41x - 51$

25. $f(x) = x^3 - 4x^2 - 5x + 20$

27. $f(x) = x^5 - 8x^4 + 23x^3 - 32x^2 + 22x - 4$

29. The conjugate of the given imaginary zeros was not included.

$$\begin{aligned} f(x) &= (x - 2)[x - (1 + i)][x - (1 - i)] \\ &= (x - 2)[(x - 1) - i][(x - 1) + i] \\ &= (x - 2)[(x - 1)^2 - i^2] \\ &= (x - 2)[(x^2 - 2x + 1) - (-1)] \\ &= (x - 2)(x^2 - 2x + 2) \\ &= x^3 - 2x^2 + 2x - 2x^2 + 4x - 4 \\ &= x^3 - 4x^2 + 6x - 4 \end{aligned}$$

31. *Sample answer:* $y = x^6 - 4x^4 - x^2 + 4$;

$$\begin{aligned} y &= (x - 1)(x + 1)(x - 2)(x + 2)(x - i)(x + i) \\ &= (x^2 - 1)(x^2 - 4)(x^2 + 1) \\ &= (x^4 - 5x^2 + 4)(x^2 + 1) \\ &= x^6 + x^4 - 5x^4 - 5x^2 + 4x^2 + 4 \\ &= x^6 - 4x^4 - x^2 + 4 \end{aligned}$$

33.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	1	2	4

35.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	1	0	3
0	1	2	3

37.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	2	0	5
3	0	2	5
1	2	2	5
1	0	4	5

39.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
3	3	0	6
3	1	2	6
1	3	2	6
1	1	4	6

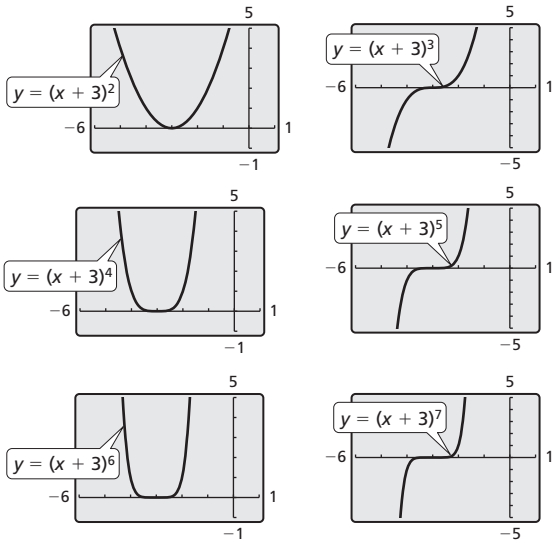
41. C; There are two sign changes in the coefficients of $f(-x)$. So, the number of negative real zeros is two or zero, not four.

43. in the year 1958 **45.** in the 3rd year and the 9th year

47. $x = 4.2577$

49. no; The Fundamental Theorem of Algebra applies to functions of degree greater than zero. Because the function $f(x) = 2$ is equivalent to $f(x) = 2x^0$, it has degree 0, and does not fall under the Fundamental Theorem of Algebra.

51.



Selected Answers

a. For all functions, $f(x) \to \infty$ as $x \to \infty$. When n is even, $f(x) \to \infty$ as $x \to -\infty$, but when n is odd, $f(x) \to -\infty$ as $x \to -\infty$.

b. As n increases, the graph becomes more flat near the zero $x = -3$.

c. The graph of g becomes more vertical and straight near $x = 4$.

53. a.

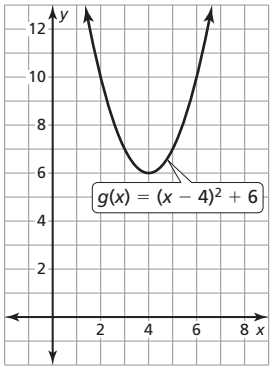
Deposit	Year 1	Year 2	Year 3	Year 4
1st Deposit	1000	$1000g$	$1000g^2$	$1000g^3$
2nd Deposit		1000	$1000g$	$1000g^2$
3rd Deposit			1000	$1000g$
4th Deposit				1000

b. $v = 1000g^3 + 1000g^2 + 1000g + 1000$

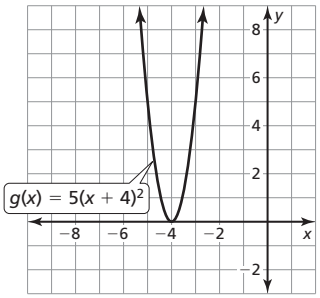
c. about 1.0484; about 4.84%

4.6 Maintaining Mathematical Proficiency *(p. 204)*

55. The function is a translation 4 units right and 6 units up of the parent quadratic function.



57. The function is a vertical stretch by a factor of 5 followed by a translation 4 units left of the parent quadratic function.



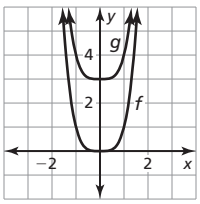
59. $g(x) = \left|\frac{1}{9}x + 1\right| - 3$

4.7 Vocabulary and Core Concept Check *(p. 209)*

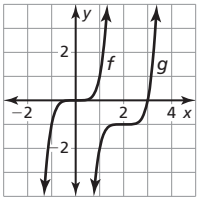
1. horizontal

4.7 Monitoring Progress and Modeling with Mathematics *(pp. 209–210)*

3. The graph of g is a translation 3 units up of the graph of f.



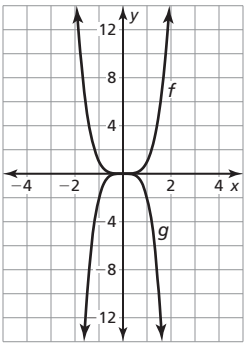
5. The graph of g is a translation 2 units right and 1 unit down of the graph of f.



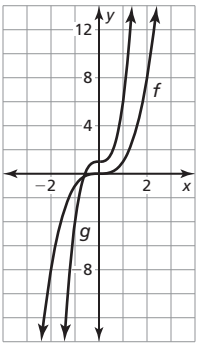
7. B; The graph has been translated 2 units right.

9. D; The graph has been translated 2 units right and 2 units up.

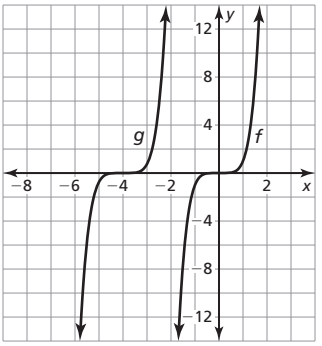
11. The graph of g is a vertical stretch by a factor of 2 followed by a reflection in the x-axis of the graph of f.



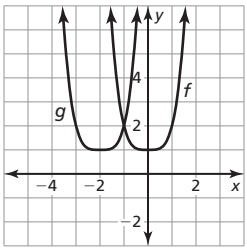
13. The graph of g is a vertical stretch by a factor of 5 followed by a translation 1 unit up of the graph of f.



15. The graph of g is a vertical shrink by a factor of $\frac{3}{4}$ followed by a translation 4 units left of the graph of f.

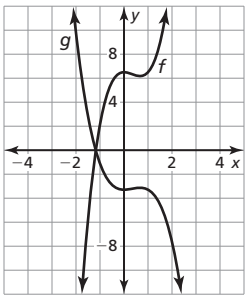


17. $g(x) = (x + 2)^4 + 1$;



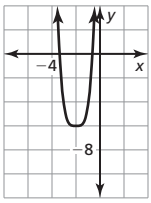
The graph of g is a translation 2 units left of the graph of f.

19. $g(x) = -x^3 + x^2 - 3$;



The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ followed by a reflection in the x-axis of the graph of f.

21. The graph has been translated horizontally to the right 2 units instead of to the left 2 units.



23. $g(x) = -x^3 + 9x^2 - 27x + 21$

25. $g(x) = -27x^3 - 18x^2 + 7$

27. $W(x) = 27x^3 - 12x$; $W(5) = 3315$; When x is 5 yards, the volume of the pyramid is 3315 cubic feet.

29. *Sample answer:* If the function is translated up and then reflected in the x-axis, the order is important; If the function is translated left and then reflected in the x-axis, the order is not important; Reflecting a graph in the x-axis does not affect its x-coordinate, but it does affect its y-coordinate. So, the order is only important if the translation is vertical.

31. **a.** 0 m, 4 m, and 7 m

b. $g(x) = -\frac{2}{5}(x - 2)(x - 6)^2(x - 9)$

33. $V(x) = 3\pi x^2(x + 3)$; $W(x) = \frac{\pi}{3}x^2\left(\frac{1}{3}x + 3\right)$;

$W(3) = 12\pi \approx 37.70$; When x is 3 feet, the volume of the cone is about 37.70 cubic yards.

4.7 Maintaining Mathematical Proficiency *(p. 210)*

35. The maximum value is 4; The domain is all real numbers and the range is $y \le 4$. The function is increasing to the left of $x = 0$ and decreasing to the right of $x = 0$.

37. The maximum value is 9; The domain is all real numbers and the range is $y \le 9$. The function is increasing to the left of $x = -5$ and decreasing to the right of $x = -5$.

39. The maximum value is 1; The domain is all real numbers and the range is $y \le 1$. The function is increasing to the left of $x = 1$ and decreasing to the right of $x = 1$.

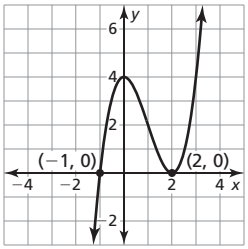
4.8 Vocabulary and Core Concept Check *(p. 216)*

1. turning

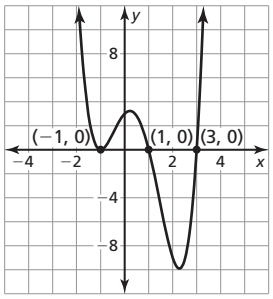
4.8 Monitoring Progress and Modeling with Mathematics *(pp. 216–218)*

3. A **5.** B

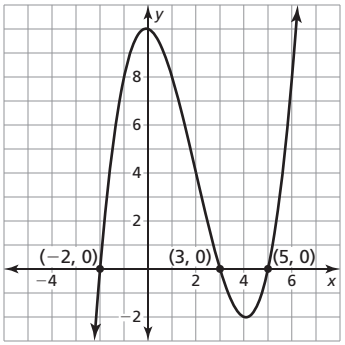
7.



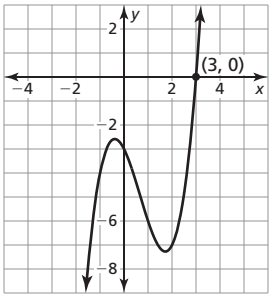
9.



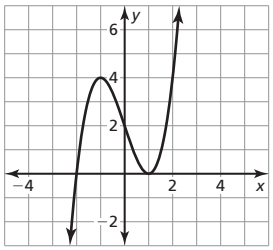
11.



13.

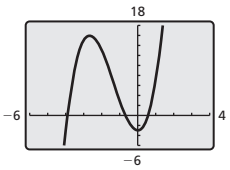


15. The x-intercepts should be -2 and 1.



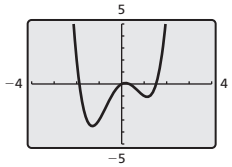
17. -1, 1, and 4 **19.** -4, $-\frac{1}{2}$, and 1 **21.** -4, $\frac{3}{4}$, and 3

23.



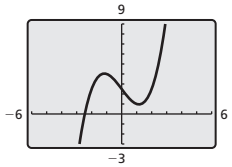
The x-intercepts of the graph are $x \approx -3.90$, $x \approx -0.67$, and $x \approx 0.57$. The function has a local maximum at $(-2.67, 15.96)$ and a local minimum at $(0, -3)$; The function is increasing when $x < -2.67$ and $x > 0$ and is decreasing when $-2.67 < x < 0$.

25.



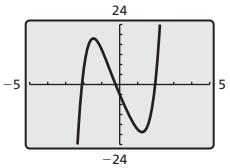
The x-intercepts of the graph are $x \approx -1.88$, $x = 0$, $x \approx 0.35$, and $x \approx 1.53$. The function has a local maximum at $(0.17, 0.08)$ and local minimums at $(-1.30, -3.51)$ and $(1.13, -1.07)$; The function is increasing when $-1.30 < x < 0.17$ and $x > 1.13$ and is decreasing when $x < -1.30$ and $0.17 < x < 1.13$.

27.



The x-intercept of the graph is $x \approx -2.46$. The function has a local maximum at $(-1.15, 4.04)$ and a local minimum at $(1.15, 0.96)$; The function is increasing when $x < -1.15$ and $x > 1.15$ and is decreasing when $-1.15 < x < 1.15$.

29.



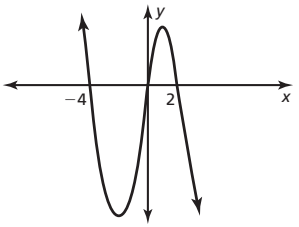
The x-intercepts of the graph are $x \approx -2.10$, $x \approx -0.23$, and $x \approx 1.97$. The function has a local maximum at $(-1.46, 18.45)$ and a local minimum at $(1.25, -19.07)$; The function is increasing when $x < -1.46$ and $x > 1.25$ and is decreasing when $-1.46 < x < 1.25$.

31. $(-0.29, 0.48)$ and $(0.29, -0.48)$; $(-0.29, 0.48)$ corresponds to a local maximum and $(0.29, -0.48)$ corresponds to a local minimum; The real zeros are -0.5, 0, and 0.5. The function is of at least degree 3.

33. $(1, 0)$, $(3, 0)$, and $(2, -2)$; $(1, 0)$ and $(3, 0)$ correspond to local maximums, and $(2, -2)$ corresponds to a local minimum; The real zeros are 1 and 3. The function is of at least degree 4.

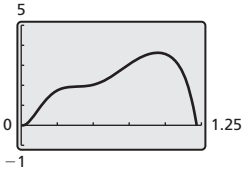
35. $(-1.25, -10.65)$; $(-1.25, -10.65)$ corresponds to a local minimum; The real zeros are -2.07 and 1.78. The function is of at least degree 4.

37.



39. odd **41.** even **43.** neither **45.** even

47.



about 1 second into the stroke

49. A quadratic function only has one turning point, and it is always the maximum or minimum value of the function.

51. no; When multiplying two odd functions, the exponents of each term will be added, creating an even exponent. So, the product will not be an odd function.

53. **a.** $\dfrac{1100 - \pi r^2}{\pi r}$ **b.** $V = 550r - \dfrac{\pi}{2}r^3$ **c.** about 10.8 ft

55. $V(h) = 64\pi h - \dfrac{\pi}{4}h^3$; about 9.24 in.; about 1238.22 in.^{3}

4.8 Maintaining Mathematical Proficiency *(p. 218)*

57. quadratic; The second differences are constant.

4.9 Vocabulary and Core Concept Check *(p. 223)*

1. finite differences

4.9 Monitoring Progress and Modeling with Mathematics *(pp. 223–224)*

3. $f(x) = (x + 1)(x - 1)(x - 2)$

5. $f(x) = \frac{1}{7}(x + 5)(x - 1)(x - 4)$

7. $3; f(x) = \frac{2}{3}x^3 + 4x^2 - \frac{1}{3}x - 4$

9. $4; f(x) = -3x^4 - 5x^3 + 9x^2 + 3x - 1$

11. $4; f(x) = x^4 - 15x^3 + 81x^2 - 183x + 142$

13. The sign in each parentheses is wrong. The x-intercepts should have been subtracted from zero, not added.

$(-6, 0), (1, 0), (3, 0), (0, 54)$

$54 = a(0 + 6)(0 - 1)(0 - 3)$

$54 = 18a$

$a = 3$

$f(x) = 3(x + 6)(x - 1)(x - 3)$

15. *Sample answer:*

$y = (x - 3)(x - 4)(x + 1)$,

$y = 3(x - 3)(x - 4)(x - 1)$,

$y = \frac{1}{2}(x - 3)(x - 4)(x + 4)$;

$$y = a(x - 3)(x - 4)(x - c)$$
$$6 = a(2 - 3)(2 - 4)(2 - c)$$
$$6 = 2a(2 - c)$$
$$3 = a(2 - c)$$
$$\frac{3}{2 - c} = a$$

Any combination of a and c that fit the equation will contain these points.

17. $y = 0.002x^2 + 0.60x - 2.5$; about 15.9 mph

19. $d = \frac{1}{2}n^2 - \frac{3}{2}n$; 35

21. With real-life data sets, the numbers rarely fit a model perfectly. Because of this, the differences are rarely constant.

23. C, A, B, D

4.9 Maintaining Mathematical Proficiency *(p. 224)*

25. $x = \pm 6$ **27.** $x = 3 \pm 2\sqrt{3}$ **29.** $x = 1$ and $x = -2.5$

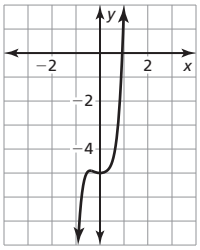
31. $x = \dfrac{-3 \pm \sqrt{29}}{10}$

Chapter 4 Review *(pp. 226–230)*

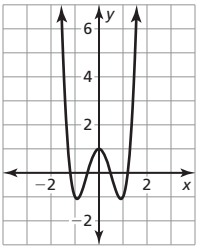
1. polynomial function; $h(x) = -15x^7 - x^3 + 2x^2$; It has degree 7 and has a leading coefficient of -15.

2. not a polynomial

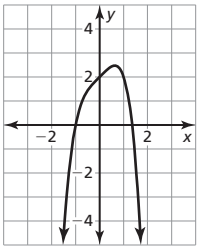
3.



4.



5.



6. $4x^3 - 4x^2 - 4x - 8$ **7.** $3x^4 + 3x^3 - x^2 - 3x + 15$

8. $2x^2 + 11x + 1$ **9.** $2y^3 + 10y^2 + 5y - 21$

10. $8m^3 + 12m^2n + 6mn^2 + n^3$ **11.** $s^3 + 3s^2 - 10s - 24$

12. $m^4 + 16m^3 + 96m^2 + 256m + 256$

13. $243s^5 + 810s^4 + 1080s^3 + 720s^2 + 240s + 32$

14. $z^6 + 6z^5 + 15z^4 + 20z^3 + 15z^2 + 6z + 1$

15. $x - 1 + \dfrac{4x - 3}{x^2 + 2x + 1}$ **16.** $x^2 + 2x - 10 + \dfrac{7x + 43}{x^2 + x + 4}$

17. $x^3 - 4x^2 + 15x - 60 + \dfrac{233}{x + 4}$ **18.** $g(5) = 546$

19. $8(2x - 1)(4x^2 + 2x + 1)$ **20.** $2z(z^2 - 5)(z - 1)(z + 1)$

21. $(a - 2)(a + 2)(2a - 7)$

22.

−2	1	2	0	−27	−54
		−2	0	0	54
	1	0	0	−27	0

$f(x) = (x + 2)(x - 3)(x^2 + 3x + 9)$

23. $x = -4$, $x = -2$, and $x = 3$

24. $x = -4$, $x = -3$, and $x = 2$

25. $f(x) = x^3 - 5x^2 + 5x - 1$

26. $f(x) = x^4 - 5x^3 + x^2 + 25x - 30$

27. $f(x) = x^4 - 9x^3 + 11x^2 + 51x - 30$

28. The length is 6 inches, the width is 2 inches, and the height is 20 inches; When $\ell(\ell - 4)(3\ell + 2) = 240$, $\ell = 6$.

29. $f(x) = x^3 - 5x^2 + 11x - 15$

30. $f(x) = x^4 - x^3 + 14x^2 - 16x - 32$

31. $f(x) = x^4 + 7x^3 + 6x^2 - 4x + 80$

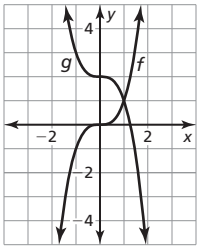
32.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
2	0	2	4
0	0	4	4

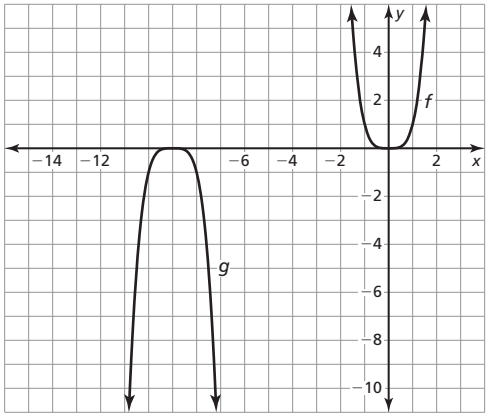
33.

Positive real zeros	Negative real zeros	Imaginary zeros	Total zeros
1	3	0	4
1	1	2	4

34. The graph of g is a reflection in the y-axis followed by a translation 2 units up of the graph of f.



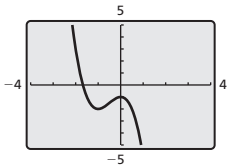
35. The graph of g is a reflection in the x-axis followed by a translation 9 units left of the graph of f.



36. $g(x) = \frac{1}{1024}(x - 3)^5 + \frac{3}{4}(x - 3) - 5$

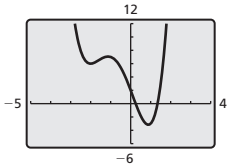
37. $g(x) = x^4 + 2x^3 - 7$

38.



The x-intercept of the graph is $x \approx -1.68$. The function has a local maximum at $(0, -1)$ and a local minimum at $(-1, -2)$; The function is increasing when $-1 < x < 0$ and decreasing when $x < -1$ and $x > 0$.

39.



The x-intercepts of the graph are $x \approx 0.25$ and $x \approx 1.34$. The function has a local maximum at $(-1.13, 7.06)$ and local minimums at $(-2, 6)$ and $(0.88, -3.17)$; The function is increasing when $-2 < x < -1.13$ and $x > 0.88$ and is decreasing when $x < -2$ and $-1.13 < x < 0.88$.

40. odd **41.** even **42.** neither

43. $f(x) = \frac{3}{16}(x + 4)(x - 4)(x - 2)$

44. $3; f(x) = 2x^3 - 7x^2 - 6x$

Chapter 5

Chapter 5 Maintaining Mathematical Proficiency *(p. 235)*

1. y^7 **2.** n **3.** $\frac{1}{x^3}$ **4.** $3x^3$ **5.** $\frac{8w^9}{z^6}$ **6.** $\frac{m^{10}}{z^4}$

7. $y = 2 - 4x$ **8.** $y = 3 + 3x$ **9.** $y = \frac{13}{2}x + \frac{9}{2}$

10. $y = \frac{5}{x + 3}$ **11.** $y = \frac{8x - 3}{4x}$ **12.** $y = \frac{15 - 6x}{7x}$

13. yes; *Sample answer:* When simplifying $x^3 \cdot (x^2)^2$, you must first apply the Power of a Power Property and then apply the Product of Powers Property.

5.1 Vocabulary and Core Concept Check *(p. 241)*

1. $\frac{1}{(\sqrt[t]{a})^s}$; t

3. When a is positive, it has two real fourth roots, $\pm\sqrt[4]{a}$, and one real fifth root $\sqrt[5]{a}$. When a is negative, it has no real fourth roots and one real fifth root, $\sqrt[5]{a}$.

5.1 Monitoring Progress and Modeling with Mathematics *(pp. 241–242)*

5. 2 **7.** 0 **9.** -2 **11.** 2 **13.** 125

15. -3 **17.** $\frac{1}{4}$

19. The cube root of 27 was calculated incorrectly;
$27^{2/3} = (27^{1/3})^2 = 3^2 = 9$

21. B; The denominator of the exponent is 3 and the numerator is 4.

23. A; The denominator of the exponent is 4 and the exponent is negative.

25. 8 **27.** 0.34 **29.** 2840.40 **31.** 50.57

33. $r \approx 3.72$ ft **35.** $x = 5$ **37.** $x \approx -7.66$

39. $x \approx -2.17$ **41.** $x = \pm 2$ **43.** $x = \pm 3$

45. potatoes: 2.4%; ham: 3.7%; eggs: 1.7%

47. 3, 4; $\sqrt[4]{81} = 3$ and $\sqrt[4]{256} = 4$ **49.** about 753 ft^3/sec

5.1 Maintaining Mathematical Proficiency *(p. 242)*

51. 5^5 **53.** $\frac{1}{z^6}$ **55.** 5000 **57.** 0.82

5.2 Vocabulary and Core Concept Check *(p. 248)*

1. No radicands have perfect nth powers as factors other than 1, no radicands contain fractions, and no radicals appear in the denominator of a fraction.

5.2 Monitoring Progress and Modeling with Mathematics *(pp. 248–250)*

3. $9^{2/3}$ **5.** $6^{3/4}$ **7.** $\frac{5}{4}$ **9.** $3^{1/3}$ **11.** 4

13. 12 **15.** $2\sqrt[4]{3}$ **17.** 3 **19.** 6 **21.** $3\sqrt[4]{7}$

23. $\frac{\sqrt[3]{10}}{2}$ **25.** $\frac{\sqrt{6}}{4}$ **27.** $\frac{4\sqrt[3]{7}}{7}$ **29.** $\frac{1 - \sqrt{3}}{-2}$

31. $\frac{15 + 5\sqrt{2}}{7}$ **33.** $\frac{9\sqrt{3} - 9\sqrt{7}}{-4}$ **35.** $\frac{3\sqrt{2} + \sqrt{30}}{-2}$

37. $12\sqrt[3]{11}$ **39.** $12(11^{1/4})$ **41.** $-9\sqrt{3}$ **43.** $5\sqrt[5]{7}$

45. $6(3^{1/3})$

47. The radicand should not change when the expression is factored;
$3\sqrt[3]{12} + 5\sqrt[3]{12} = (3 + 5)\sqrt[3]{12} = 8\sqrt[3]{12}$

49. $3y^2$ **51.** $\frac{m^2}{n}$ **53.** $\frac{|g|}{|h|}$

55. Absolute value was not used to ensure that all variables are positive;
$\frac{\sqrt[6]{2^6(h^2)^6}}{\sqrt[6]{g^6}} = \frac{2h^2}{|g|}$

57. $9a^3b^6c^4\sqrt{ac}$ **59.** $\frac{2m\sqrt[5]{5mn^3}}{n^2}$ **61.** $\frac{\sqrt[6]{w^5}}{5w^6}$

63. $\frac{2v^{3/4}}{3w}$, $v \neq 0$ **65.** $21\sqrt[3]{y}$

67. $-2x^{7/2}$ **69.** $4w^2\sqrt{w}$

71. $P = 2x^3 + 4x^{2/3}$
$A = 2x^{11/3}$

73. about 0.45 mm

75. no; The second radical can be simplified to $18\sqrt{11}$. The difference is $-11\sqrt{11}$.

77. $10 + 6\sqrt{5}$

79. **a.** $r = \sqrt[3]{\frac{3V}{4\pi}}$

b. $S = 4\pi\left(\sqrt[3]{\frac{3V}{4\pi}}\right)^2$

$S = \frac{4\pi(3V)^{2/3}}{(4\pi)^{2/3}}$

$S = (4\pi)^{3/3 - 2/3}(3V)^{2/3}$

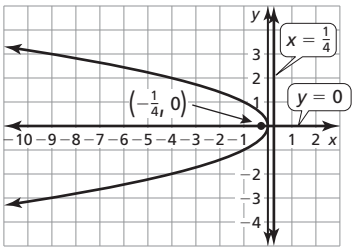
$S = (4\pi)^{1/3}(3V)^{2/3}$

c. The surface area of the larger balloon is $2^{2/3} \approx 1.59$ times as large as the surface area of the smaller balloon.

81. when n is even and $\frac{m}{n}$ is odd

5.2 Maintaining Mathematical Proficiency *(p. 250)*

83. The focus is $\left(-\frac{1}{4}, 0\right)$. The directrix is $x = \frac{1}{4}$. The axis of symmetry is $y = 0$.



85. $g(x) = -x^4 + 3x^2 + 2x$; The graph of g is a reflection in the x-axis of the graph of f.

87. $g(x) = (x - 2)^3 - 4$; The graph of g is a translation 2 units right of the graph of f.

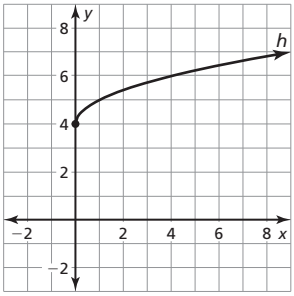
5.3 Vocabulary and Core Concept Check *(p.256)*

1. radical

5.3 Monitoring Progress and Modeling with Mathematics *(pp. 256–258)*

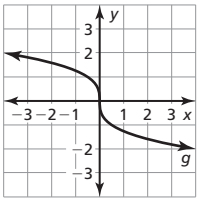
3. B **5.** F **7.** E

9.



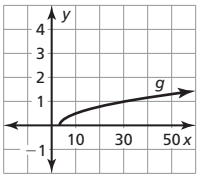
The domain is $x \geq 0$. The range is $y \geq 4$.

11.



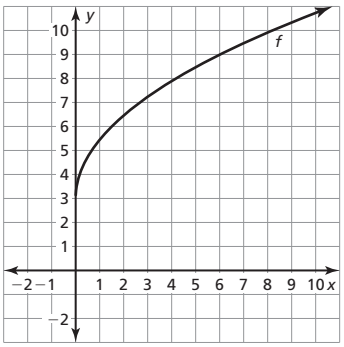
The domain and range are all real numbers.

13.



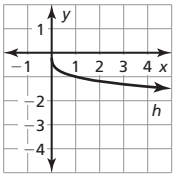
The domain is $x \geq 3$. The range is $y \geq 0$.

15.



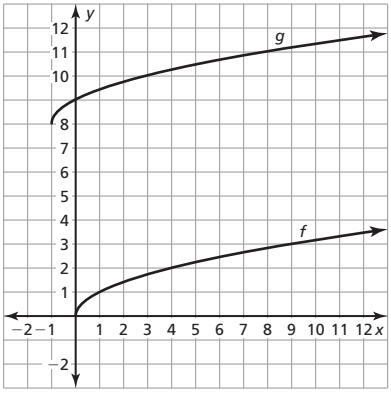
The domain is $x \geq 0$. The range is $y \geq 3$.

17.

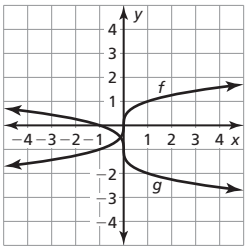


The domain is $x \geq 0$. The range is $y \leq 0$.

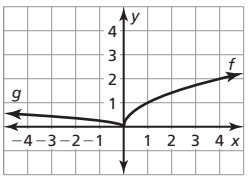
19. The graph of g is a translation 1 unit left and 8 units up of the graph of f.



21. The graph of g is a reflection in the x-axis followed by a translation 1 unit down of the graph of f.

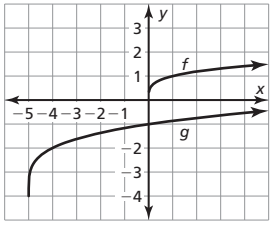


23. The graph of g is a vertical shrink by a factor of $\frac{1}{4}$ followed by a reflection in the y-axis of the graph of f.

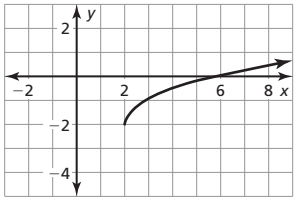


Selected Answers

25. The graph of g is a vertical stretch by a factor of 2 followed by a translation 5 units left and 4 units down of the graph of f.



27. The graph was translated 2 units left but it should be translated 2 units right.



29. The domain is $x \le -1$ and $x \ge 0$. The range is $y \ge 0$.

31. The domain is all real numbers. The range is $y \ge -\dfrac{\sqrt[3]{2}}{2}$.

33. The domain is all real numbers. The range is $y \ge \dfrac{\sqrt{14}}{4}$.

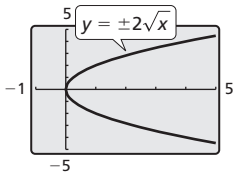
35. always **37.** always

39. $M(n) = 0.915\sqrt{n}$; about 91.5 mi **41.** $g(x) = 2\sqrt{x} + 8$

43. $g(x) = \sqrt{9x + 36}$ **45.** $g(x) = 2\sqrt{x + 1}$

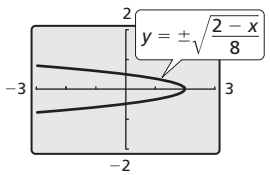
47. $g(x) = 2\sqrt{x + 3}$ **49.** $g(x) = 2\sqrt{(x + 5)^2 - 2}$

51.



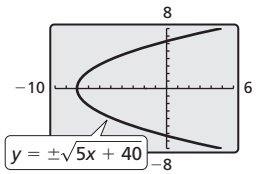
(0, 0), right

53.



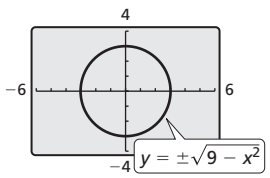
(2, 0), left

55.



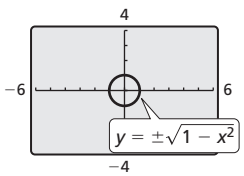
(−8, 0), right

57.



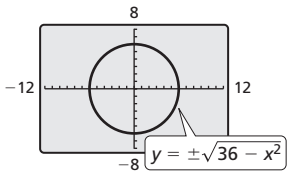
The radius is 3 units. The x-intercepts are ± 3. The y-intercepts are ± 3.

59.



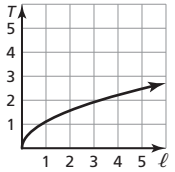
The radius is 1 unit. The x-intercepts are ± 1. The y-intercepts are ± 1.

61.



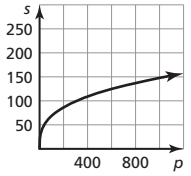
The radius is 6 units. The x-intercepts are ± 6. The y-intercepts are ± 6.

63.



about 3 ft; *Sample answer:* Locate the T-value 2 on the graph and estimate the ℓ-value.

65.



a. about 2468 hp **b.** about 0.04 mph/hp

67. **a.** the 165-lb skydiver

b. When $A = 1$, the diver is most likely vertical. When $A = 7$, the diver is most likely horizontal.

5.3 Maintaining Mathematical Proficiency *(p. 258)*

69. $x = 1$ and $x = -\frac{7}{3}$ **71.** $x = 2$ and $x = 6$

73. $-4 < x < -3$ **75.** $x < 0.5$ and $x > 6$

5.4 Vocabulary and Core Concept Check *(p. 266)*

1. no; The radicand does not contain a variable.

5.4 Monitoring Progress and Modeling with Mathematics *(pp. 266–268)*

3. $x = 7$ **5.** $x = 24$ **7.** $x = 6$ **9.** $x = -\dfrac{1000}{3}$

11. $x = 1024$ **13.** about 21.7 yr **15.** $x = 12$

17. $x = 14$ **19.** $x = 0$ and $x = \frac{1}{2}$ **21.** $x = 3$

23. $x = -1$ **25.** $x = 4$ **27.** $x = \pm 8$

29. no real solution **31.** $x = 3$ **33.** $x = 5$

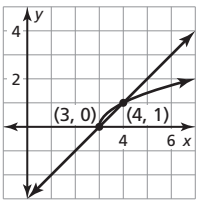
35. Only one side of the equation was cubed;

$$\sqrt[3]{3x - 8} = 4$$
$$\left(\sqrt[3]{3x - 8}\right)^3 = 4^3$$
$$3x - 8 = 64$$
$$x = 24$$

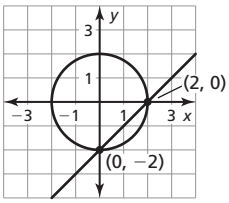
37. $x \ge 64$ **39.** $x > 27$ **41.** $0 \le x \le \frac{25}{4}$

43. $x > -220$ **45.** about 0.15 in.

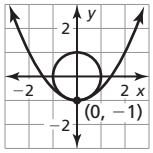
47. (3, 0) and (4, 1);



49. (0, −2) and (2, 0);



51. (0, −1);

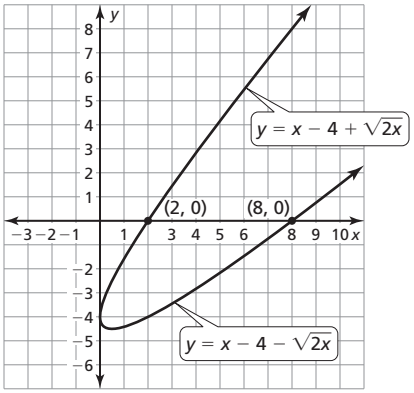


53. **a.** The greatest stopping distance is 450 feet on ice. On wet asphalt and snow, the stopping distance is 225 feet. The least stopping distance is 90 feet on dry asphalt.

b. about 272.2 ft; When $s = 35$ and $f = 0.15$, $d \approx 272.2$.

55. **a.** When solving the first equation, the solution is $x = 8$ with $x = 2$ as an extraneous solution. When solving the second equation, the solution is $x = 2$ with $x = 8$ as an extraneous solution.

b.



57. The square root of a quantity cannot be negative.

59. Raising the price would decrease demand.

61. $36\pi \approx 113.1$ ft^2

63. **a.** $h = h_0 - \dfrac{kt}{\pi r^2}$ **b.** about 5.75 in.

5.4 Maintaining Mathematical Proficiency *(p. 268)*

65. $x^5 + x^4 - 4x^2 + 3$ **67.** $x^3 + 11x - 8$

69. $g(x) = \frac{1}{2}x^3 - 2x^2$; The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 3 units down of the graph of f.

5.5 Vocabulary and Core Concept Check *(p. 273)*

1. You can add, subtract, multiply, or divide f and g.

5.5 Monitoring Progress and Modeling with Mathematics *(pp. 273–274)*

3. $(f + g)(x) = 14\sqrt[4]{x}$ and the domain is $x \geq 0$; $(f - g)(x) = -24\sqrt[4]{x}$ and the domain is $x \geq 0$; $(f + g)(16) = 28$; $(f - g)(16) = -48$

5. $(f + g)(x) = -7x^3 + 5x^2 + x$ and the domain is all real numbers; $(f - g)(x) = -7x^3 - 13x^2 + 11x$ and the domain is all real numbers; $(f + g)(-1) = 11$; $(f - g)(-1) = -17$

7. $(fg)(x) = 2x^{10/3}$ and the domain is all real numbers; $\left(\dfrac{f}{g}\right)(x) = 2x^{8/3}$ and the domain is $x \neq 0$; $(fg)(-27) = 118{,}098$; $\left(\dfrac{f}{g}\right)(-27) = 13{,}122$

9. $(fg)(x) = 36x^{3/2}$ and the domain is $x \geq 0$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{4}{9}x^{1/2}$ and the domain is $x > 0$; $(fg)(9) = 972$; $\left(\dfrac{f}{g}\right)(9) = \dfrac{4}{3}$

11. $(fg)(x) = -98x^{11/6}$ and the domain is $x \geq 0$; $\left(\dfrac{f}{g}\right)(x) = -\dfrac{1}{2}x^{7/6}$; and the domain is $x > 0$; $(fg)(64) = -200{,}704$; $\left(\dfrac{f}{g}\right)(64) = -64$

13. 2541.04; 2458.96; 102,598.56; 60.92

15. 7.76; −14.60; −38.24; −0.31

17. Because the functions have an even index, the domain is restricted; The domain of $(fg)(x)$ is $x \geq 0$.

19. **a.** $(F + M)(t) = 0.0001t^3 - 0.016t^2 + 0.21t + 7.4$

b. the total number of employees from the ages of 16 to 19 in the United States

21. yes; When adding or multiplying functions, the order in which they appear does not matter.

23. $(f + g)(3) = -21$; $(f - g)(1) = -1$; $(fg)(2) = 0$; $\left(\dfrac{f}{g}\right)(0) = 2$

25. $r(x) = x^2 - \frac{1}{2}x^2 = \frac{1}{2}x^2$

27. **a.** $r(x) = \dfrac{20 - x}{6.4}$; $s(x) = \dfrac{\sqrt{x^2 + 144}}{0.9}$

b. $t(x) = \dfrac{20 - x}{6.4} + \dfrac{\sqrt{x^2 + 144}}{0.9}$

c. $x \approx 1.7$; If Elvis runs along the shore until he is about 1.7 meters from point C then swims to point B, the time taken to get there will be a minimum.

5.5 Maintaining Mathematical Proficiency *(p. 274)*

29. $n = \dfrac{5z}{7 + 8z}$ **31.** $n = \dfrac{3}{7b - 4}$

33. no; −1 has two outputs. **35.** no; 2 has two outputs.

5.6 Vocabulary and Core Concept Check *(p. 281)*

1. Inverse functions are functions that undo each other.

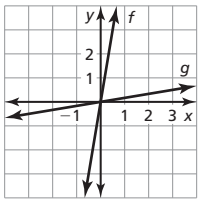
3. x; x

5.6 Monitoring Progress and Modeling with Mathematics *(pp. 281–284)*

5. $x = \dfrac{y - 5}{3}$; $-\dfrac{8}{3}$ **7.** $x = 2y + 6$; 0 **9.** $x = \sqrt[3]{\dfrac{y}{3}}$; −1

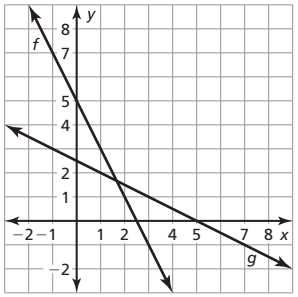
11. $x = 2 \pm \sqrt{y + 7}$; 0, 4

13. $g(x) = \frac{1}{6}x$;

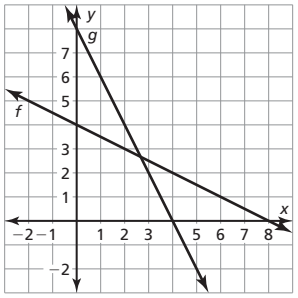


Selected Answers

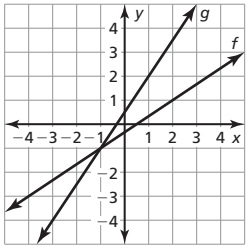
15. $g(x) = \dfrac{x-5}{-2}$;



17. $g(x) = -2x + 8$;

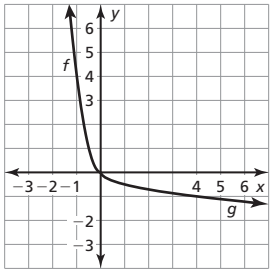


19. $g(x) = \dfrac{3x+1}{2}$;

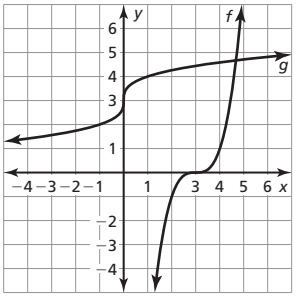


21. $g(x) = \dfrac{x-4}{-3}$; *Sample answer:* switching x and y; You can graph the inverse to check your answer.

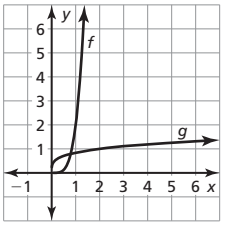
23. $g(x) = -\dfrac{\sqrt{x}}{2}$;



25. $g(x) = \sqrt[3]{x} + 3$



27. $g(x) = \sqrt[4]{\dfrac{x}{2}}$;



29. When switching x and y, the negative should not be switched with the variables;

$y = -x + 3$

$x = -y + 3$

$-x + 3 = y$

31. no; The function does not pass the horizontal line test.

33. no; The function does not pass the horizontal line test.

35. yes; $g(x) = \sqrt[3]{x+1}$

37. yes; $g(x) = x^2 - 4$, where $x \geq 0$ **39.** yes; $g(x) = \dfrac{x^3}{8} + 5$

41. no; $y = \pm\sqrt[4]{x-2}$ **43.** yes; $g(x) = \dfrac{x^3}{27} - 1$

45. yes; $g(x) = \sqrt[5]{2x}$ **47.** B

49. The functions are not inverses.

51. The functions are inverses.

53. $\ell = \left(\dfrac{v}{1.34}\right)^2$; about 31.3 ft **55.** B **57.** A

59. 5; When $x = 5$, $2x^2 + 3 = 53$.

61. **a.** $w = 2\ell - 6$; the weight of an object on a stretched spring of length ℓ

b. 5 lb **c.** $0.5(2\ell - 6) + 3 = \ell$; $2(0.5w + 3) - 6 = w$

63. **a.** $F = \frac{9}{5}C + 32$; The equation converts temperatures in Celsius to Fahrenheit.

b. start: 41° F; end: 14° F **c.** −40°

65. B **67.** A

69. **a.** false; All functions of the form $f(x) = x^n$, where n is an even integer, fail the horizontal line test.

b. true; All functions of the form $f(x) = x^n$, where n is an odd integer, pass the horizontal line test.

71. The inverse $y = \dfrac{1}{m}x - \dfrac{b}{m}$ has a slope of $\dfrac{1}{m}$ and a y-intercept of $-\dfrac{b}{m}$.

5.6 Maintaining Mathematical Proficiency *(p. 284)*

73. $-\frac{1}{3^3}$ **75.** 4^2

77. The function is increasing when $x > 1$ and decreasing when $x < 1$. The function is positive when $x < 0$ and when $x > 2$, and negative when $0 < x < 2$.

79. The function is increasing when $-2.89 < x < 2.89$ and decreasing when $x < -2.89$ and $x > 2.89$. The function is positive when $x < -5$ and $0 < x < 5$ and negative when $-5 < x < 0$ and $x > 5$.

Chapter 5 Review *(pp. 286–288)*

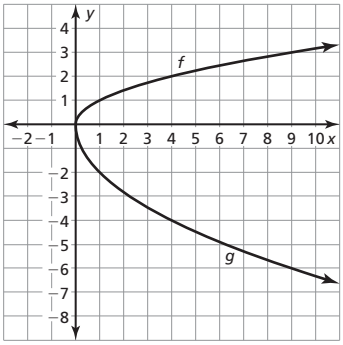
1. 128 **2.** 243 **3.** $\frac{1}{9}$ **4.** $x \approx 1.78$ **5.** $x = 3$

6. $x = -10$ and $x = -6$ **7.** $\frac{1}{6^{3/5}}$ **8.** 4 **9.** $2 + \sqrt{3}$

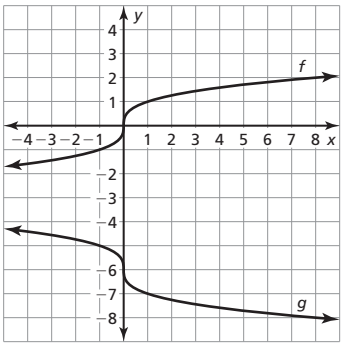
10. $7\sqrt[5]{8}$ **11.** $7\sqrt{3}$ **12.** $5^{1/3} \cdot 2^{3/4}$ **13.** $5z^3$

14. $\frac{\sqrt[4]{2z}}{6}$ **15.** $-z^2\sqrt{10z}$

16. The graph of g is a vertical stretch by a factor of 2 followed by a reflection in the x-axis of the graph of f;

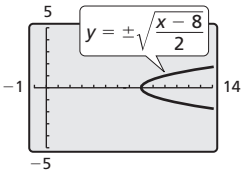


17. The graph of g is a reflection in the y-axis followed by a translation 6 units down of the graph of f.



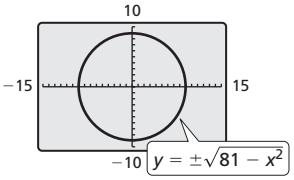
18. $g(x) = \sqrt[3]{-x + 7}$

19.



(8, 0); right

20.



The radius is 9. The x-intercepts are ± 9. The y-intercepts are ± 9.

21. $x = 62$ **22.** $x = 2$ and $x = 10$ **23.** $x = \pm 36$

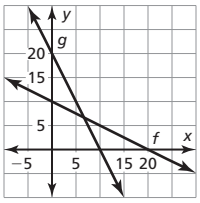
24. $x > 9$ **25.** $8 \le x < 152$ **26.** $x \ge 30$

27. about 4082 m

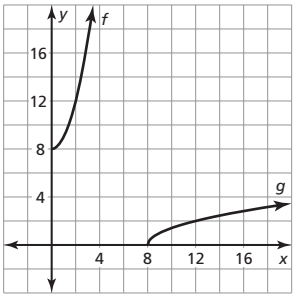
28. $(fg)(x) = 8(3 - x)^{5/6}$ and the domain is $x \le 3$; $\left(\frac{f}{g}\right)(x) = \frac{1}{2}(3 - x)^{1/6}$ and the domain is $x < 3$; $(fg)(2) = 8$; $\left(\frac{f}{g}\right)(2) = \frac{1}{2}$

29. $(f + g)(x) = 3x^2 + x + 5$ and the domain is all real numbers; $(f - g)(x) = 3x^2 - x - 3$ and the domain is all real numbers; $(f + g)(-5) = 75$; $(f - g)(-5) = 77$

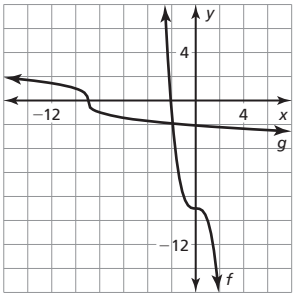
30. $g(x) = -2x + 20$;



31. $g(x) = \sqrt{x - 8}$;

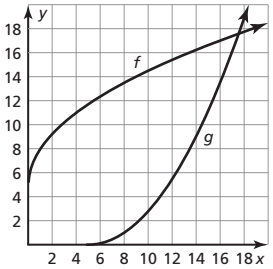


32. $g(x) = \sqrt[3]{-x - 9}$;



Selected Answers

33. $g(x) = \frac{1}{9}(x - 5)^2, x \geq 5$;



34. no **35.** yes **36.** $p = \frac{d}{1.587}$; about 63£

Chapter 6

Chapter 6 Maintaining Mathematical Proficiency *(p. 293)*

1. 48 **2.** -32 **3.** $-\frac{25}{36}$ **4.** $\frac{27}{64}$

5. domain: $-5 \leq x \leq 5$, range: $0 \leq y \leq 5$

6. domain: $\{-2, -1, 0, 1, 2\}$, range: $\{-5, -3, -1, 1, 3\}$

7. domain: all real numbers, range: $y \leq 0$

8. all values, odd values; no values, even values; The exponent of -4^n is evaluated first, then the result is multiplied by -1, so the value will always remain negative. The product of an odd number of negative values is negative. After the exponent of -4^n is evaluated, the result is multiplied by -1, so it will never be positive. The product of an even number of negative values is positive.

6.1 Vocabulary and Core Concept Check *(p. 300)*

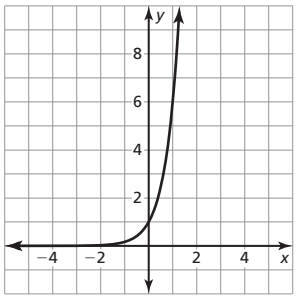
1. The initial amount is 2.4, the growth factor is 1.5, and the percent increase is 0.5 or 50%.

6.1 Monitoring Progress and Modeling with Mathematics *(pp. 300–302)*

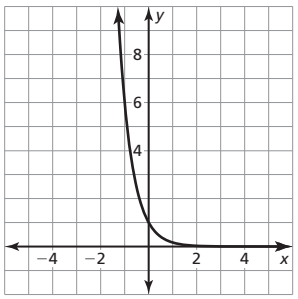
3. **a.** $\frac{1}{4}$ **b.** 8 **5.** **a.** $\frac{8}{9}$ **b.** 216

7. **a.** $\frac{46}{9}$ **b.** 32

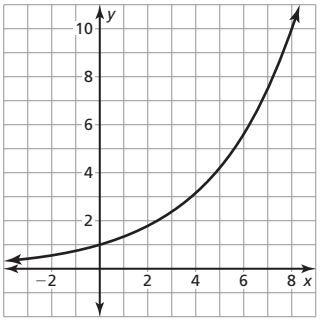
9. exponential growth



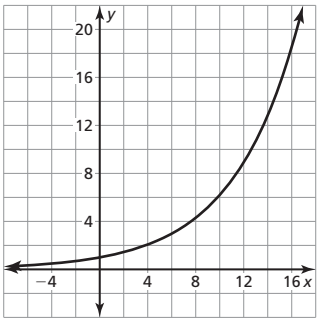
11. exponential decay



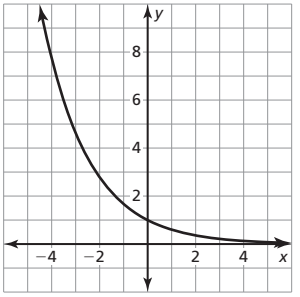
13. exponential growth



15. exponential growth



17. exponential decay



19. $b = 3$

21. **a.** exponential decay **b.** 25% decrease
c. in about 4.8 years

23. **a.** $y = 233(1.06)^t$; about 261.8 million **b.** 2009

25. Power of a Power Property; Evaluate power; Rewrite in form $y = a(1 + r)^t$.

27. about 0.01% **29.** $y = a(1 + 0.26)^t$; 26% growth

31. $y = a(1 - 0.06)^t$; 6% decay

33. $y = a(1 - 0.04)^t$; 4% decay

35. $y = a(1 + 255)^t$; 25,500% growth **37.** \$5593.60

39. The percent decrease needs to be subtracted from 1 to produce the decay factor;
$y = \begin{pmatrix}\text{Initial}\\ \text{amount}\end{pmatrix}\begin{pmatrix}\text{Decay}\\ \text{factor}\end{pmatrix}^t$; $y = 500(1 - 0.02)^t$; $y = 500(0.98)^t$

41. \$3982.92 **43.** \$3906.18

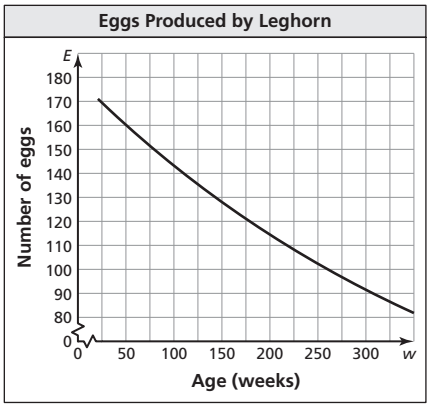
45. a represents the number of referrals it received at the start of the model. b represents the growth factor of the number of referrals each year; 50%; 1.50 can be rewritten as $(1 + 0.50)$, showing the percent increase of 50%.

47. no; $f(x) = 2^x$ eventually increases at a faster rate than $g(x) = x^2$ but not for all $x \geq 0$.

49. 221.5; The curve contains the points (0, 6850) and (6, 8179.26) and $\frac{8179.26 - 6850}{6 - 0} \approx 221.5$.

51. **a.** The decay factor is 0.9978. The percent decrease is 0.22%.

b.



c. about 134 eggs per year

d. Replace $\frac{w}{52}$ with y, where y represents the age of the chicken in years.

6.1 Maintaining Mathematical Proficiency *(p. 302)*

53. x^{11} **55.** $24x^2$ **57.** $2x$ **59.** $3 + 5x$

6.2 Vocabulary and Core Concept Check *(p. 307)*

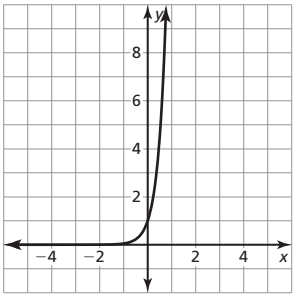
1. an irrational number that is approximately 2.718281828

6.2 Monitoring Progress and Modeling with Mathematics *(pp. 307–308)*

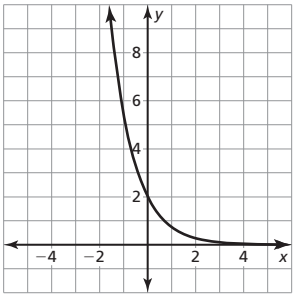
3. e^8 **5.** $\frac{1}{2e}$ **7.** $625e^{28x}$ **9.** $3e^{3x}$ **11.** e^{-5x+8}

13. The 4 was not squared; $(4e^{3x})^2 = 4^2e^{(3x)(2)} = 16e^{6x}$

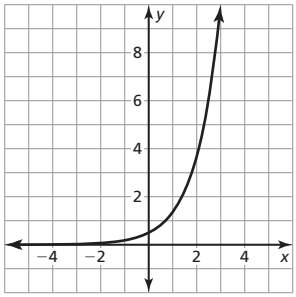
15. exponential growth



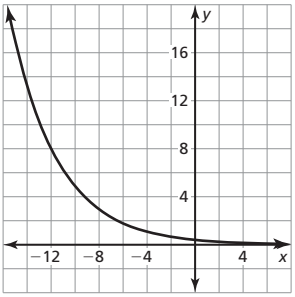
17. exponential decay



19. exponential growth



21. exponential decay



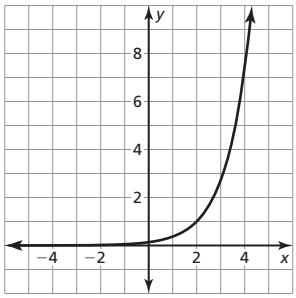
23. D; The graph shows growth and has a y-intercept of 1.

25. B; The graph shows decay and has a y-intercept of 4.

27. $y = (1 - 0.221)^t$; 22.1% decay

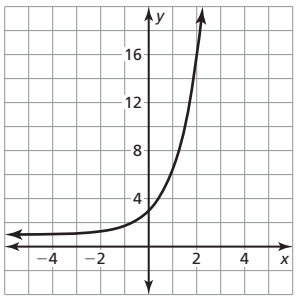
29. $y = 2(1 + 0.492)^t$; 49.2% growth

31.



domain: all real numbers, range: $y > 0$

33.



domain: all real numbers, range: $y > 1$

35. the education fund; the education fund

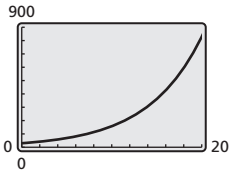
37. *Sample answer:* $a = 6$, $b = 2$, $r = -0.2$, $q = -0.7$

39. no; e is an irrational number. Irrational numbers cannot be expressed as a ratio of two integers.

41. account 1; With account 1, the balance would be $A = 2500\left(1 + \frac{0.06}{4}\right)^{4 \cdot 10} = \$4535.05$. With account 2, the balance would be $A = 2500e^{0.04 \cdot 10} = \$3729.56$.

43. **a.** $N(t) = 30e^{0.166t}$

b.

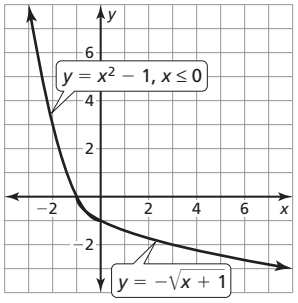


c. At 3:45 P.M., it has been 2 hours and 45 minutes, or 2.75 hours, since 1:00 P.M. Using the *trace* feature of the calculator, type 2.75 to find the point (2.75, 47.356183). At 3:45 P.M., there are about 47 cells.

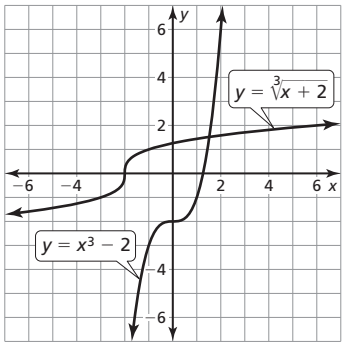
6.2 Maintaining Mathematical Proficiency *(p. 308)*

45. 5×10^3 **47.** 4.7×10^{-8}

49. $y = -\sqrt{x + 1}$



51. $y = \sqrt[3]{x + 2}$



6.3 Vocabulary and Core Concept Check *(p. 314)*

1. common **3.** They are inverse equations.

6.3 Monitoring Progress and Modeling with Mathematics *(pp. 314–316)*

5. $3^2 = 9$ **7.** $6^0 = 1$ **9.** $\left(\frac{1}{2}\right)^{-4} = 16$

11. $\log_6 36 = 2$ **13.** $\log_{16} \frac{1}{16} = -1$ **15.** $\log_{125} 25 = \frac{2}{3}$

17. 4 **19.** 1 **21.** -4 **23.** -1

25. $\log_7 8$, $\log_5 23$, $\log_6 38$, $\log_2 10$ **27.** 0.778

29. -1.099 **31.** -2.079 **33.** 4603 m **35.** x

37. 4 **39.** $2x$

41. -3 and $\frac{1}{64}$ are in the wrong position; $\log_4 \frac{1}{64} = -3$

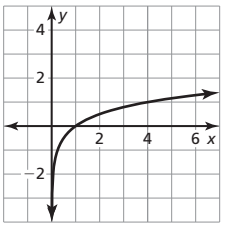
43. $y = \log_{0.3} x$ **45.** $y = 2^x$ **47.** $y = e^x + 1$

49. $y = \frac{1}{3} \ln x$ **51.** $y = \log_5(x + 9)$

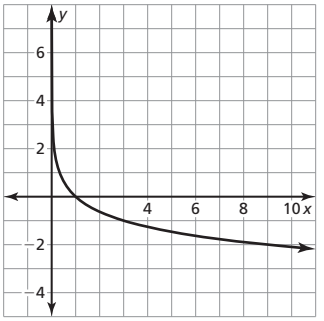
53. **a.** about 283 mi/h

b. $d = 10^{(s - 65)/93}$; The inverse gives the distance a tornado will travel given the wind speed, s.

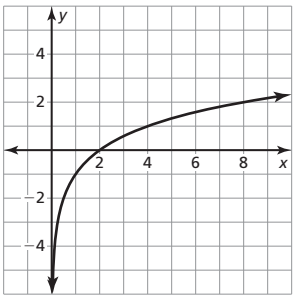
55.



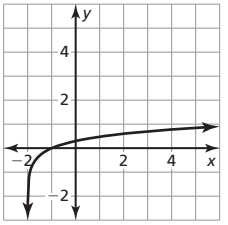
57.



59.

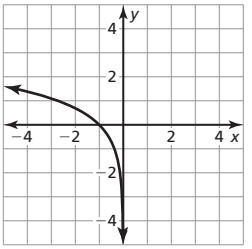


61.



domain: $x > -2$, range: all real numbers, asymptote: $x = -2$

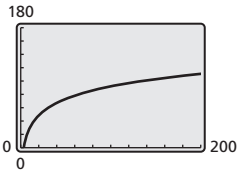
63.



domain: $x < 0$; range: all real numbers, asymptote: $x = 0$

65. no; Any logarithmic function of the form $g(x) = \log_b x$ will pass through (1, 0), but if the function has been translated or reflected in the x-axis, it may not pass through (1, 0).

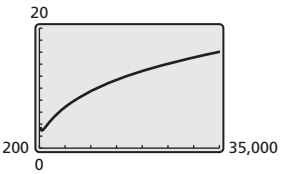
67. **a.**



b. about 281 lb

c. (3.4, 0); no; The x-intercept shows that an alligator with a weight of 3.4 pounds has no length. If an object has weight, it must have length.

69. **a.**



b. 15 species **c.** about 3918 m^2

d. The number of species of fish increases; *Sample answer:* This makes sense because in a smaller pool or lake, one species could dominate another more easily and feed on the weaker species until it became extinct.

71. **a.** $\frac{2}{3}$ **b.** $\frac{5}{3}$ **c.** $\frac{4}{3}$ **d.** $\frac{7}{2}$

6.3 Maintaining Mathematical Proficiency *(p. 316)*

73. $g(x) = \sqrt[3]{\frac{1}{2}x}$ **75.** $g(x) = \sqrt[3]{x + 2}$

77. quadratic; The graph is a translation 2 units left and 1 unit down of the parent quadratic function.

6.4 Vocabulary and Core Concept Check *(p. 322)*

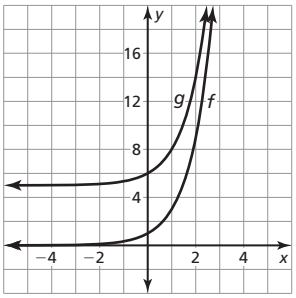
1. Positive values of a vertically stretch ($a > 1$) or shrink ($a < 1$) the graph of f, h translates the graph of f left ($h < 0$) or right ($h > 0$), and k translates the graph of f up ($k > 0$) or down ($k < 0$). When a is negative, the graph of f is reflected in the x-axis.

6.4 Monitoring Progress and Modeling with Mathematics *(pp. 322–324)*

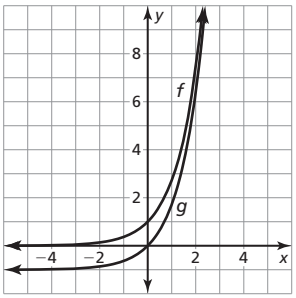
3. C; The graph of f is a translation 2 units left and 2 units down of the graph of the parent function $y = 2^x$.

5. A; The graph of h is a translation 2 units right and 2 units down of the graph of the parent function $y = 2^x$.

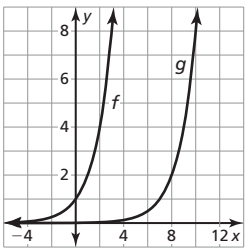
7. The graph of g is a translation 5 units up of the graph of f.



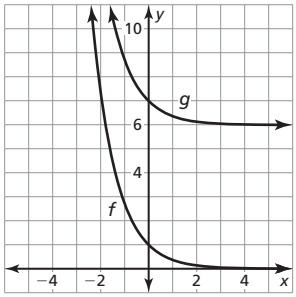
9. The graph of g is a translation 1 unit down of the graph of f.



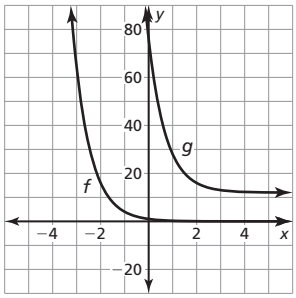
11. The graph of g is a translation 7 units right of the graph of f.



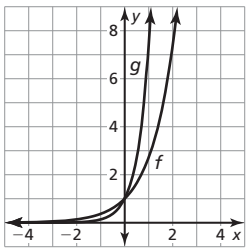
13. The graph of g is a translation 6 units up of the graph of f.



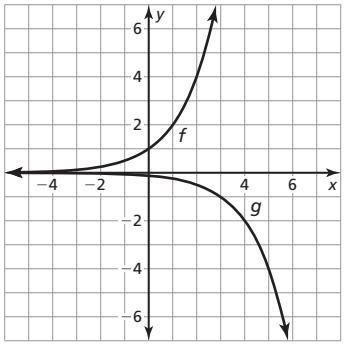
15. The graph of g is a translation 3 units right and 12 units up of the graph of f.



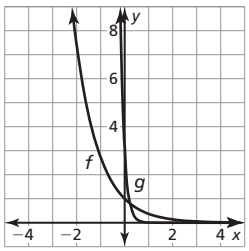
17. The graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ of the graph of f.



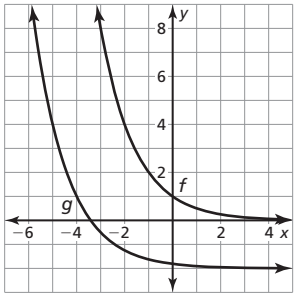
19. The graph of g is reflection in the x-axis followed by a translation 3 units right of the graph of f.



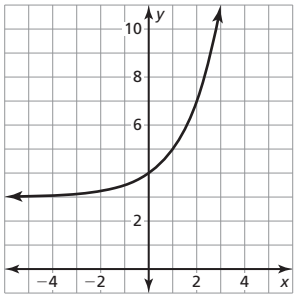
21. The graph of g is a horizontal shrink by a factor of $\frac{1}{6}$ followed by a vertical stretch by a factor of 3 of the graph of f.



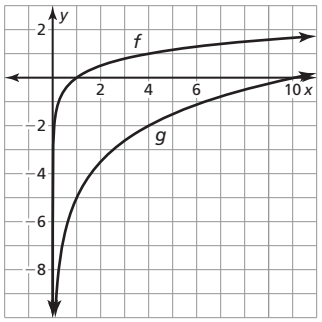
23. The graph of g is a vertical stretch by a factor of 6 followed by a translation 5 units left and 2 units down of the graph of f.



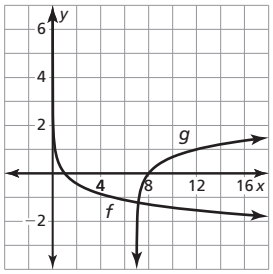
25. The graph of the parent function $f(x) = 2^x$ was translated 3 units left instead of up.



27. The graph of g is a vertical stretch by a factor of 3 followed by a translation 5 units down of the graph of f.



29. The graph of g is a reflection in the x-axis followed by a translation 7 units right of the graph of f.



31. A; The graph of f has been translated 2 units right.

33. C; The graph of f has been stretched vertically by a factor of 2.

35. $g(x) = 5^{-x} - 2$ **37.** $g(x) = e^{2x} + 5$

39. $g(x) = 6 \log_6 x - 5$ **41.** $g(x) = \log_{1/2}(-x + 3) + 2$

43. Multiply the output by -1; Substitute $\log_7 x$ for $f(x)$. Subtract 6 from the output; Substitute $-\log_7 x$ for $h(x)$.

45. The graph of g is a translation 4 units up of the graph of f; $y = 4$

47. The graph of g is a translation 6 units left of the graph of f; $x = -6$

49. The graph of S is a vertical shrink by a factor of 0.118 followed by a translation 0.159 unit up of the graph of f; For fine sand, the slope of the beach is about 0.05. For medium sand, the slope of the beach is about 0.09. For coarse sand, the slope of the beach is about 0.12. For very coarse sand, the slope of the beach is about 0.16.

51. yes; *Sample answer:* If the graph is reflected in the y-axis, the graphs will never intersect because there are no values of x where $\log x = \log(-x)$.

53. **a.** never; The asymptote of $f(x) = \log x$ is a vertical line and would not change by shifting the graph vertically.

b. always; The asymptote of $f(x) = e^x$ is a horizontal line and would be changed by shifting the graph vertically.

c. always; The domain of $f(x) = \log x$ is $x > 0$ and would not be changed by a horizontal shrink.

d. sometimes; The graph of the parent exponential function does not intersect the x-axis, but if it is shifted down, the graph would intersect the x-axis.

55. The graph of h is a translation 2 units left of the graph of f; The graph of h is a reflection in the y-axis followed by a translation 2 units left of the graph of g; x has been replaced with $x + 2$. x has been replaced with $-(x + 2)$.

6.4 Maintaining Mathematical Proficiency *(p. 324)*

57. $(fg)(x) = x^6$; $(fg)(3) = 729$

59. $(f + g)(x) = 14x^3$; $(f + g)(2) = 112$

6.5 Vocabulary and Core Concept Check *(p. 331)*

1. Product

6.5 Monitoring Progress and Modeling with Mathematics *(pp. 331–332)*

3. 0.565 **5.** 1.424 **7.** -0.712

9. B; Quotient Property **11.** A; Power Property

13. $\log_3 4 + \log_3 x$ **15.** $1 + 5 \log x$

17. $\ln x - \ln 3 - \ln y$ **19.** $\log_7 5 + \frac{1}{2} \log_7 x$

21. The two expressions should be added, not multiplied; $\log_2 5x = \log_2 5 + \log_2 x$

23. $\log_4 \frac{7}{10}$ **25.** $\ln x^6y^4$ **27.** $\log_5 4\sqrt[3]{x}$

29. $\ln 32x^7y^4$

31. B;

$\log_5 \dfrac{y^4}{3x} = \log_5 y^4 - \log_5 3x$ Quotient Property

$= 4 \log_5 y - (\log_5 3 + \log_5 x)$ Power and Product Properties

$= 4 \log_5 y - \log_5 3 - \log_5 x$ Distributive Property

33. 1.404 **35.** 1.232 **37.** 1.581 **39.** -0.860

41. yes; Using the change-of-base formula, the equation can be graphed as $y = \dfrac{\log x}{\log 3}$.

43. 60 decibels

45. **a.** $2 \ln 2 \approx 1.39$ knots

b.
$$\begin{aligned} s(h) &= 2 \ln 100h \\ s(h) &= \ln(100h)^2 \\ e^{s(h)} &= e^{\ln(100h)^2} \\ e^{s(h)} &= (100h)^2 \\ \log e^{s(h)} &= \log(100h)^2 \\ s(h) \log e &= 2 \log(100h) \\ s(h) \log e &= 2(\log 100 + \log h) \\ s(h) \log e &= 2(2 + \log h) \\ s(h) &= \frac{2}{\log e}(\log h + 2) \end{aligned}$$

47. Rewrite each logarithm in exponential form to obtain $a = b^x$, $c = b^y$, and $a = c^z$. So,
$$\frac{\log_b a}{\log_b c} = \frac{\log_b c^z}{\log_b c} = \frac{z \log_b c}{\log_b c} = z = \log_c a.$$

6.5 Maintaining Mathematical Proficiency *(p. 332)*

49. $x < -2$ or $x > 2$ **51.** $-7 < x < -6$

53. $x \approx -0.76$ and $x \approx 2.36$ **55.** $x \approx -1.79$ and $x \approx 1.12$

6.6 Vocabulary and Core Concept Check *(p. 338)*

1. exponential

3. The domain of a logarithmic function is positive numbers only, so any quantity that results in taking the log of a non-positive number will be an extraneous solution.

6.6 Monitoring Progress and Modeling with Mathematics *(pp. 338–340)*

5. $x = -1$ **7.** $x = 7$ **9.** $x \approx 1.771$ **11.** $x = -\frac{5}{3}$

13. $x \approx 0.255$ **15.** $x \approx 0.173$ **17.** about 17.6 years old

19. about 50 min **21.** $x = 6$ **23.** $x = 3$ **25.** $x = 6$

27. $x = 10$ **29.** $x = 1$

31. $x = \dfrac{1 + \sqrt{41}}{2} \approx 3.7$ and $x = \dfrac{1 - \sqrt{41}}{2} \approx -2.7$

33. $x = 4$ **35.** $x \approx 6.04$ **37.** $x = \pm 1$

39. $x \approx 10.24$

41. 3 should be the base on both sides of the equation;
$$\begin{aligned} \log_3(5x - 1) &= 4 \\ 3^{\log_3(5x-1)} &= 3^4 \\ 5x - 1 &= 81 \\ 5x &= 82 \\ x &= 16.4 \end{aligned}$$

43. **a.** 39.52 years **b.** 38.66 years **c.** 38.38 years **d.** 38.38 years

45. **a.** $x \approx 3.57$ **b.** $x = 0.8$

47. $x > 1.815$ **49.** $x \geq 20.086$ **51.** $x < 1.723$

53. $x \geq \frac{1}{5}$

55. $0 < x < 25$; *Sample answer:* algebraically; Converting the equation to exponential form is the easiest method because it isolates the variable.

57. $r > 0.0718$ or $r > 7.18\%$ **59.** $x \approx 1.78$

61. no solution

63. **a.** $a = -\dfrac{1}{0.09} \ln\left(\dfrac{45 - \ell}{25.7}\right)$

b. 36 cm footprint: 11.7 years old; 32 cm footprint: 7.6 years old; 28 cm footprint: 4.6 years old; 24 cm footprint: 2.2 years old

65. *Sample answer:* $2^x = 16$; $\log_3(-x) = 1$ **67.** $x \approx 0.89$

69. $x \approx 10.61$ **71.** $x = 2$ and $x = 3$

73. To solve exponential equations with different bases, take a logarithm of each side. Then use the Power Property to move the exponent to the front of the logarithm, and solve for x. To solve logarithmic equations of different bases, find a common multiple of the bases, and exponentiate each side with this common multiple as the base. Rewrite the base as a power that will cancel out the given logarithm and solve the resulting equation.

6.6 Maintaining Mathematical Proficiency *(p. 340)*

75. $y + 2 = 4(x - 1)$ **77.** $y + 8 = -\frac{1}{3}(x - 3)$

79. 3; $y = 2x^3 - x + 1$

81. 4; $y = -3x^4 + 2x^3 - x^2 + 5x - 6$

6.7 Vocabulary and Core Concept Check *(p. 346)*

1. exponential

6.7 Monitoring Progress and Modeling with Mathematics *(pp. 346–348)*

3. exponential; The data have a common ratio of 4.

5. quadratic; The second differences are constant.

7. $y = 0.75(4)^x$ **9.** $y = \frac{1}{8}(2)^x$ **11.** $y = \frac{2}{5}(5)^x$

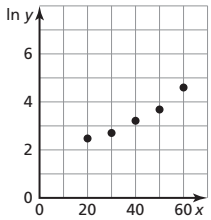
13. $y = 5(0.5)^x$ **15.** $y = 0.25(2)^x$

17. Data are linear when the first differences are constant; The outputs have a common ratio of 3, so the data represents an exponential function.

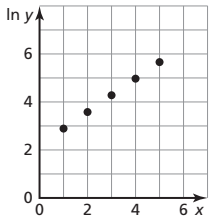
19. *Sample answer:* $y = 7.20(1.39)^x$

21. yes; *Sample answer:* $y = 8.88(1.21)^x$

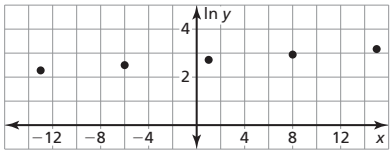
23. no; *Sample answer:* $y = -0.8x + 66$

25.


Sample answer: $y = 3.25(1.052)^x$

27.


yes; $y = 9.14(1.99)^x$

29.


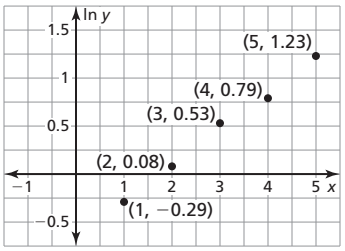
yes; $y = 14.73(1.03)^x$

31. $y = 6.70(1.41)^x$; about 208 scooters

33. $t = 12.59 - 2.55 \ln d$; 2.6 h

Selected Answers

35. **a.**



Sample answer: $y = 0.50(1.47)^x$

b. about 47%; The base is 1.47 which means that the function shows 47% growth.

37. no; When d is the independent variable and t is the dependent variable, the data can be modeled with a logarithmic function. When the variables are switched, the data can be modeled with an exponential function.

39. **a.** 5.9 weeks

b.

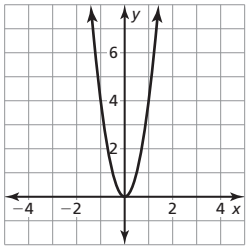
The asymptote is the line $y = 256$ and represents the maximum height of the sunflower.

6.7 Maintaining Mathematical Proficiency *(p. 348)*

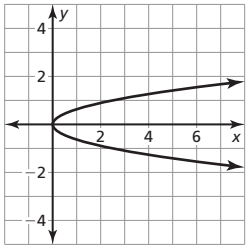
41. no; When one variable is increased by a factor, the other variable does not increase by the same factor.

43. yes; When one variable is increased by a factor, the other variable increases by the same factor.

45. The focus is $\left(0, \frac{1}{16}\right)$, the directrix is $y = -\frac{1}{16}$, and the axis of symmetry is $x = 0$.

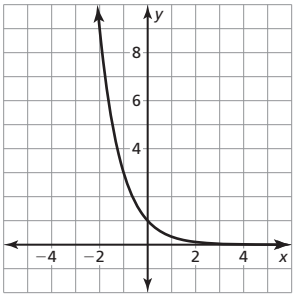


47. The focus is (0.1, 0), the directrix is $x = -0.1$, and the axis of symmetry is $y = 0$.

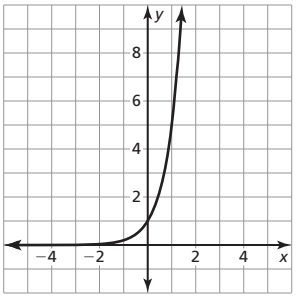


Chapter 6 Review *(pp. 350–352)*

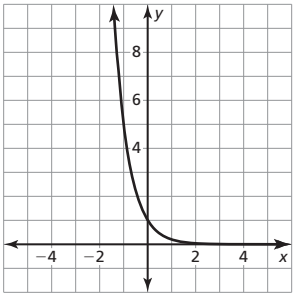
1. exponential decay; 66.67% decrease



2. exponential growth; 400% increase

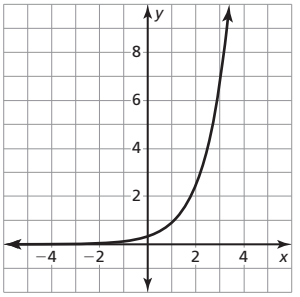


3. exponential decay; 80% decrease

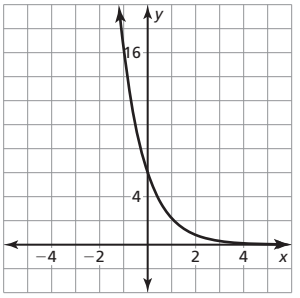


4. \$1725.39 **5.** e^{15} **6.** $\frac{2}{e^3}$ **7.** $\frac{9}{e^{10x}}$

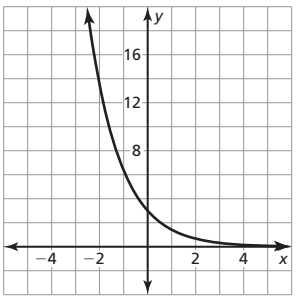
8. exponential growth



9. exponential decay



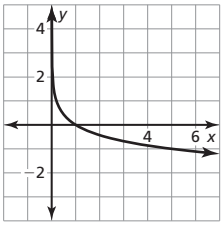
10. exponential decay



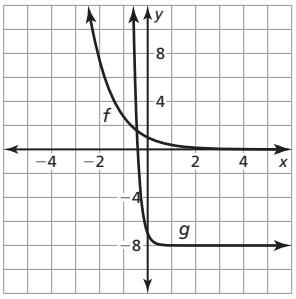
11. 3 **12.** -2 **13.** 0

14. $g(x) = \log_8 x$ **15.** $y = e^x + 4$ **16.** $y = 10^x - 9$

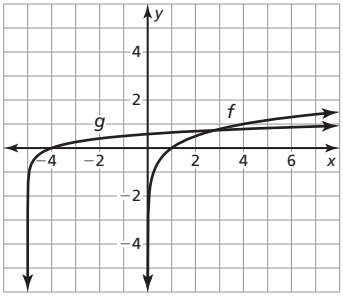
17.



18. The graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ followed by a translation 8 units down of the graph of f.



19. The graph of g is a vertical shrink by a factor of $\frac{1}{2}$ followed by a translation 5 units left of the graph of f.



20. $g(x) = 3e^{x+6} + 3$ **21.** $g(x) = \log(-x) - 2$

22. $\log_8 3 + \log_8 x + \log_8 y$ **23.** $1 + 3 \log x + \log y$

24. $\ln 3 + \ln y - 5 \ln x$ **25.** $\log_7 384$ **26.** $\log_2 \frac{12}{x^2}$

27. $\ln 4x^2$ **28.** about 3.32 **29.** about 1.13

30. about 1.19 **31.** $x \approx 1.29$ **32.** $x = 7$

33. $x \approx 3.59$ **34.** $x > 1.39$ **35.** $0 < x \leq 8103.08$

36. $x \geq 1.19$ **37.** $y = 64\left(\frac{1}{2}\right)^x$

38. *Sample answer:* $y = 3.60(1.43)^x$

39. $s = 3.95 + 27.48 \ln t$; 53 pairs

Chapter 7

Chapter 7 Maintaining Mathematical Proficiency *(p. 357)*

1. $\frac{19}{15}$, or $1\frac{4}{15}$ **2.** $-\frac{17}{42}$ **3.** $\frac{1}{3}$ **4.** $\frac{11}{12}$ **5.** $-\frac{3}{7}$

6. $-\frac{1}{20}$ **7.** $\frac{9}{20}$ **8.** $-\frac{7}{20}$ **9.** $\frac{8}{11}$

10. 0; Division by zero is not possible.

7.1 Vocabulary and Core Concept Check *(p. 363)*

1. The ratio of the variables is constant in a direct variation equation, and the product of the variables is constant in an inverse variation equation.

7.1 Monitoring Progress and Modeling with Mathematics *(pp. 363–364)*

3. inverse variation **5.** direct variation **7.** neither

9. direct variation **11.** direct variation

13. inverse variation **15.** $y = -\frac{20}{x}$; $y = -\frac{20}{3}$

17. $y = -\frac{24}{x}$; $y = -8$ **19.** $y = \frac{21}{x}$; $y = 7$

21. $y = \frac{2}{x}$; $y = \frac{2}{3}$

23. The equation for direct variation was used; Because $5 = \frac{a}{8}$, $a = 40$. So, $y = \frac{40}{x}$.

25. **a.**

Size	2	2.5	3	5
Number of songs	5000	4000	3333	2000

b. The number of songs decreases.

27. $A = \frac{26{,}000}{c}$; about 321 chips per wafer

29. yes; The product of the number of hats and the price per hat is \$50, which is constant.

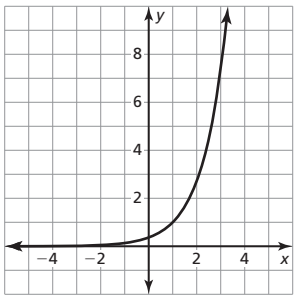
31. *Sample answer:* As the speed of your car increases, the number of minutes per mile decreases.

33. cat: 4 ft, dog: 2 ft; The inverse equations are $d = \frac{a}{7}$ and $6 - d = \frac{a}{14}$. Because the constant is the same, solve the equation $7d = 14(6 - d)$ for d.

7.1 Maintaining Mathematical Proficiency *(p. 364)*

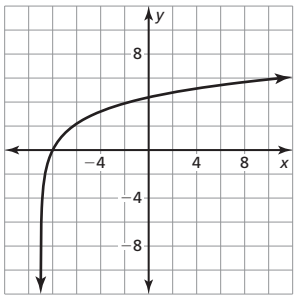
35. $x^2 - 6$

37.



domain: all real numbers, range: $y > 0$

39.



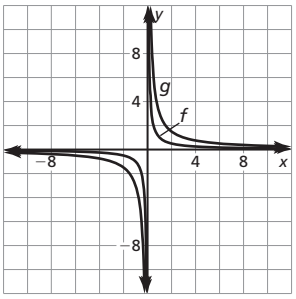
domain: $x > -9$, range: all real numbers

7.2 Vocabulary and Core Concept Check *(p. 370)*

1. range; domain

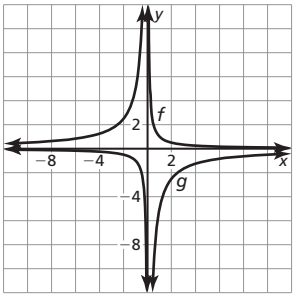
7.2 Monitoring Progress and Modeling with Mathematics *(pp. 370–372)*

3.



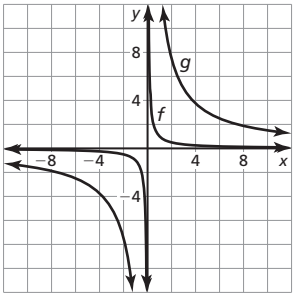
The graph of g lies farther from the axes. Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.

5.



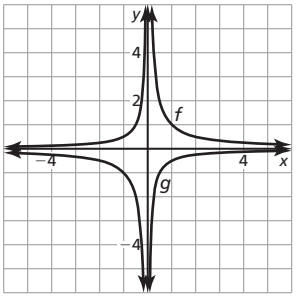
The graph of g lies farther from the axes and is reflected over the x-axis. Both graphs have the same asymptotes, domain, and range.

7.



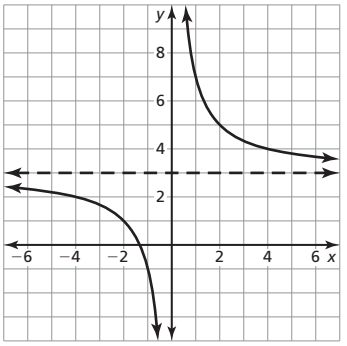
The graph of g lies farther from the axes. Both graphs lie in the first and third quadrants and have the same asymptotes, domain, and range.

9.



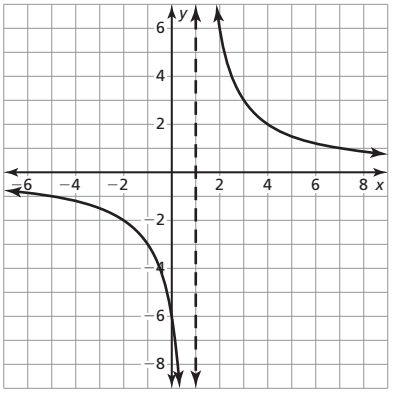
The graph of g lies closer to the axes and is reflected over the x-axis. Both graphs have the same asymptotes, domain, and range.

11.



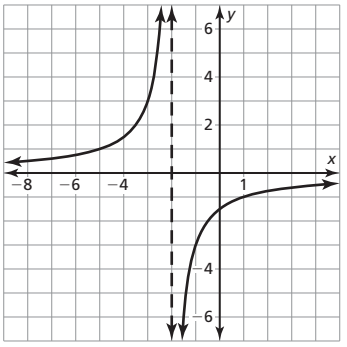
domain: all real numbers except 0; range: all real numbers except 3

13.



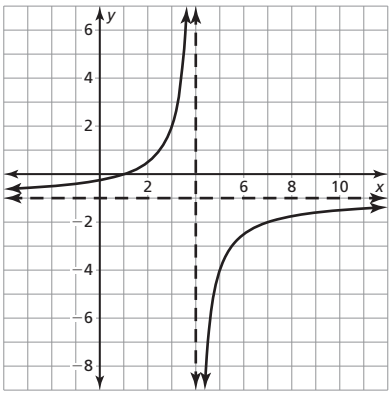
domain: all real numbers except 1; range: all real numbers except 0

15.



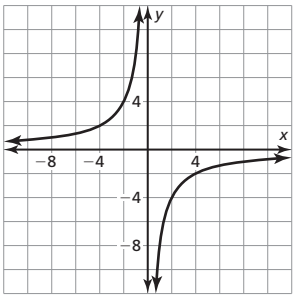
domain: all real numbers except -2; range: all real numbers except 0

17.



domain: all real numbers except 4; range: all real numbers except -1

19.

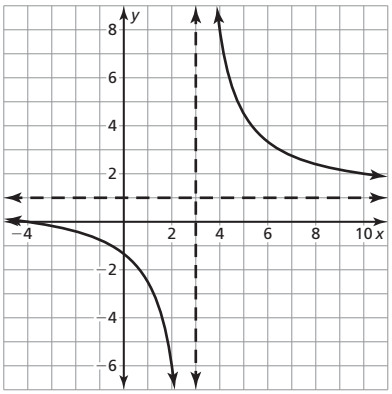


The graph should lie in the second and fourth quadrants instead of the first and third quadrants.

21. A; The asymptotes are $x = 3$ and $y = 1$.

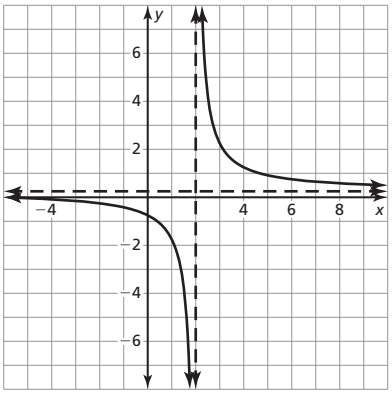
23. B; The asymptotes are $x = 3$ and $y = -1$.

25.



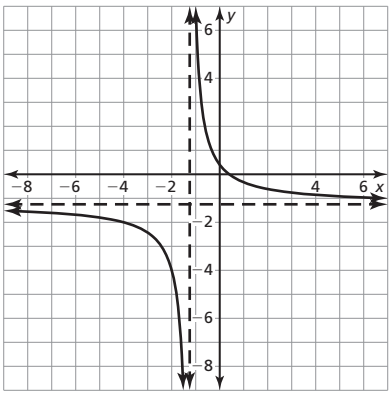
domain: all real numbers except 3; range: all real numbers except 1

27.



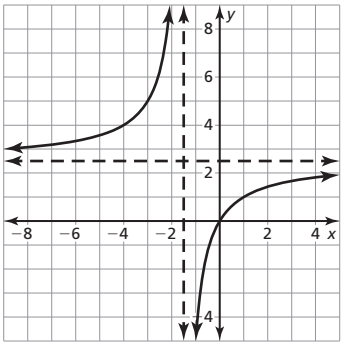
domain: all real numbers except 2; range: all real numbers except $\frac{1}{4}$

29.



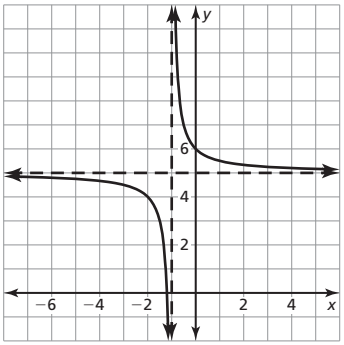
domain: all real numbers except $-\frac{5}{4}$; range: all real numbers except $-\frac{5}{4}$

31.



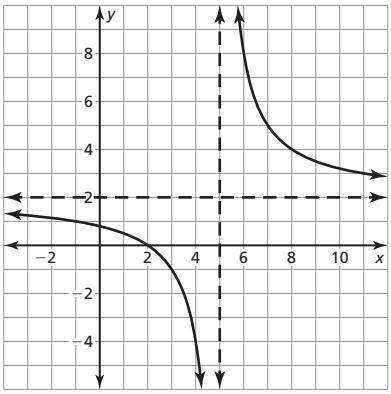
domain: all real numbers except $-\frac{3}{2}$; range: all real numbers except $\frac{5}{2}$

33. $g(x) = \dfrac{1}{x + 1} + 5$



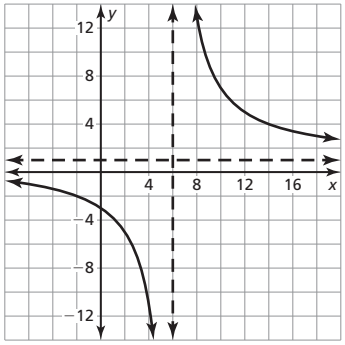
translation 1 unit left and 5 units up

35. $g(x) = \dfrac{6}{x - 5} + 2$



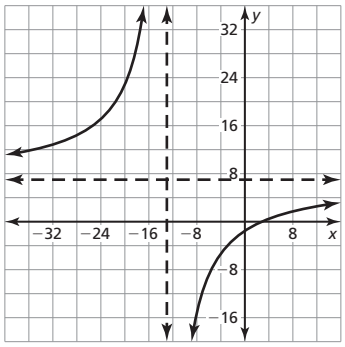
translation 5 units right and 2 units up

37. $g(x) = \dfrac{24}{x - 6} + 1$



translation 6 units right and 1 unit up

39. $g(x) = \dfrac{-111}{x + 13} + 7$



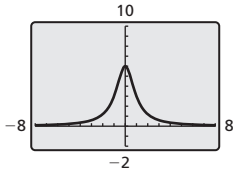
translation 13 units left and 7 units up

41. **a.** 50 students

b. The average cost per student approaches \$20.

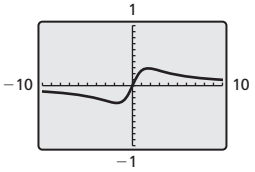
43. B **45.** **a.** about 23°C **b.** −0.005 sec/°C

47.



even

49.



odd

51. yes; A rational function can have more than one vertical asymptote when the denominator is zero for more than one value of x, such as $y = \dfrac{3}{(x + 1)(x - 1)}$.

53. $y = x$, $y = -x$; The function and its inverse are the same.

55. (4, 3); The point (2, 1) is one unit left and one unit down from (3, 2), so a point on the other branch is one unit right and one unit up from (3, 2).

57. The competitor is a better choice for less than 18 months of service; The cost of Internet service is modeled by $C = \dfrac{50 + 43x}{x}$. The competitor's cost is lesser when $x = 6$ and $x = 12$, and greater when $x = 18$ and $x = 24$.

7.2 Maintaining Mathematical Proficiency *(p. 372)*

59. $4(x - 5)(x + 4)$ **61.** $2(x - 3)(x + 2)$

63. 3^6 **65.** $6^{2/3}$

7.3 Vocabulary and Core Concept Check *(p. 380)*

1. To multiply rational expressions, multiply numerators, then multiply denominators, and write the new fraction in simplified form. To divide one rational expression by another, multiply the first rational expression by the reciprocal of the second rational expression.

7.3 Monitoring Progress and Modeling with Mathematics *(pp. 380–382)*

3. $\dfrac{2x}{3x - 4}, x \neq 0$ **5.** $\dfrac{x + 3}{x - 1}, x \neq 6$

7. $\dfrac{x + 9}{x^2 - 2x + 4}, x \neq -2$ **9.** $\dfrac{2(4x^2 + 5)}{x - 3}, x \neq \pm\sqrt{\dfrac{5}{4}}$

11. $\dfrac{y^3}{2x^2}, y \neq 0$ **13.** $\dfrac{(x - 4)(x + 6)}{x}, x \neq 3$

15. $(x - 3)(x + 3), x \neq 0, x \neq 2$ **17.** $\dfrac{2x(x + 4)}{(x + 2)(x - 3)}, x \neq 1$

19. $\dfrac{(x + 9)(x - 4)^2}{(x + 7)}, x \neq 7$

21. The polynomials need to be factored first, and then the common factors can divide out; $\dfrac{x + 12}{x + 4}$

23. B

25. The expressions have the same simplified form, but the domain of f is all real numbers except $x \neq \frac{7}{3}$, and the domain of g is all real numbers.

27. $\dfrac{256x^7}{y^{14}}, x \neq 0$ **29.** $2, x \neq -2, x \neq 0, x \neq 3$

31. $\dfrac{(x + 2)}{(x + 4)(x - 3)}$ **33.** $\dfrac{(x + 6)(x - 2)}{(x + 2)(x - 6)}, x \neq -4, x \neq -3$

35. **a.** $\dfrac{2(r + h)}{rh}$

b. soup: about 0.784, coffee: about 0.382, paint: about 0.341

From most efficient to least efficient, paint can, coffee can, and soup can

37. $M = \dfrac{171{,}000t + 1{,}361{,}000}{(1 + 0.018t)(2.96t + 278.649)}$; \$8443

39. **a.** The population is increasing by 2,960,000 people each year.

b. The population was 278,649,000 people in 2000.

41.

x	y
−3.5	−0.1333
−3.8	−0.1282
−3.9	−0.1266
−4.1	−0.1235
−4.2	−0.1220

The graph does not have a value for y when $x = -4$ and approaches $y = -0.125$.

43. $\dfrac{4}{7x}$ **45.** $9(x + 3), x \neq -\frac{3}{2}, x \neq \frac{5}{2}, x \neq 7$

47. Galapagos: about 0.371, King: about 0.203; King; The King penguin has a smaller surface area to volume ratio, so it is better equipped to live in a colder environment.

49. $f(x) = \dfrac{x(x-1)}{x+2}$, $g(x) = \dfrac{x(x+2)}{x-1}$

7.3 Maintaining Mathematical Proficiency *(p. 382)*

51. $x = -\frac{24}{5}$ **53.** $x = \frac{32}{15}$ **55.** $7 \cdot 13$ **57.** prime

7.4 Vocabulary and Core Concept Check *(p. 388)*

1. complex fraction

7.4 Monitoring Progress and Modeling with Mathematics *(pp. 388–390)*

3. $\dfrac{5}{x}$ **5.** $\dfrac{9-2x}{x+1}$ **7.** $5, x \neq -3$ **9.** $3x(x-2)$

11. $2x(x-5)$ **13.** $(x+5)(x-5)$

15. $(x-5)(x+8)(x-8)$

17. The LCM of $5x$ and x^2 is $5x^2$, so multiply the first term by $\dfrac{x}{x}$ and the second term by $\dfrac{5}{5}$ before adding the numerators; $\dfrac{2(x+10)}{5x^2}$

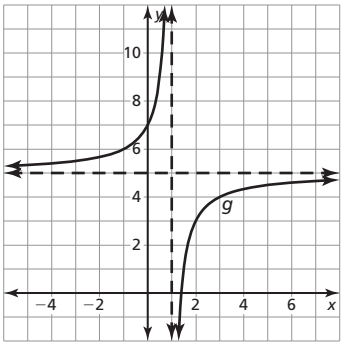
19. $\dfrac{37}{30x}$ **21.** $\dfrac{2(x+7)}{(x+4)(x+6)}$ **23.** $\dfrac{3(x+12)}{(x+8)(x-3)}$

25. $\dfrac{8x^3 - 9x^2 - 28x + 8}{x(x-4)(3x-1)}$

27. sometimes; When the denominators have no common factors, the product of the denominators is the LCD. When the denominators have common factors, use the LCM to find the LCD.

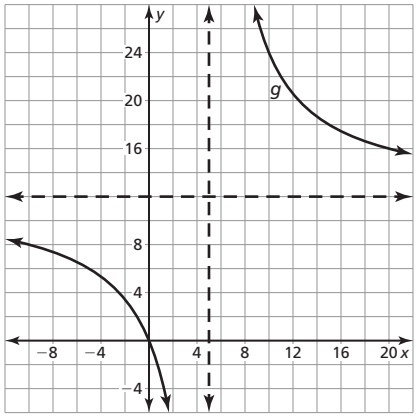
29. A

31. $g(x) = \dfrac{-2}{x-1} + 5$



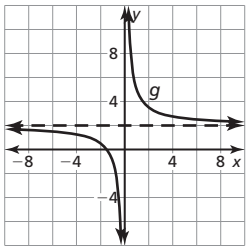
The graph of g is a translation 1 unit to the right and 5 units up of the graph of $f(x) = \dfrac{-2}{x}$.

33. $g(x) = \dfrac{60}{x-5} + 12$



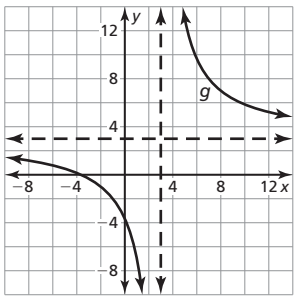
The graph of g is a translation 5 units to the right and 12 units up of the graph of $f(x) = \dfrac{60}{x}$.

35. $g(x) = \dfrac{3}{x} + 2$



The graph of g is a translation 2 units up of the graph of $f(x) = \dfrac{3}{x}$.

37. $g(x) = \dfrac{20}{x-3} + 3$



The graph of g is a translation 3 units to the right and 3 units up of the graph of $f(x) = \dfrac{20}{x}$.

39. $\dfrac{x(x-18)}{6(5x+2)}, x \neq 0$ **41.** $-\dfrac{3}{4x}, x \neq \dfrac{5}{2}$

43. $\dfrac{x-4}{12(x-6)(x-1)}, x \neq -1, x \neq 4$

45. $T = \dfrac{2ad}{(a+j)(a-j)}$; about 10.2 h **47.** $y = \dfrac{20(7x+60)}{x(x+30)}$

49. no; The LCM of 2 and 4 is 4, which is greater than one number and equal to the other number.

51. **a.** $M = \dfrac{Pi}{1 - \left(\dfrac{1}{1+i}\right)^{12t}}$

$= \dfrac{Pi}{1 - \dfrac{1}{(1+i)^{12t}}} \cdot \dfrac{(1+i)^{12t}}{(1+i)^{12t}}$

$= \dfrac{Pi(1+i)^{12t}}{(1+i)^{12t} - 1}$

b. \$364.02

53. $g(x) = \dfrac{2.3058}{x + 12.2} + 0.003$; translation 12.2 units to the left and 0.003 unit up of the graph of f

55. **a.** $R_1 = \dfrac{1}{40}, R_2 = \dfrac{1}{x}, R_3 = \dfrac{1}{x + 10}$

b. $R = \dfrac{x^2 + 90x + 400}{40x(x + 10)}$

c. about 0.0758 car/min; about 4.5 cars/h; Multiply the number of cars washed per minute by the rate 60 min/h to obtain an answer in cars per hour.

57. $1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{2}}}}}, 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{2}}}}}}$

1.4, about 1.4167, about 1.4138, about 1.4143, about 1.4142; $\sqrt{2}$

7.4 Maintaining Mathematical Proficiency *(p. 390)*

59. $\left(\frac{1}{2}, -1\right)$ and $\left(\frac{9}{4}, \frac{27}{8}\right)$ **61.** no solution

7.5 Vocabulary and Core Concept Check *(p. 396)*

1. when each side of the equation is a single rational expression; *Sample answer:* The equation is a proportion.

7.5 Monitoring Progress and Modeling with Mathematics *(pp. 396–398)*

3. $x = 4$ **5.** $x = 5$ **7.** $x = -5, x = 7$

9. $x = -1, x = 0$ **11.** 26 serves **13.** 20.5 oz

15. $x(x + 3)$ **17.** $2(x + 1)(x + 4)$ **19.** $x = 2$

21. $x = \frac{7}{2}$ **23.** $x = -\frac{3}{2}, x = 2$ **25.** no solution

27. $x = -2, x = 3$ **29.** $x = \dfrac{-3 \pm \sqrt{129}}{4}$

31. Both sides of the equation should be multiplied by the same expression;

$3x^3 \cdot \dfrac{5}{3x} + 3x^3 \cdot \dfrac{2}{x^2} = 3x^3 \cdot 1$

33. **a.**

	Work rate	Time	Work done
You	$\dfrac{1 \text{ room}}{8 \text{ hours}}$	5 hours	$\dfrac{5}{8}$ room
Friend	$\dfrac{1 \text{ room}}{t \text{ hours}}$	5 hours	$\dfrac{5}{t}$ room

b. The sum is the amount of time it would take for you and your friend to paint the room together; $\dfrac{5}{8} + \dfrac{5}{t} = 1$,

$t = 13.\overline{3}$ h $= 13$ h 20 min

35. *Sample answer:* $\dfrac{x+1}{x+2} = \dfrac{3}{x+4}$, Cross multiplication can be used when each side of the equation is a single rational expression; *Sample answer:* $\dfrac{x+1}{x+2} + \dfrac{3}{x+4} = \dfrac{1}{x+3}$; Multiplying by the LCD can be used when there is more than one rational expression on one side of the equation.

37. yes; $y = \dfrac{2}{x} + 4$ **39.** yes; $y = \dfrac{3}{x+2}$

41. yes; $y = \dfrac{-2}{x} + \dfrac{11}{2}$ **43.** no; $y = \pm\sqrt{\dfrac{1}{x-4}}$

45. **a.** about 21 mi/gal **b.** about 21 mi/gal

47. $x \approx \pm 0.8165$ **49.** $x \approx 1.3247$

51. $\dfrac{1 + \sqrt{5}}{2}$ **53.** $g(x) = \dfrac{4x + 1}{x - 3}$ **55.** $y = \dfrac{b - xd}{xc - a}$

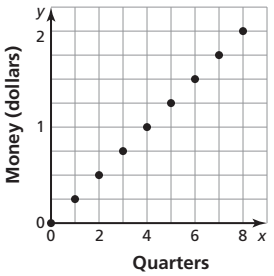
57. **a.** always true; When $x = a$, the denominators of the fractions are both zero.

b. sometimes true; The equation will have exactly one solution except when $a = 3$.

c. always true; $x = a$ is an extraneous solution, so the equation has no solution.

7.5 Maintaining Mathematical Proficiency *(p. 398)*

59. discrete; The number of quarters in your pocket is an integer.



61. 3 **63.** 15

Chapter 7 Review *(pp. 400–402)*

1. inverse variation **2.** direct variation

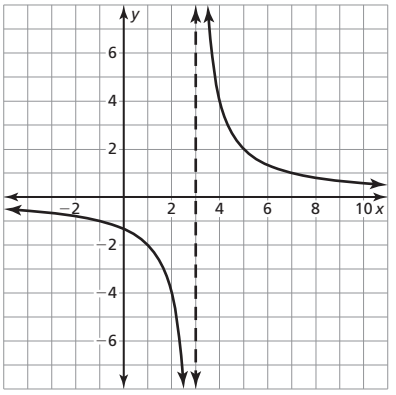
3. direct variation **4.** neither **5.** direct variation

6. inverse variation **7.** $y = \dfrac{5}{x}; y = -\dfrac{5}{3}$

8. $y = \dfrac{24}{x}; y = -8$ **9.** $y = \dfrac{45}{x}; y = -15$

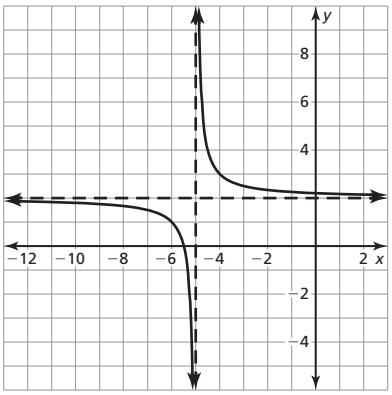
10. $y = \dfrac{-8}{x}; y = \dfrac{8}{3}$

11.



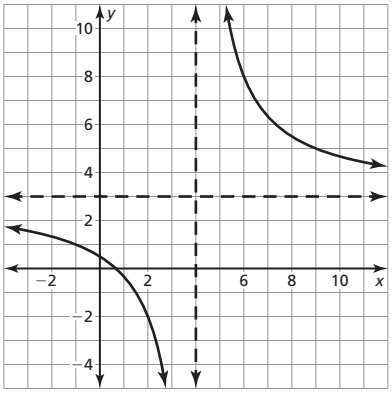
domain: all real numbers except 3; range: all real numbers except 0

12.



domain: all real numbers except -5; range: all real numbers except 2

13.



domain: all real numbers except 4; range: all real numbers except 3

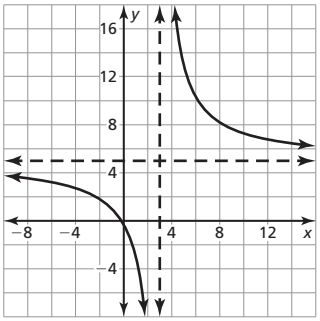
14. $\dfrac{16x^3}{y^2}, x \neq 0$ **15.** $\dfrac{3(x+4)}{x+3}, x \neq 3, x \neq 4$

16. $\dfrac{3x(4x-1)}{(x-4)(x-3)}, x \neq 0, x \neq \dfrac{1}{4}$

17. $\dfrac{1}{(x+3)^2}, x \neq 5, x \neq 8$ **18.** $\dfrac{3x^2+26x+36}{6x(x+3)}$

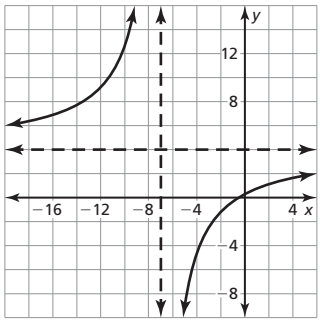
19. $\dfrac{5x^2-11x-9}{(x+8)(x-3)}$ **20.** $\dfrac{-2(2x^2+3x+3)}{(x-3)(x+3)(x+1)}$

21. $g(x) = \dfrac{16}{x-3} + 5$



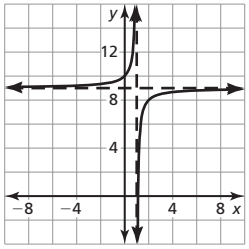
translation 3 units right and 5 units up of the graph of f

22. $g(x) = \dfrac{-26}{x+7} + 4$



translation 7 units left and 4 units up of the graph of f

23. $g(x) = \dfrac{-1}{x-1} + 9$



translation 1 unit right and 9 units up of the graph of f

24. $\dfrac{pq}{p+q}, p \neq 0, q \neq 0$ **25.** $x = 5$ **26.** $x = 0$

27. no solution **28.** yes; $g(x) = \dfrac{3}{x} - 6$

29. yes; $g(x) = \dfrac{10}{x} + 7$ **30.** yes; $g(x) = \dfrac{1}{x-8}$

31. **a.** 4 games **b.** 4 games

Chapter 8

Chapter 8 Maintaining Mathematical Proficiency *(p. 407)*

1.

x	y
1	1
2	−1
3	−5

2.

x	y
2	21
3	46
4	81

3.

x	y
5	4
10	−16
15	−36

4. $x = 4$ **5.** $x = 6$ **6.** $x = 66$ **7.** $x = 7$

8. $x = 100$ **9.** $x = 3$

10. *Sample answer:* The points on the scatterplot are increasing and f is decreasing; Both level off as x increases.

8.1 Vocabulary and Core Concept Check *(p. 414)*

1. sigma notation

3. A sequence is an ordered list of numbers and a series is the sum of the terms of a sequence.

8.1 Monitoring Progress and Modeling with Mathematics *(pp. 414–416)*

5. 3, 4, 5, 6, 7, 8 **7.** 1, 4, 9, 16, 25, 36

9. 1, 4, 16, 64, 256, 1024 **11.** −4, −1, 4, 11, 20, 31

13. $\frac{2}{3}, 1, \frac{6}{5}, \frac{4}{3}, \frac{10}{7}, \frac{3}{2}$

15. arithmetic; $a_5 = 5(5) - 4 = 21$; $a_n = 5n - 4$

17. arithmetic; $a_5 = 0.7(5) + 2.4 = 5.9$; $a_n = 0.7n + 2.4$

19. arithmetic; $a_5 = -1.6(6) + 7.4 = -2.2$; $a_n = -1.6n + 7.4$

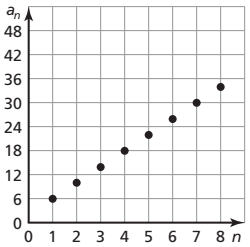
21. arithmetic; $a_5 = \frac{1}{4}(5) = \frac{5}{4}$; $a_n = \frac{n}{4}$

23. $\frac{2}{3(1)}, \frac{2}{3(2)}, \frac{2}{3(3)}, \frac{2}{3(4)}$; $a_5 = \frac{2}{3(5)} = \frac{2}{15}$; $a_n = \frac{2}{3n}$

25. $(1)^3 + 1, (2)^3 + 1, (3)^3 + 1, (4)^3 + 1$; $a_5 = 5^3 + 1 = 126$; $a_n = n^3 + 1$

27. D; The number of squares in the nth figure is equal to the sum of the first positive n integers which is equal to the equation shown in D.

29. $a_n = 4n + 2$



31. $\sum_{i=1}^{5} (3i + 4)$ **33.** $\sum_{i=1}^{\infty} (i^2 + 3)$ **35.** $\sum_{i=1}^{\infty} \frac{1}{3^i}$

37. $\sum_{i=1}^{5} (-1)^i(i + 2)$ **39.** 42 **41.** 100 **43.** 82

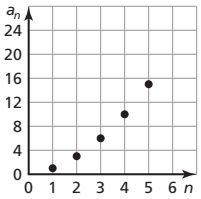
45. $\frac{481}{140}$ **47.** 35 **49.** 280

51. There should be ten terms in the series;

$$\sum_{n=1}^{10} (3n - 5) = -2 + 1 + 4 + 7 + 10 + 13 + 16 + 19 + 22 + 25 = 115$$

53. **a.** \$50.50 **b.** 316 days

55. $a_n = \frac{1}{2}(n)(n + 1)$



57. yes; Subtract 3 from the sum.

59. **a.** true;

$$\sum_{i=1}^{n} ca_i = ca_1 + ca_2 + ca_3 + \cdots + ca_n = c(a_1 + a_2 + a_3 + \cdots + a_n) = c\sum_{i=1}^{n} a_i$$

b. true;

$$\sum_{i=1}^{n} (a_i + b_i) = (a_1 + b_1) + (a_2 + b_2) + \cdots + (a_n + b_n) = a_1 + a_2 + \cdots + a_n + b_1 + b_2 + \cdots + b_n = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

c. false; $\sum_{i=1}^{2} (2i)(3i) = 30$, $\left(\sum_{i=1}^{2} 2i\right)\left(\sum_{i=1}^{2} 3i\right) = 54$

d. false; $\sum_{i=1}^{2} (2i)^2 = 20$, $\left(\sum_{i=1}^{2} 2i\right)^2 = 36$

61. **a.** $a_n = 2^n - 1$ **b.** 63; 127; 255

8.1 Maintaining Mathematical Proficiency *(p. 416)*

63. (3, 1, 1)

8.2 Vocabulary and Core Concept Check *(p. 422)*

1. common difference

8.2 Monitoring Progess and Modeling with Mathematics *(pp. 422–424)*

3. arithmetic; The common difference is −2.

5. not arithmetic; The differences are not constant.

7. not arithmetic; The differences are not constant.

9. arithmetic; The common difference is $\frac{1}{4}$.

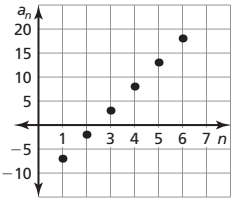
11. **a.** $a_n = -6n + 3$ **b.** $a_n = 5n + 2$

13. $a_n = 8n + 4$; 164 **15.** $a_n = -3n + 54$; −6

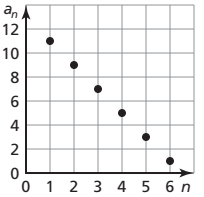
17. $a_n = \frac{2}{3}n - \frac{5}{3}$; $\frac{35}{3}$ **19.** $a_n = -0.8n + 3.1$; −12.9

21. The formula should be $a_n = a_1 + (n - 1)d$; $a_n = 35 - 13n$

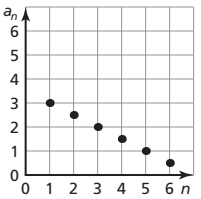
23. $a_n = 5n - 12$



25. $a_n = -2n + 13$



27. $a_n = -\frac{1}{2}n + \frac{7}{2}$



29. C **31.** $a_n = 11n - 14$ **33.** $a_n = -6n + 28$

35. $a_n = -4n + 13$ **37.** $a_n = \frac{5}{4}n + 2$

39. $a_n = -3n + 12$ **41.** $a_n = 3n - 7$

43. $a_n = 4n + 9$

45. The graph of a_n consists of discrete points and the graph of f consists of a continuous line.

47. 360 **49.** −924 **51.** −8.2 **53.** −1026

55. **a.** $a_n = 2n + 1$ **b.** 63 band members

57. $1 + \sum_{i=1}^{4} 8i$; 81

59. no; Doubling the difference does not necessarily double the terms.

61. 22,500; $\sum_{i=1}^{150} (2i - 1) = 150\left(\frac{1 + 299}{2}\right)$

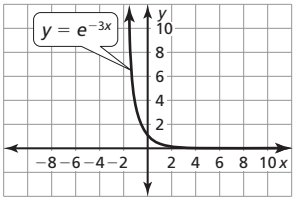
63. $\left(\frac{2y}{n} - x\right)$ seats

65. $\frac{7}{16}, \frac{9}{16}, \frac{11}{16}, \frac{13}{16}, \frac{15}{16}, \frac{17}{16}, \frac{19}{16}, \frac{21}{16}, \frac{23}{16}$, and $\frac{25}{16}$

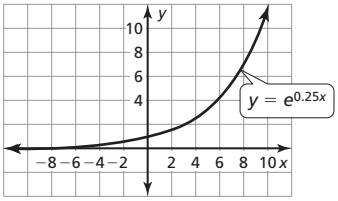
8.2 Maintaining Mathematical Proficiency *(p. 424)*

67. 3^2 **69.** $5^{3/4}$

71. exponential decay



73. exponential growth



8.3 Vocabulary and Core Concept Check *(p. 430)*

1. common ratio **3.** a_1r^{n-1}

8.3 Monitoring Progress and Modeling with Mathematics *(pp. 430–432)*

5. geometric; The common ratio is $\frac{1}{2}$.

7. not geometric; The ratios are not constant.

9. not geometric; The ratios are not constant.

11. geometric; The common ratio is $\frac{1}{3}$.

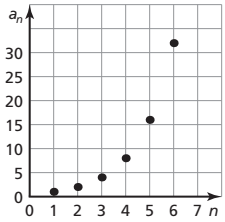
13. **a.** $a_n = -3(5)^{n-1}$ **b.** $a_n = 72\left(\frac{1}{3}\right)^{n-1}$

15. $a_n = 4(5)^{n-1}$; $a_7 = 62{,}500$

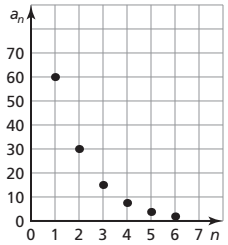
17. $a_n = 112\left(\frac{1}{2}\right)^{n-1}$; $a_7 = \frac{7}{4}$ **19.** $a_n = 4\left(\frac{3}{2}\right)^{n-1}$; $a_7 = \frac{729}{16}$

21. $a_n = 1.3(-3)^{n-1}$; $a_7 = 947.7$

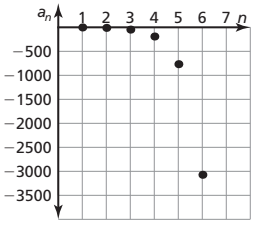
23. $a_n = 2^{n-1}$



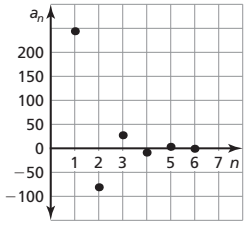
25. $a_n = 60\left(\frac{1}{2}\right)^{n-1}$



27. $a_n = -3(4)^{n-1}$



29. $a_n = 243\left(-\frac{1}{3}\right)^{n-1}$



31. The formula should be $a_n = a_1r^{n-1}$; $a_n = 8(6)^{n-1}$

33. $a_n = 7(4)^{n-1}$ **35.** $a_n = -6(3)^{n-1}$ or $a_n = -6(-3)^{n-1}$

37. $a_n = 512\left(\frac{1}{8}\right)^{n-1}$ or $a_n = -512\left(-\frac{1}{8}\right)^{n-1}$

39. $a_n = -432\left(\frac{1}{6}\right)^{n-1}$ or $a_n = 432\left(-\frac{1}{6}\right)^{n-1}$

41. $a_n = 4(2)^{n-1}$ **43.** $a_n = 5\left(\frac{1}{2}\right)^{n-1}$

45. $a_n = 6(-2)^{n-1}$ **47.** 40,353,606 **49.** $\frac{989{,}527}{65{,}536}$

51. $\frac{32{,}312}{6561}$ **53.** $-262{,}140$

55. The graph of a_n consists of discrete points and the graph of f is continuous.

57. \$276.25

59. **a.** $a_n = 32\left(\frac{1}{2}\right)^{n-1}$; $1 \le n \le 6$; The number of games must be a whole number.

b. 63 games

61. **a.** $a_n = 8^{n-1}$; 2,396,745 squares

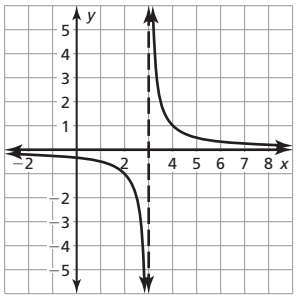
b. $b_n = \left(\frac{8}{9}\right)^n$; about 0.243 square units

63. \$141,521.58

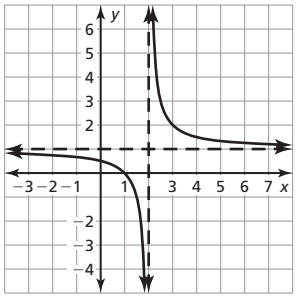
65. no; The total amount repaid for loan 1 is about \$205,000 and the total amount repaid for loan 2 is about \$284,000.

8.3 Maintaining Mathematical Proficiency *(p. 432)*

67. domain: all real numbers except 3; range: all real numbers except 0



69. domain: all real numbers except 2; range: all real numbers except 1

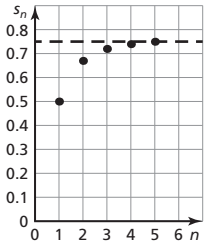


8.4 Vocabulary and Core Concept Check *(p. 439)*

1. partial sum

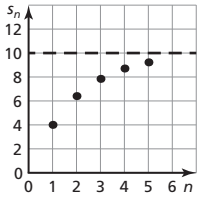
8.4 Monitoring Progress and Modeling with Mathematics *(pp. 439–440)*

3. $S_1 = 0.5$, $S_2 = 0.67$, $S_3 \approx 0.72$, $S_4 \approx 0.74$, $S_5 \approx 0.75$; S_n appears to approach 0.75.



Selected Answers

5. $S_1 = 4$, $S_2 = 6.4$, $S_3 = 7.84$, $S_4 \approx 8.70$, $S_5 \approx 9.22$; S_n appears to approach 10.



7. 10 **9.** $\frac{88}{15}$ **11.** 8 **13.** 18

15. Because $\left|\frac{7}{2}\right| > 1$, the sum does not exist. **17.** 56 ft

19. $\frac{2}{9}$ **21.** $\frac{16}{99}$ **23.** $\frac{3200}{99} = 32\frac{32}{99}$

25. *Sample answer:* $\sum_{i=1}^{\infty} 3\left(\frac{1}{2}\right)^{i-1}$; $\sum_{i=1}^{\infty} 2\left(\frac{2}{3}\right)^{i-1}$; $\dfrac{3}{1-\frac{1}{2}} = 6$ and $\dfrac{2}{1-\frac{2}{3}} = 6$

27. \$5000

29. yes; At 2 seconds, both distances are 40 feet.

31. **a.** $a_n = \frac{1}{4}\left(\frac{3}{4}\right)^{n-1}$

b. 1 ft^2; As n increases, the area of the removed triangles gets closer to the area of the original triangle.

8.4 Maintaining Mathematical Proficiency *(p. 440)*

33. quadratic **35.** neither

8.5 Vocabulary and Core Concept Check *(p. 447)*

1. equation

8.5 Monitoring Progress and Modeling with Mathematics *(pp. 447–450)*

3. $a_1 = 1, a_2 = 4, a_3 = 7, a_4 = 10, a_5 = 13, a_6 = 16$

5. $f(0) = 4, f(1) = 8, f(2) = 16, f(3) = 32, f(4) = 64, f(5) = 128$

7. $a_1 = 2, a_2 = 5, a_3 = 26, a_4 = 677, a_5 = 458{,}330,$ $a_6 = 210{,}066{,}388{,}901$

9. $f(0) = 2, f(1) = 4, f(2) = 2, f(3) = -2, f(4) = -4,$ $f(5) = -2$

11. $a_1 = 21, a_n = a_{n-1} - 7$ **13.** $a_1 = 3, a_n = 4a_{n-1}$

15. $a_1 = 44, a_n = \dfrac{a_{n-1}}{4}$

17. $a_1 = 2, a_2 = 5, a_n = a_{n-2} \cdot a_{n-1}$

19. $a_1 = 1, a_2 = 4, a_n = a_{n-2} + a_{n-1}$

21. $a_1 = 6, a_n = n \cdot a_{n-1}$ **23.** $f(1) = 1, f(n) = f(n-1) + 1$

25. $f(1) = -2, f(n) = f(n-1) + 3$

27. A recursive rule needs to include the values of the first terms; $a_1 = 5, a_2 = 2, a_n = a_{n-2} - a_{n-1}$

29. $a_1 = 7, a_n = a_{n-1} + 4$ **31.** $a_1 = 2, a_n = a_{n-1} - 10$

33. $a_1 = 12, a_n = 11a_{n-1}$ **35.** $a_1 = 1.9, a_n = a_{n-1} - 0.6$

37. $a_1 = -\frac{1}{2}, a_n = \frac{1}{4}a_{n-1}$ **39.** $a_1 = 112, a_n = a_{n-1} + 30$

41. $a_n = -6n + 9$ **43.** $a_n = -2(3)^{n-1}$

45. $a_n = 9.1n - 21.1$ **47.** $a_n = -\frac{1}{3}n + \frac{16}{3}$

49. $a_n = -2n + 22$ **51.** B; An explicit rule is $a_n = 6n - 2$.

53. **a.** $a_1 = 50{,}000, a_n = 0.8a_{n-1} + 5000$

b. 35,240 members

c. The number stabilizes at about 25,000 people.

55. *Sample answer:* You have saved \$100 for a vacation. Each week, you save \$5 more. $a_1 = 100, a_n = a_{n-1} + 5$

57. **a.** \$1612.38 **b.** \$91.39

59. 144 rabbits; When $n = 12$, each formula produces 144.

61. **a.** $a_1 = 9000, a_n = 0.9a_{n-1} + 800$

b. The number stabilizes at 8000 trees.

63. **a.** 1, 2, 4, 8, 16, 32, 64; geometric

b. $a_n = 2^{n-1}; a_1 = 1, a_n = 2a_{n-1}$

65. 15 months; \$213.60; $a_1 = 3000,$ $a_n = \left(1 + \dfrac{0.1}{12}\right)a_{n-1} - 213.59$

67. **a.** 3, 10, 21, 36, 55 **b.** quadratic **c.** $a_n = 2n^2 + n$

69. **a.** $T_n = \frac{1}{2}n^2 + \frac{1}{2}n; S_n = n^2$

b. $T_1 = 1, T_n = T_{n-1} + n; S_1 = 1, S_n = S_{n-1} + 2n - 1$

c. $S_n = T_{n-1} + T_n$

8.5 Maintaining Mathematical Proficiency *(p. 450)*

71. $x = 25$ **73.** $x = 27$

75. $y = \dfrac{18}{x}; y = \dfrac{9}{2}$ **77.** $y = \dfrac{320}{x}; y = 80$

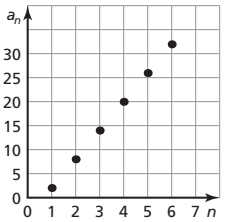
Chapter 8 Review *(pp. 452–454)*

1. $a_n = n^2 + n$ **2.** $\sum_{i=1}^{12}(3i + 4)$ **3.** $\sum_{i=0}^{\infty}(i^2 + i)$

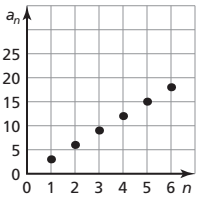
4. -729 **5.** 1081 **6.** 650 **7.** 15

8. yes; The terms have a common difference of -8.

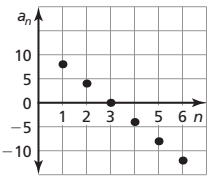
9. $a_n = 6n - 4$



10. $a_n = 3n$



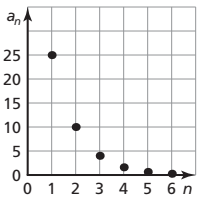
11. $a_n = -4n + 12$



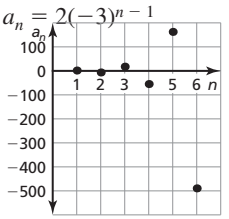
12. 2070 **13.** $a_n = 1500n + 35{,}500$; \$244,500

14. yes; The terms have a common ratio of 2.

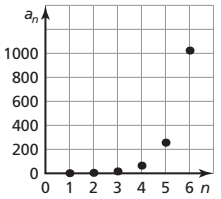
15. $a_n = 25\left(\frac{2}{5}\right)^{n-1}$

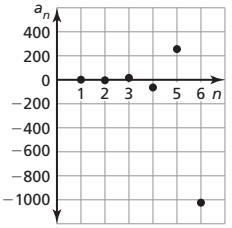


16. $a_n = 2(-3)^{n-1}$



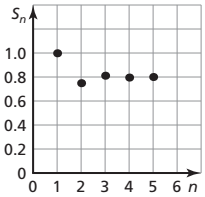
17. $a_n = 4^{n-1}$ or $a_n = (-4)^{n-1}$





18. 855

19. $S_1 = 1, S_2 = 0.75, S_3 \approx 0.81, S_4 \approx 0.80, S_5 \approx 0.80$; S_n approaches 0.80.



20. -1.6 **21.** $\frac{4}{33}$

22. $a_1 = 7, a_2 = 18, a_3 = 29, a_4 = 40, a_5 = 51, a_6 = 62$

23. $a_1 = 6, a_2 = 24, a_3 = 96, a_4 = 384, a_5 = 1536, a_6 = 6144$

24. $f(0) = 4, f(1) = 6, f(2) = 10, f(3) = 16, f(4) = 24, f(5) = 34$

25. $a_1 = 9, a_n = \frac{2}{3}a_{n-1}$ **26.** $a_1 = 2, a_n = a_{n-1}(n-1)$

27. $a_1 = 7, a_2 = 3, a_n = a_{n-2} - a_{n-1}$

28. $a_1 = 105, a_n = \frac{3}{5}a_{n-1}$ **29.** $a_n = 26n - 30$

30. $a_n = 8(-5)^{n-1}$ **31.** $a_n = 26\left(\frac{2}{5}\right)^{n-1}$

32. $P_1 = 11{,}120, P_n = 1.04P_{n-1}$

33. $a_1 = 1, a_n = a_{n-1} + 4n - 3$

Chapter 9

Chapter 9 Maintaining Mathematical Proficiency *(p. 459)*

1. $-|7|, |4|, |2-9|, |6+4|$ **2.** $|0|, \frac{|-5|}{|2|}, |-4|, |9-3|$

3. $|9-1|, |9| + |-2| - |1|, |-2 \cdot 8|, |-8^3|$

4. $-|4^2|, |5| - |3 \cdot 2|, |-15|, |-4+20|$ **5.** 13 m **6.** 24 ft

7. 12 mm **8.** 28 km **9.** $11\frac{2}{3}$ in. **10.** 0.4 yd

11. yes; The line passing through the points (x_1, y_1) and (x_2, y_1) is horizontal. The line passing through the points (x_2, y_1) and (x_2, y_2) is vertical. Horizontal and vertical lines are perpendicular, so the triangle formed by the line segments connecting (x_1, y_1), (x_2, y_1), and (x_2, y_2) contains a right angle.

9.1 Vocabulary and Core Concept Check *(p. 466)*

1. cosine and secant

3. To solve a right triangle, the missing angles and side lengths must be found.

9.1 Monitoring Progress and Modeling with Mathematics *(pp. 466–468)*

5. $\sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5}, \tan\theta = \frac{4}{3}, \csc\theta = \frac{5}{4}, \sec\theta = \frac{5}{3}, \cot\theta = \frac{3}{4}$

7. $\sin\theta = \frac{5}{7}, \cos\theta = \frac{2\sqrt{6}}{7}, \tan\theta = \frac{5\sqrt{6}}{12}, \csc\theta = \frac{7}{5}, \sec\theta = \frac{7\sqrt{6}}{12}, \cot\theta = \frac{2\sqrt{6}}{5}$

9. $\sin\theta = \frac{2\sqrt{14}}{9}, \cos\theta = \frac{5}{9}, \tan\theta = \frac{2\sqrt{14}}{5}, \csc\theta = \frac{9\sqrt{14}}{28}, \sec\theta = \frac{9}{5}, \cot\theta = \frac{5\sqrt{14}}{28}$

11. $\sin\theta = \frac{4\sqrt{97}}{97}, \cos\theta = \frac{9\sqrt{97}}{97}, \csc\theta = \frac{\sqrt{97}}{4}, \cot\theta = \frac{9}{4}$

13. $\cos\theta = \frac{6\sqrt{2}}{11}, \tan\theta = \frac{7\sqrt{2}}{12}, \csc\theta = \frac{11}{7}, \sec\theta = \frac{11\sqrt{2}}{12}, \cot\theta = \frac{6\sqrt{2}}{7}$

15. $\sin\theta = \frac{7\sqrt{85}}{85}, \cos\theta = \frac{6\sqrt{85}}{85}, \csc\theta = \frac{\sqrt{85}}{7}, \sec\theta = \frac{\sqrt{85}}{6}, \cot\theta = \frac{6}{7}$

17. $\sin\theta = \frac{\sqrt{115}}{14}, \cos\theta = \frac{9}{14}, \tan\theta = \frac{\sqrt{115}}{9}, \csc\theta = \frac{14\sqrt{115}}{115}, \cot\theta = \frac{9\sqrt{115}}{115}$

19. The adjacent side was used instead of the opposite; $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17}$

21. $x = 4.5$ **23.** $x = 6$ **25.** $x = 8$

27. 0.9703 **29.** 1.1666 **31.** 9.5144

33. $A = 54°, b \approx 16.71, c \approx 28.43$

35. $B = 35°, b \approx 11.90, c \approx 20.75$

37. $B = 47°, a \approx 28.91, c \approx 42.39$

39. $A = 18°, a \approx 3.96, b \approx 12.17$ **41.** $w \approx 514$ m

43. about 427 m **45.** **a.** about 451 ft **b.** about 5731 ft

47. **a.** about 22,818 mi **b.** about 7263 mi

49. **a.** about 59,155 ft **b.** about 53,613 ft

c. about 39,688 ft; Use the tangent function to find the horizontal distance, $x + y$, from the airplane to the second town to be about 93,301 ft. Subtract 53,613 ft to find the distance between the two towns.

51. yes; The triangle must be a 45-45-90 triangle because both acute angles would be the same and have the same cosine value.

53. **a.** $x = 0.5$; 6 units

b. *Sample answer:* Each side is part of two right triangles, with opposing angles $\left(\frac{180°}{n}\right)$. So, each side length is $2\sin\left(\frac{180°}{n}\right)$, and there are n sides.

c. $n \cdot \sin\left(\frac{180°}{n}\right)$; about 3.14

9.1 Maintaining Mathematical Proficiency *(p. 468)*

55. 1.5 gal **57.** $C \approx 37.7$ cm, $A \approx 113.1$ cm^2

59. $C \approx 44.0$ ft, $A \approx 153.9$ ft^2

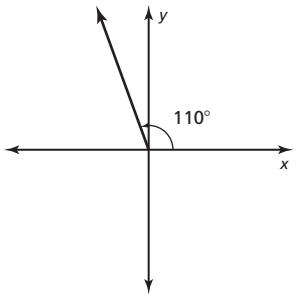
9.2 Vocabulary and Core Concept Check *(p. 474)*

1. origin; initial side

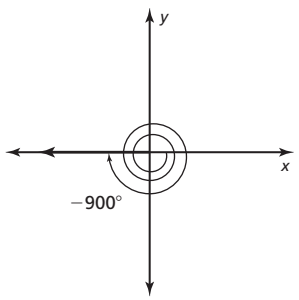
3. *Sample answer:* A radian is a measure of an angle that is approximately equal to 57.3° and there are 2π radians in a circle.

9.2 Monitoring Progress and Modeling with Mathematics *(pp. 474–476)*

5.



7.



9. 430°; −290° **11.** 235°; −485° **13.** $\frac{2\pi}{9}$

15. $-\frac{13\pi}{9}$ **17.** 20° **19.** about −286.5°

21. A full revolution is 360° or 2π radians. The terminal side rotates one-sixth of a revolution from the positive x-axis, so multiply by $\frac{1}{6}$ to get $\frac{1}{6} \cdot 360° = 60°$ and $\frac{1}{6} \cdot 2\pi = \frac{\pi}{3}$.

23. B **25.** A **27.** about 15.7 yd, about 78.5 yd^2

29. The wrong conversion was used;

$$24° = 24 \text{ degrees}\left(\frac{\pi \text{ radians}}{180 \text{ degrees}}\right)$$

$$= \frac{24\pi}{180} \text{ radians} \approx 0.42 \text{ radians}$$

31. 72,000°, 400π **33.** −0.5 **35.** 3.549

37. −0.138 **39.** 528 in.^{2} **41.** 60°, $\frac{\pi}{3}$

43. about 6.89 in.^{2}, about 0.76 in.^{2}, about 0.46 in.^{2}

45. yes; When the arc length is equal to the radius, the equation $s = r\theta$ shows that $\theta = 1$ and $A = \frac{1}{2}r^2\theta$ is equivalent to $A = \frac{s^2}{2}$ for $r = s$ and $\theta = 1$.

47. **a.** 70°33′ **b.** 110.76°; $110 + \frac{45}{60} + \frac{30}{3600} \approx 110.76°$

9.2 Maintaining Mathematical Proficiency *(p. 476)*

49. about 27.02 **51.** about 18.03 **53.** about 18.68

9.3 Vocabulary and Core Concept Check *(p. 482)*

1. quadrantal angle

9.3 Monitoring Progress and Modeling with Mathematics *(pp. 482–484)*

3. $\sin\theta = -\frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = -\frac{3}{4}$, $\csc\theta = -\frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

5. $\sin\theta = -\frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = -\frac{5}{4}$, $\sec\theta = -\frac{5}{3}$ $\cot\theta = \frac{3}{4}$

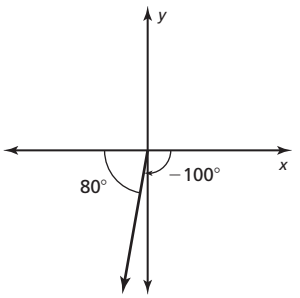
7. $\sin\theta = -\frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\csc\theta = -\frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = \frac{4}{3}$

9. $\sin\theta = 0$, $\cos\theta = 1$, $\tan\theta = 0$, $\csc\theta =$ undefined, $\sec\theta = 1$, $\cot\theta =$ undefined

11. $\sin\theta = 1$, $\cos\theta = 0$, $\tan\theta =$ undefined, $\csc\theta = 1$, $\sec\theta =$ undefined, $\cot\theta = 0$

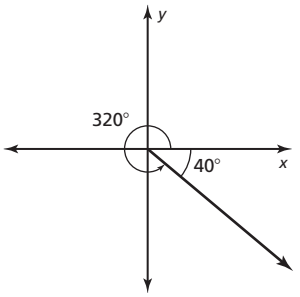
13. $\sin\theta = 1$, $\cos\theta = 0$, $\tan\theta =$ undefined, $\csc\theta = 1$, $\sec\theta =$ undefined, $\cot\theta = 0$

15.



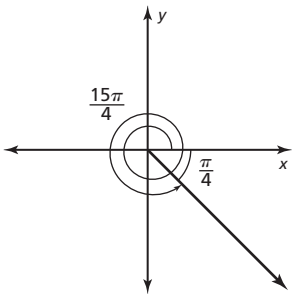
; 80°

17.



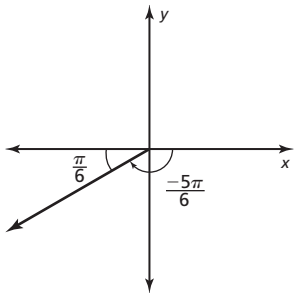
; 40°

19.



; $\frac{\pi}{4}$

21.



; $\frac{\pi}{6}$

23. The equation for tangent is $\tan\theta = \frac{y}{x}$; $\tan\theta = \frac{y}{x} = -\frac{2}{3}$

25. $-\sqrt{2}$ **27.** $-\frac{1}{2}$ **29.** 1 **31.** $\frac{\sqrt{2}}{2}$ **33.** 65 ft

35. about 16.5 ft/sec **37.** about 10.7 ft

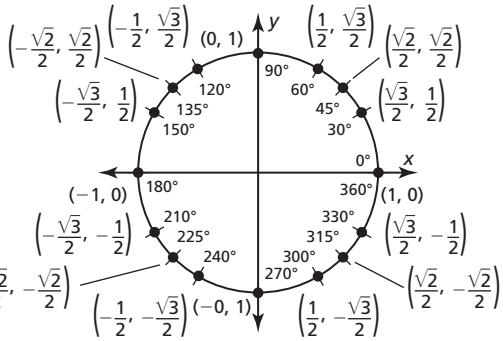
39. **a.**

Angle of sprinkler, θ	Horizontal distance water travels, d
30°	16.9
35°	18.4
40°	19.2
45°	19.5
50°	19.2
55°	18.4
60°	16.9

b. 45°; Because $\frac{v^2}{32}$ is constant in this situation, the maximum distance traveled will occur when $\sin 2\theta$ is as large as possible. The maximum value of $\sin 2\theta$ occurs when $2\theta = 90°$, that is, when $\theta = 45°$.

c. The distances are the same.

41.



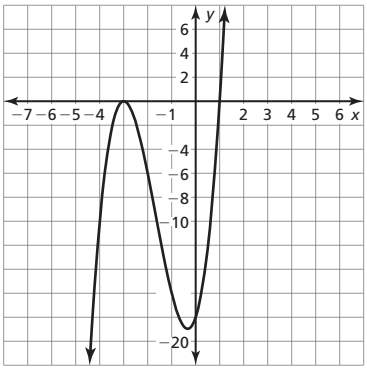
43. $\tan\theta = \frac{\sin\theta}{\cos\theta}$; $\sin 90° = 1$ and $\cos 90° = 0$, so $\tan 90°$ is undefined because you cannot divide by 0, but $\cot 90° = \frac{0}{1} = 0$.

45. $m = \tan\theta$ **47. a.** $(-58.1, 114)$ **b.** about 218 pm

9.3 Maintaining Mathematical Proficiency *(p. 484)*

49. $x = -3$ and $x = 1$

51.



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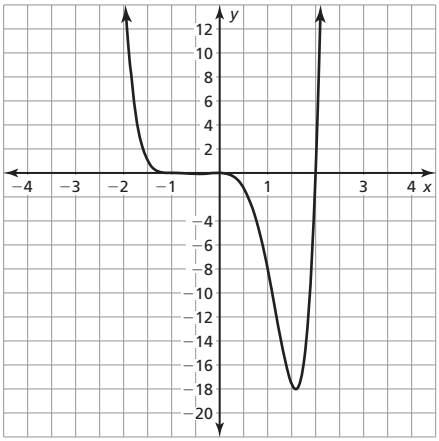


9.4 Vocabulary and Core Concept Check *(p. 491)*

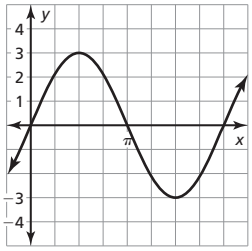
1. cycle

3. A phase shift is a horizontal translation of a periodic function; *Sample answer:* $y = \sin\left(x - \frac{\pi}{2}\right)$

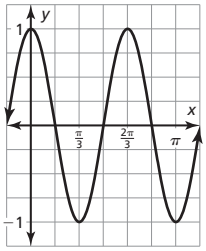
9.4 Monitoring Progress and Modeling with Mathematics *(pp. 491–494)*

5. yes; 2 **7.** no **9.** $1, 6\pi$ **11.** $4, \pi$

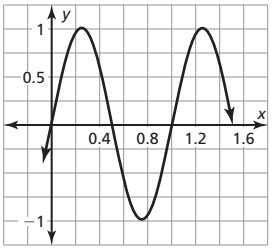
13. $3, 2\pi$; The graph of g is a vertical stretch by a factor of 3 of the graph of $f(x) = \sin x$.



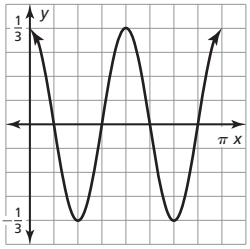
15. $1, \frac{2\pi}{3}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \cos x$.



17. 1, 1; The graph of g is a horizontal shrink by a factor of $\frac{1}{2\pi}$ of the graph of $f(x) = \sin x$.



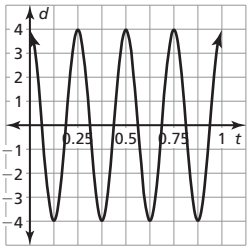
19. $\frac{1}{3}, \frac{\pi}{2}$; The graph of g is a horizontal shrink by a factor of $\frac{1}{4}$ and a vertical shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \cos x$.



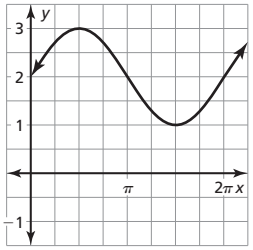
21. B, D

Selected Answers

23. The period is $\frac{1}{4}$ and represents the amount of time, in seconds, that it takes for the pendulum to go back and forth and return to the same position. The amplitude is 4 and represents the maximum distance, in inches, the pendulum will be from its resting position.



25.



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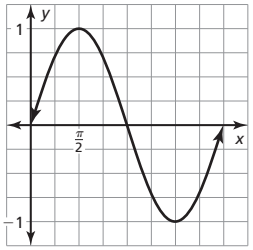

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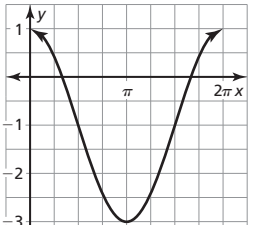

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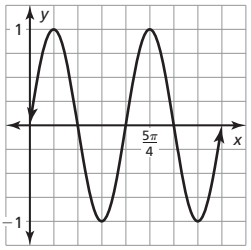

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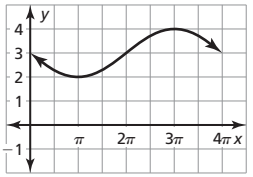


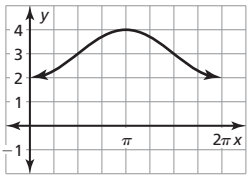
35. To find the period, use the expression $\frac{2\pi}{|b|}$;

Period: $\frac{2\pi}{|b|} = \frac{2\pi}{\frac{2}{3}} = 3\pi$

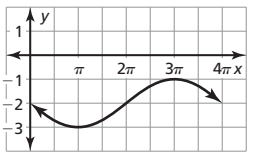
37. The graph of g is a vertical stretch by a factor of 2 followed by a translation $\frac{\pi}{2}$ units right and 1 unit up of the graph of f.

39. The graph of g is a horizontal shrink by a factor of $\frac{1}{3}$ followed by a translation 3π units left and 5 units down of the graph of f.

41.



43.



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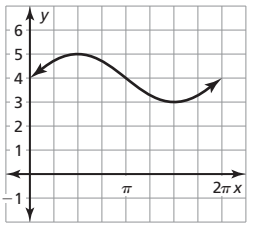
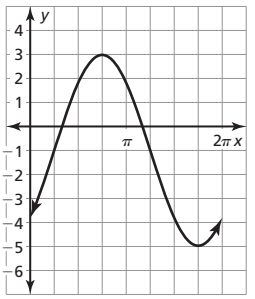


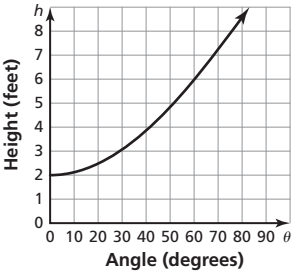
47.



49. A **51.** $g(x) = 3\sin(x - \pi) + 2$

53. $g(x) = -\frac{1}{3}\cos \pi x - 1$

55.

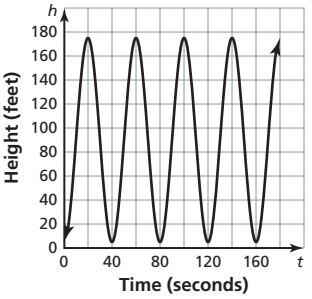


; 4.3 ft

57. days 205 and 328; When the function is graphed with the line $y = 10$, the two points of intersection are (205.5, 10) and (328.7, 10).

59. **a.** about -1.27 **b.** about 0.64 **c.** about 0.64

61. **a.**



b. 4.5 **c.** 175 ft, 5 ft

63. The x-intercepts occur when $x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \ldots$.

Sample answer: The x-intercepts can be represented by the expression $(2n + 1)\frac{\pi}{4}$, where n is an integer.

65. The graph of $g(x) = \cos x$ is a translation $\frac{\pi}{2}$ units to the right of the graph of $f(x) = \sin x$.

67. 80 beats per minute

9.4 Maintaining Mathematical Proficiency *(p. 494)*

69. $x - 2, x \neq -3$ **71.** $\frac{(x - 5)(x + 1)}{(x + 5)(x - 1)}$

73. $2x(x - 5)$ **75.** $(x + 6)(x + 2)$

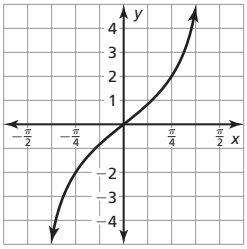
9.5 Vocabulary and Core Concept Check *(p. 502)*

1. The graphs of the tangent, cotangent, secant and cosecant functions have no amplitude because the ranges do not have minimum or maximum values.

3. 2π; π

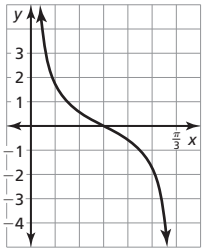
9.5 Monitoring Progress and Modeling with Mathematics *(pp. 502–504)*

5.



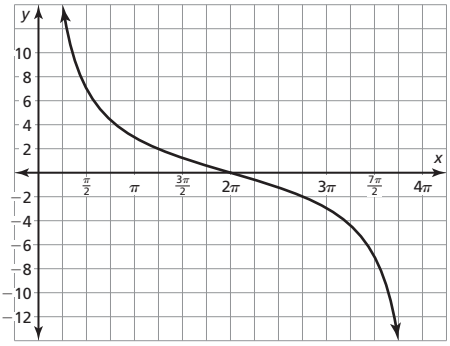
The graph of g is a vertical stretch by a factor of 2 of the graph of $f(x) = \tan x$.

7.



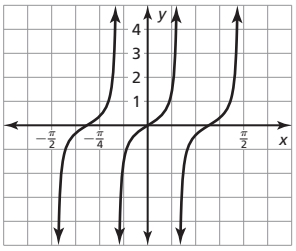
The graph of g is a horizontal shrink by a factor of $\frac{1}{3}$ of the graph of $f(x) = \cot x$.

9.



The graph of g is a horizontal stretch by a factor of 4 and a vertical stretch by a factor of 3 of the graph of $f(x) = \cot x$.

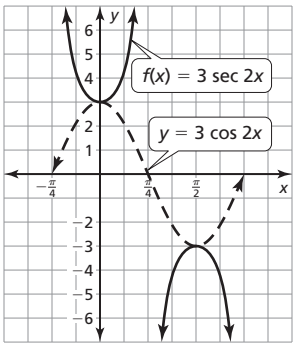
11.



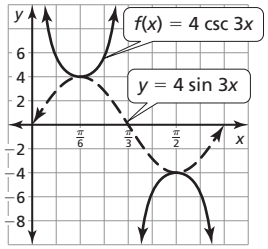
The graph of g is a horizontal shrink by a factor of $\frac{1}{\pi}$ and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \tan x$.

13. To find the period, use the expression $\frac{\pi}{|b|}$; Period: $\frac{\pi}{|b|} = \frac{\pi}{3}$

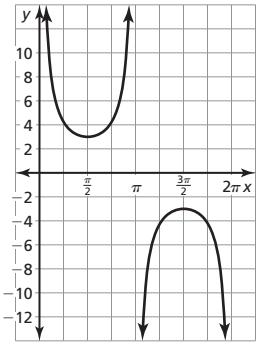
15. a.



b.

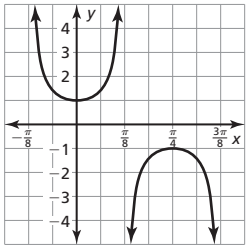


17.



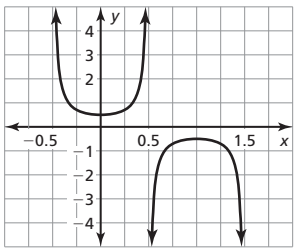
The graph of g is a vertical stretch by a factor of 3 of the graph of $f(x) = \csc x$.

19.



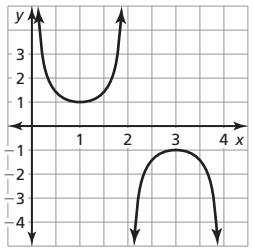
The graph of g is a horizontal shrink by a factor of $\frac{1}{4}$ of the graph of $f(x) = \sec x$.

21.



The graph of g is a horizontal shrink by a factor of $\frac{1}{\pi}$ and a vertical shrink by a factor of $\frac{1}{2}$ of the graph of $f(x) = \sec x$.

23.



The graph of g is a horizontal stretch by a factor of $\frac{2}{\pi}$ of the graph of $f(x) = \csc x$.

25. $y = 6 \tan x$ **27.** $y = 2 \tan \pi x$

29. B; The parent function is the tangent function and the graph has an asymptote at $x = \frac{\pi}{2}$.

31. D; The parent function is the cosecant function and the graph has an asymptote at $x = 1$.

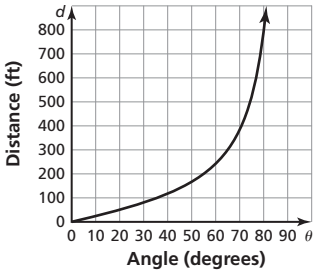
33. A; The parent function is the secant function and the graph has an asymptote at $x = \frac{\pi}{4}$.

35. The tangent function that passes through the origin and has asymptotes at $x = \pi$ and $x = -\pi$ can be stretched or shrunk vertically to create more tangent functions with the same characteristics.

37. $g(x) = \cot\left(2x + \frac{\pi}{2}\right) + 3$ **39.** $g(x) = -5 \sec(x - \pi) + 2$

41. Function B has a local maximum value of -5 so Function A's local maximum value of $-\frac{1}{4}$ is greater. Function A has a local minimum of $\frac{1}{4}$ so Function B's local minimum value of 5 is greater.

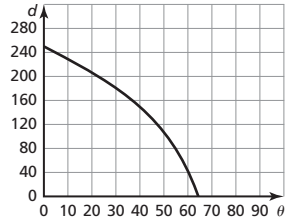
43.



As d increases, θ increases because, as the car gets farther away, the angle required to see the car gets larger.

45. **a.** $d = 260 - 120 \tan \theta$

b.



The graph shows a negative correlation meaning that as the angle gets larger, the distance from your friend to the top of the building gets smaller. As the angle gets smaller, the distance from your friend to the top of the building gets larger.

47. no; The graph of cosecant can be translated $\frac{\pi}{2}$ units right to create the same graph as $y = \sec x$.

49. $a \sec bx = \frac{a}{\cos bx}$

Because the cosine function is at most 1, $y = a \cos bx$ will produce a maximum when $\cos bx = 1$ and $y = a \sec bx$ will produce a minimum. When $\cos bx = -1$, $y = a \cos bx$ will produce a minimum and $y = a \sec bx$ will produce a maximum.

51. *Sample answer:* $y = 5 \tan\left(\frac{1}{2}x - \frac{3\pi}{4}\right)$

9.5 Maintaining Mathematical Proficiency *(p. 504)*

53. $y = -x^3 + 2x^2 + 5x - 6$ **55.** $y = \frac{1}{5}x^3 + \frac{1}{5}x^2 - \frac{9}{5}x - \frac{9}{5}$

57. 3, π

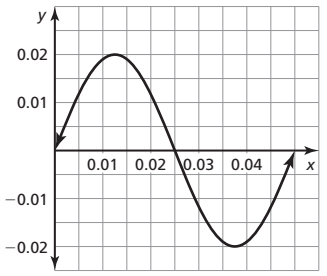
9.6 Vocabulary and Core Concept Check *(p. 510)*

1. sinusoids

9.6 Monitoring Progress and Modeling with Mathematics *(pp. 510–512)*

3. $\frac{1}{2\pi}$ **5.** $\frac{2}{\pi}$ **7.** $\frac{3}{2}$ **9.** $\frac{3}{8\pi}$

11. $P = 0.02 \sin 40\pi t$



13. $y = 3 \sin 2x$ **15.** $y = -2 \cos \frac{\pi}{2}(x + 4)$

17. To find the amplitude, take half of the difference between the maximum and the minimum; $\frac{10 - (-6)}{2} = 8$

19. $h = -2.5 \cos \pi t + 6.5$

21. $D = 19.81 \sin(0.549t - 2.40) + 79.8$; The period of the graph represents the amount of time it takes for the weather to repeat its cycle, which is about 11.4 months.

23. $V = 100 \sin 4\pi t$

25. **a.** $N = 3.68 \sin(0.776t - 0.70) + 20.4$

b. about 23,100 employees

27. **a. and b.** A cosine function because it does not require determining a horizontal shift.

c. A sine function because it does not require determining a horizontal shift.

29. $y = 2.5 \sin 4\left(x - \frac{\pi}{8}\right) + 5.5$, $y = -2.5 \cos 4x + 5.5$

31. **a.** $d = -6.5 \cos \frac{\pi}{6}t + 10$

b. low tide: 12:00 A.M., 12:00 P.M., high tide: 6:00 A.M., 6:00 P.M.

c. It is a horizontal shift to the left by 3.

9.6 Maintaining Mathematical Proficiency *(p. 512)*

33. $\frac{6 + 3\sqrt{6}}{2}$ **35.** $\frac{13\sqrt{11} - 13\sqrt{3}}{8}$ **37.** $\ln 2 + \ln x$

39. $\ln 4 + 6 \ln x - \ln y$

9.7 Vocabulary and Core Concept Check *(p. 517)*

1. A trigonometric equation is true for some values of a variable but a trigonometric identity is true for all values of the variable for which both sides of the equation are defined.

9.7 Monitoring Progress and Modeling with Mathematics *(pp. 517–518)*

3. $\cos\theta = \frac{2\sqrt{2}}{3}$, $\tan\theta = \frac{\sqrt{2}}{4}$, $\csc\theta = 3$, $\sec\theta = \frac{3\sqrt{2}}{4}$, $\cot\theta = 2\sqrt{2}$

5. $\sin\theta = \frac{3\sqrt{58}}{58}$, $\cos\theta = -\frac{7\sqrt{58}}{58}$, $\csc\theta = \frac{\sqrt{58}}{3}$, $\sec\theta = -\frac{\sqrt{58}}{7}$, $\cot\theta = -\frac{7}{3}$

7. $\sin\theta = -\frac{\sqrt{11}}{6}$, $\tan\theta = \frac{\sqrt{11}}{5}$, $\csc\theta = -\frac{6\sqrt{11}}{11}$, $\sec\theta = -\frac{6}{5}$, $\cot\theta = \frac{5\sqrt{11}}{11}$

9. $\sin\theta = -\frac{\sqrt{10}}{10}$, $\cos\theta = \frac{3\sqrt{10}}{10}$, $\tan\theta = -\frac{1}{3}$, $\csc\theta = -\sqrt{10}$, $\sec\theta = \frac{\sqrt{10}}{3}$

11. $\cos x$ **13.** $-\tan\theta$ **15.** $\sin^2 x$ **17.** $-\sec x$

19. 1

21. $\sin^2\theta = 1 - \cos^2\theta$;
$1 - \sin^2\theta = 1 - (1 - \cos^2\theta) = 1 - 1 + \cos^2\theta = \cos^2\theta$

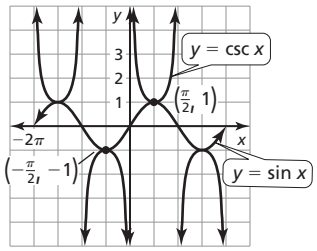
23. $\sin x \csc x = \sin x \cdot \frac{1}{\sin x} = 1$

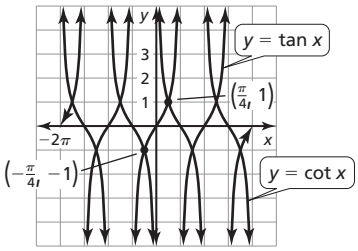
25. $\cos\left(\frac{\pi}{2} - x\right)\cot x = \sin x \cdot \frac{\cos x}{\sin x} = \cos x$

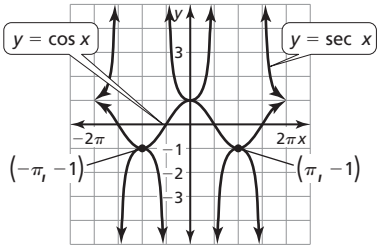
27.
$$\begin{aligned}\frac{\cos\left(\frac{\pi}{2} - \theta\right) + 1}{1 - \sin(-\theta)} &= \frac{\sin\theta + 1}{1 - \sin(-\theta)}\\ &= \frac{\sin\theta + 1}{1 - (-\sin\theta)}\\ &= \frac{\sin\theta + 1}{1 + \sin\theta}\\ &= 1\end{aligned}$$

29.
$$\begin{aligned}\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} &= \frac{1 + \cos x}{\sin x} + \frac{\sin x(1 - \cos x)}{(1 + \cos x)(1 - \cos x)}\\ &= \frac{1 + \cos x}{\sin x} + \frac{\sin x(1 - \cos x)}{1 - \cos^2 x}\\ &= \frac{1 + \cos x}{\sin x} + \frac{\sin x(1 - \cos x)}{\sin^2 x}\\ &= \frac{\sin x(1 + \cos x)}{\sin^2 x} + \frac{\sin x(1 - \cos x)}{\sin^2 x}\\ &= \frac{\sin x(1 + \cos x) + \sin x(1 - \cos x)}{\sin^2 x}\\ &= \frac{\sin x(1 + \cos x + 1 - \cos x)}{\sin^2 x}\\ &= \frac{\sin x(2)}{\sin^2 x}\\ &= \frac{2}{\sin x}\\ &= 2\csc x\end{aligned}$$

31. $\sin x$, $\csc x$, $\tan x$, $\cot x$; $\cos x$, $\sec x$;
$\sin(-\theta) = -\sin\theta$
$\csc(-\theta) = \frac{1}{\sin(-\theta)} = -\frac{1}{\sin\theta} = -\csc\theta$
$\tan(-\theta) = -\tan\theta$
$\cot(-\theta) = \frac{1}{\tan(-\theta)} = -\frac{1}{\tan\theta} = -\cot\theta$







33. yes; $\sec x \tan x - \sin x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} - \sin x$
$$\begin{aligned}&= \frac{\sin x}{\cos^2 x} - \sin x\\ &= \sec^2 x \sin x - \sin x\\ &= \sin x(\sec^2 x - 1)\\ &= \sin x \tan^2 x\end{aligned}$$

35. $s = \frac{h\sin(90° - \theta)}{\sin\theta}$
$s = \frac{h\cos\theta}{\sin\theta}$
$s = h\cot\theta$

37. **a.** $u = \tan\theta$
b. u starts at 0 and increases without bound.

39. You can obtain the graph of $y = \cos x$ by reflecting the graph of $f(x) = \sin x$ in the y-axis and translating it $\frac{\pi}{2}$ units right.

9.7 Maintaining Mathematical Proficiency *(p. 518)*

41. $x = 11$ **43.** $x = \frac{14\sqrt{3}}{3}$

9.8 Vocabulary and Core Concept Check *(p. 523)*

1. $\cos 170°$

9.8 Monitoring Progress and Modeling with Mathematics *(pp. 523–524)*

3. $\sqrt{3} - 2$ **5.** $\dfrac{\sqrt{2} - \sqrt{6}}{4}$ **7.** $\dfrac{\sqrt{2} - \sqrt{6}}{4}$ **9.** $\sqrt{3} + 2$

11. $-\frac{36}{85}$ **13.** $-\frac{13}{85}$ **15.** $-\frac{36}{77}$ **17.** $\tan x$

19. $\cos x$ **21.** $\cos x$

23. The sign in the denominator should be negative when using the sum formula;

$$\frac{\tan x + \tan \frac{\pi}{4}}{1 - \tan x \tan \frac{\pi}{4}} = \frac{\tan x + 1}{1 - \tan x}$$

25. B, D **27.** $x = \frac{\pi}{3}, \frac{5\pi}{3}$ **29.** $x = \frac{3\pi}{2}$ **31.** $x = 0, \pi$

33. $\sin\left(\frac{\pi}{2} - \theta\right) = \sin \frac{\pi}{2} \cos \theta - \cos \frac{\pi}{2} \sin \theta$

$= (1) \cos \theta - (0) \sin \theta$

$= \cos \theta$

35. $\dfrac{35 \tan (\theta - 45°) + 35 \tan 45°}{h \tan \theta}$

$= \dfrac{35\left(\dfrac{\tan \theta - \tan 45°}{1 + \tan \theta \tan 45°}\right) + 35 \tan 45°}{h \tan \theta}$

$= \dfrac{35\left(\dfrac{\tan \theta - 1}{1 + \tan \theta}\right) + 35}{h \tan \theta}$

$= \dfrac{35(\tan \theta - 1) + 35(1 + \tan \theta)}{h \tan \theta(1 + \tan \theta)}$

$= \dfrac{35 \tan \theta - 35 + 35 + 35 \tan \theta}{h \tan \theta(1 + \tan \theta)}$

$= \dfrac{70 \tan \theta}{h \tan \theta(1 + \tan \theta)}$

$= \dfrac{70}{h(1 + \tan \theta)}$

37. $y_1 + y_2 = \cos 960\pi t + \cos 1240\pi t$

$= \cos(1100\pi t - 140\pi t) + \cos(1100\pi t + 140\pi t)$

$= \cos 1100\pi t \cos 140\pi t + \sin 1100\pi t \sin 140\pi t$

$+ \cos 1100\pi t \cos 140\pi t - \sin 1100\pi t \sin 140\pi t$

$= \cos 1100\pi t \cos 140\pi t + \cos 1100\pi t \cos 140\pi t$

$= 2 \cos 1100\pi t \cos 140\pi t$

39. **a.** $\tan(\theta_2 - \theta_1) = \dfrac{m_2 - m_1}{1 + m_2 m_1}$ **b.** 60°

9.8 Maintaining Mathematical Proficiency *(p. 524)*

41. $x = 4$ **43.** $x = -\frac{2}{3}$

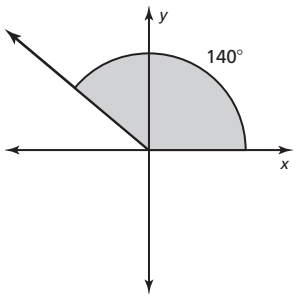
Chapter 9 Review *(pp. 526–530)*

1. $\sin \theta = \dfrac{\sqrt{85}}{11}$, $\tan \theta = \dfrac{\sqrt{85}}{6}$, $\csc \theta = \dfrac{11\sqrt{85}}{85}$, $\sec \theta = \dfrac{11}{6}$, $\cot \theta = \dfrac{6\sqrt{85}}{85}$

2. about 15 ft **3.** 22°; −338° **4.** $\dfrac{\pi}{6}$ **5.** $\dfrac{5\pi}{4}$

6. 135° **7.** 300°

8.



about 1497 m^2

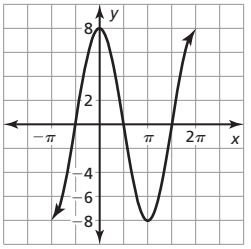
9. $\sin \theta = 1$, $\cos \theta = 0$, $\tan \theta =$ undefined, $\csc \theta = 1$, $\sec \theta =$ undefined, $\cot \theta = 0$

10. $\sin \theta = -\frac{7}{25}$, $\cos \theta = \frac{24}{25}$, $\tan \theta = -\frac{7}{24}$, $\csc \theta = -\frac{25}{7}$, $\sec \theta = \frac{25}{24}$, $\cot \theta = -\frac{24}{7}$

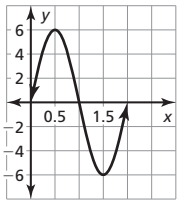
11. $\sin \theta = \dfrac{3\sqrt{13}}{13}$, $\cos \theta = -\dfrac{2\sqrt{13}}{13}$, $\tan \theta = -\dfrac{3}{2}$, $\csc \theta = \dfrac{\sqrt{13}}{3}$, $\sec \theta = -\dfrac{\sqrt{13}}{2}$, $\cot \theta = -\dfrac{2}{3}$

12. $-\dfrac{\sqrt{3}}{3}$ **13.** $\sqrt{2}$ **14.** $\dfrac{1}{2}$ **15.** 2

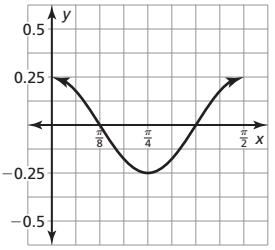
16. 8, 2π; The graph of g is a vertical stretch by a factor of 8 of the graph of $f(x) = \cos x$;



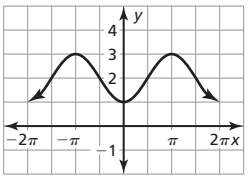
17. 6, 2; The graph of g is a horizontal shrink by a factor of $\dfrac{1}{\pi}$ and a vertical stretch by a factor of 6 of the graph of $f(x) = \sin x$;



18. $\dfrac{1}{4}$, $\dfrac{\pi}{2}$; The graph of g is a horizontal shrink by a factor of $\dfrac{1}{4}$ and a vertical shrink by a factor of $\dfrac{1}{4}$ of the graph of $f(x) = \cos x$;



19.



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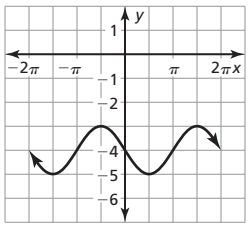

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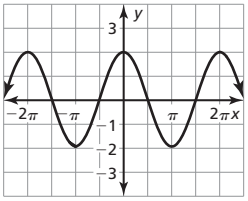

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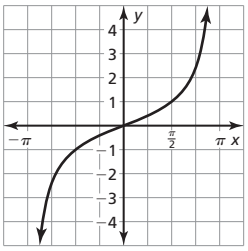
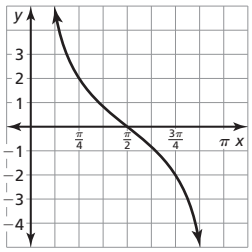


The graph of g is a horizontal stretch by a factor of 2 of the graph of $f(x) = \tan x$.

23.



The graph of g is a vertical stretch by a factor of 2 of the graph of $f(x) = \cot x$.

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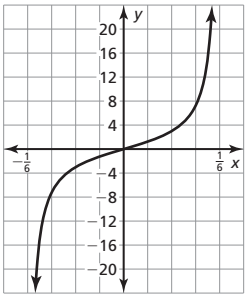

The graph of g is a horizontal shrink by a factor of $\frac{1}{3\pi}$ and a vertical stretch by a factor of 4 of the graph of $f(x) = \tan x$.

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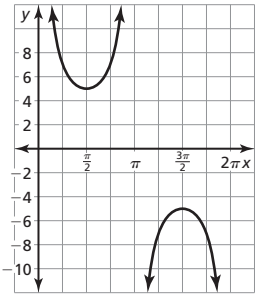

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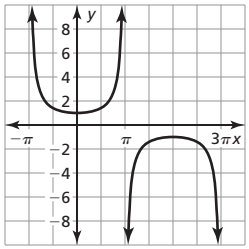

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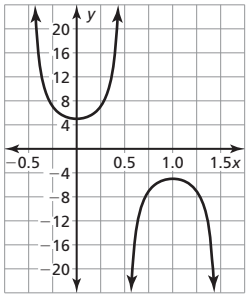

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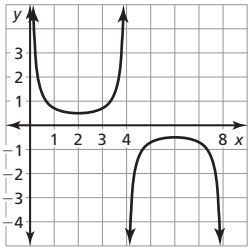


29. *Sample answer:* $y = -\sin \frac{1}{2}x$

30. *Sample answer:* $y = \cos \pi x - 2$

31. $h = -11.5 \cos 2\pi t + 13.5$

32. $P = 1.08 \sin(0.585t - 2.33) + 1.5$; The period represents the amount of time it takes for the precipitation level to complete one cycle, which is about 10.7 months.

33. $\cos^2 x$ **34.** $\tan x$ **35.** $\sin x$

36.
$$\begin{aligned}\frac{\cos x \sec x}{1 + \tan^2 x} &= \frac{\cos x \sec x}{\sec^2 x}\\ &= \frac{\cos x}{\sec x}\\ &= \cos x \cos x\\ &= \cos^2 x\end{aligned}$$

37.
$$\begin{aligned}\tan\left(\frac{\pi}{2} - x\right)\cot x &= \cot x \cot x\\ &= \cot^2 x\\ &= \csc^2 x - 1\end{aligned}$$

38. $\frac{\sqrt{2} + \sqrt{6}}{4}$ **39.** $\sqrt{3} - 2$ **40.** $\frac{\sqrt{6} + \sqrt{2}}{4}$

41. $\frac{19}{25}$ **42.** $x = \frac{3\pi}{4}, \frac{5\pi}{4}$ **43.** $x = 0, \pi$

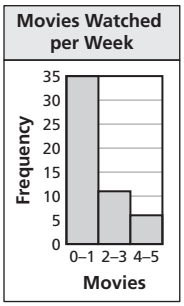
Chapter 10

Chapter 10 Maintaining Mathematical Proficiency *(p. 535)*

1. $\frac{6}{30} = \frac{p}{100}$, 20% **2.** $\frac{a}{25} = \frac{68}{100}$, 17

3. $\frac{34.4}{86} = \frac{p}{100}$, 40%

4.



5. no; The sofa will cost 80% of the retail price and the arm chair will cost 81% of the retail price.

10.1 Vocabulary and Core Concept Check *(p. 542)*

1. probability

10.1 Monitoring Progress and Modeling with Mathematics *(pp. 542–544)*

3. 48; 1HHH, 1HHT, 1HTH, 1THH, 1HTT, 1THT, 1TTH, 1TTT, 2HHH, 2HHT, 2HTH, 2THH, 2HTT, 2THT, 2TTH, 2TTT, 3HHH, 3HHT, 3HTH, 3THH, 3HTT, 3THT, 3TTH, 3TTT, 4HHH, 4HHT, 4HTH, 4THH, 4HTT, 4THT, 4TTH, 4TTT, 5HHH, 5HHT, 5HTH, 5THH, 5HTT, 5THT, 5TTH, 5TTT, 6HHH, 6HHT, 6HTH, 6THH, 6HTT, 6THT, 6TTH, 6TTT

5. 12; R1, R2, R3, R4, W1, W2, W3, W4, B1, B2, B3, B4

7. $\frac{5}{16}$, or about 31.25%

9. **a.** $\frac{11}{12}$, or about 92% **b.** $\frac{13}{18}$, or about 72%

11. There are 4 outcomes, not 3; The probability is $\frac{1}{4}$.

13. about 0.56, or about 56% **15.** 4

17. **a.** $\frac{9}{10}$, or 90% **b.** $\frac{2}{3}$, or about 67%

c. The probability in part (b) is based on trials, not possible outcomes.

19. about 0.08, or about 8% **21.** C, A, D, B

23. **a.** 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

b. 2: $\frac{1}{36}$, 3: $\frac{1}{18}$, 4: $\frac{1}{12}$, 5: $\frac{1}{9}$, 6: $\frac{5}{36}$, 7: $\frac{1}{6}$, 8: $\frac{5}{36}$, 9: $\frac{1}{9}$, 10: $\frac{1}{12}$, 11: $\frac{1}{18}$, 12: $\frac{1}{36}$

c. *Sample answer:* The probabilities are similar.

25. $\frac{\pi}{6}$, or about 52%

27. $\frac{3}{400}$, or 0.75%; about 113; (0.0075)15,000 = 112.5

10.1 Maintaining Mathematical Proficiency *(p. 544)*

29. $\frac{6x^4}{y^3}$ **31.** $\frac{x^3 - 4x^2 - 15x + 18}{x^4 - 2}$

33. $\frac{15x^2}{12x^2 + x - 11}$, $x \neq 0$

10.2 Vocabulary and Core Concept Check *(p. 550)*

1. When two events are dependent, the occurrence of one event affects the other. When two events are independent, the occurrence of one event does not affect the other. *Sample answer:* choosing two marbles from a bag without replacement; rolling two dice

10.2 Monitoring Progress and Modeling with Mathematics *(pp. 550–552)*

3. dependent; The occurrence of event A affects the occurrence of event B.

5. dependent; The occurrence of event A affects the occurrence of event B.

7. yes **9.** yes **11.** about 2.8% **13.** about 34.7%

15. The probabilities were added instead of multiplied; $P(A \text{ and } B) = (0.6)(0.2) = 0.12$

17. 0.325

19. **a.** about 1.2% **b.** about 1.0%

You are about 1.2 times more likely to select 3 face cards when you replace each card before you select the next card.

21. **a.** about 17.1% **b.** about 81.4% **23.** about 53.5%

25. **a.** *Sample answer:* Put 20 pieces of paper with each of the 20 students' names in a hat and pick one; 5%

b. *Sample answer:* Put 45 pieces of paper in a hat with each student's name appearing once for each hour the student worked. Pick one piece; about 8.9%

27. yes; The chance that it will be rescheduled is (0.7)(0.75) = 0.525, which is a greater than a 50% chance.

29. **a.** wins: 0%; loses: 1.99%; ties: 98.01%

b. wins: 20.25%; loses: 30.25%; ties: 49.5%

c. yes; Go for 2 points after the first touchdown, and then go for 1 point if they were successful the first time or 2 points if they were unsuccessful the first time; winning: 44.55%; losing: 30.25%

10.2 Maintaining Mathematical Proficiency *(p. 552)*

31. $x = 0.2$ **33.** $x = 0.15$

10.3 Vocabulary and Core Concept Check *(p. 558)*

1. two-way table

10.3 Monitoring Progress and Modeling with Mathematics *(p. 558–560)*

3. 34; 40; 4; 6; 12

5.

		Gender		
		Male	**Female**	**Total**
Response	**Yes**	132	151	283
	No	39	29	68
	Total	171	180	351

351 people were surveyed, 171 males were surveyed, 180 females were surveyed, 283 people said yes, 68 people said no.

7.

		Dominant Hand		
		Left	**Right**	**Total**
Gender	**Female**	0.048	0.450	0.498
	Male	0.104	0.398	0.502
	Total	0.152	0.848	1

9.

		Gender		
		Male	**Female**	**Total**
Response	**Yes**	0.376	0.430	0.806
	No	0.111	0.083	0.194
	Total	0.487	0.513	1

11.

		Breakfast	
		Ate	**Did Not Eat**
Feeling	**Tired**	0.091	0.333
	Not Tired	0.909	0.667

13. **a.** about 0.789 **b.** 0.168

c. The events are independent.

15. The value for P(yes) was used in the denominator instead of the value for P(Tokyo);

$$\frac{0.049}{0.39} \approx 0.126$$

17. Route B; It has the best probability of getting to school on time.

19. *Sample answer:*

		Transportation to School			
		Rides Bus	Walks	Car	Total
Gender	Male	6	9	4	19
	Female	5	2	4	11
	Total	11	11	8	30

		Transportation to School			
		Rides Bus	Walks	Car	Total
Gender	Male	0.2	0.3	0.133	0.633
	Female	0.167	0.067	0.133	0.367
	Total	0.367	0.367	0.266	1

21. Routine B is the best option, but your friend's reasoning of why is incorrect; Routine B is the best choice because there is a 66.7% chance of reaching the goal, which is higher than the chances of Routine A (62.5%) and Routine C (63.6%).

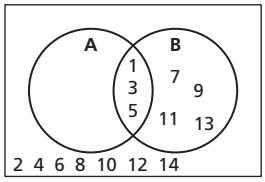
23. **a.** about 0.438 **b.** about 0.387

25. **a.** More of the current consumers prefer the leader, so they should improve the new snack before marketing it.

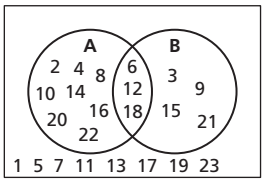
b. More of the new consumers prefer the new snack than the leading snack, so there is no need to improve the snack.

10.3 Maintaining Mathematical Proficiency *(p. 560)*

27.



29.



10.4 Vocabulary and Core Concept Check *(p. 567)*

1. yes; $\overline{A}$ is everything not in A; *Sample answer:* event A: you win the game, event $\overline{A}$: you do not win the game

10.4 Monitoring Progress and Modeling with Mathematics *(p. 567–568)*

3. 0.4 **5.** $\frac{7}{12}$, or about 0.58 **7.** $\frac{9}{20}$, or 0.45

9. $\frac{7}{10}$, or 0.7

11. forgot to subtract P(heart and face card);
P(heart) + P(face card) − P(heart and face card) = $\frac{11}{26}$

13. $\frac{2}{3}$ **15.** 10% **17.** 0.4742, or 47.42% **19.** $\frac{13}{18}$

21. $\frac{3}{20}$

23. no; Until all cards, numbers, and colors are known, the conclusion cannot be made.

10.4 Maintaining Mathematical Proficiency *(p. 568)*

25. $a_1 = 1, a_2 = 2, a_3 = 3, a_4 = 4, a_5 = 5, a_6 = 6$

10.5 Vocabulary and Core Concept Check *(p. 575)*

1. permutation

10.5 Monitoring Progress and Modeling with Mathematics *(p. 575–578)*

3. **a.** 2 **b.** 2 **5.** **a.** 24 **b.** 12

7. **a.** 720 **b.** 30 **9.** 20 **11.** 9 **13.** 20,160

15. 870 **17.** 990 **19.** $\frac{1}{56}$ **21.** 4 **23.** 20

25. 5 **27.** 1 **29.** 220 **31.** 6435 **33.** 635,376

35. The factorial in the denominator was left out;

$${}_{11}P_7 = \frac{11!}{(11-7)!} = 1{,}663{,}200$$

37. combinations; The order is not important; 45

39. permutations; The order is important; 132,600

41. ${}_{50}C_9 = {}_{50}C_{41}$; For each combination of 9 objects, there is a corresponding combination of the 41 remaining objects.

43. **a.** neither, they are the same; ${}_4P_4 = {}_4P_3 = 24$

b. 3; ${}_4C_4 = 1$, ${}_4C_3 = 4$

c. ${}_nP_n = {}_nP_{n-1}$, but ${}_nC_n < {}_nC_{n-1}$ when $n > 1$, and ${}_nC_n = {}_nC_{n-1}$ when $n = 1$.

45.

	$r = 0$	$r = 1$	$r = 2$	$r = 3$
${}_3P_r$	1	3	6	6
${}_3C_r$	1	3	3	1

${}_nP_r \geq {}_nC_r$; Because ${}_nP_r = \frac{n!}{(n-r)!}$ and ${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$, ${}_nP_r > {}_nC_r$ when $r > 1$ and ${}_nP_r = {}_nC_r$ when $r = 0$ or $r = 1$.

47. $\frac{1}{44{,}850}$ **49.** $\frac{1}{15{,}890{,}700}$ **51.** $x^3 + 6x^2 + 12x + 8$

53. $a^4 + 12a^3b + 54a^2b^2 + 108ab^3 + 81b^4$

55. $w^{12} - 12w^9 + 54w^6 - 108w^3 + 81$

57. $729u^6 + 1458u^5v^2 + 1215u^4v^4 + 540u^3v^6 + 135u^2v^8 + 18uv^{10} + v^{12}$

59. −8064 **61.** −13,608 **63.** 316,800,000

65. −337,920

67. ${}_8C_0, {}_8C_1, {}_8C_2, {}_8C_3, {}_8C_4, {}_8C_5, {}_8C_6, {}_8C_7, {}_8C_8$; 1, 8, 28, 56, 70, 56, 28, 8, 1

69. **a.** ${}_nC_{n-2} - n$ **b.** $\frac{n(n-3)}{2}$ **71.** 30 **73.** $\frac{1061}{1250}$

75. **a.** $\frac{1}{90}$ **b.** $\frac{9}{10}$ **77.** **a.** 2,598,960 **b.** 5148

79. **a.** about 0.04; about 0.12 **b.** $1 - \frac{{}_{365}P_x}{365^x}$ **c.** 23 people

10.5 Maintaining Mathematical Proficiency *(p. 578)*

81. TH

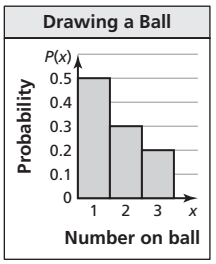
10.6 Vocabulary and Core Concept Check *(p. 583)*

1. a variable whose value is determined by the outcomes of a probability experiment

10.6 Monitoring Progress and Modeling with Mathematics *(pp. 583–584)*

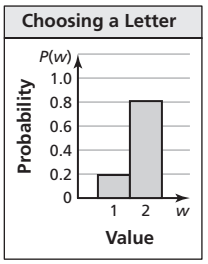
3.

x (value)	1	2	3
Outcomes	5	3	2
$P(x)$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{5}$



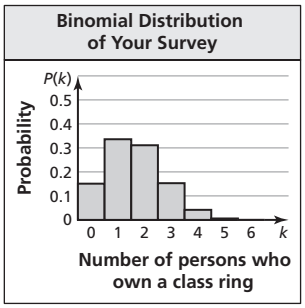
5.

w (value)	1	2
Outcomes	5	21
$P(w)$	$\frac{5}{26}$	$\frac{21}{26}$



7. **a.** 2 **b.** $\frac{5}{8}$ **9.** about 0.00002 **11.** about 0.00018

13. **a.**



b. The most likely outcome is that 1 of the 6 students owns a ring.

c. about 0.798

15. The exponents are switched;
$P(k = 3) = {}_5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{5-3} \approx 0.032$

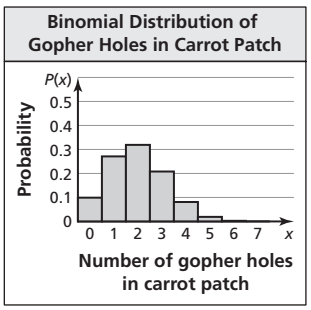
17. **a.** $P(0) \approx 0.099$, $P(1) \approx 0.271$, $P(2) \approx 0.319$, $P(3) \approx 0.208$, $P(4) \approx 0.081$, $P(5) \approx 0.019$, $P(6) \approx 0.0025$, $P(7) \approx 0.00014$

b.

x	0	1	2	3	4
$P(x)$	0.099	0.271	0.319	0.208	0.081

x	5	6	7
$P(x)$	0.019	0.0025	0.00014

c.

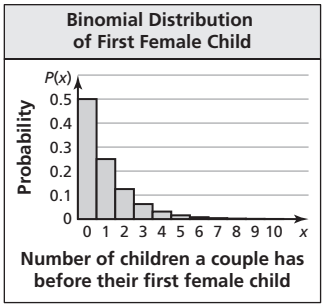


19. no; The data is skewed right, so the probability of failure is greater.

21. **a.** The statement is not valid, because having a male and having a female are independent events.

b. 0.03125

c.



skewed right

10.6 Maintaining Mathematical Proficiency *(p. 584)*

23. FFF, FFM FMF, FMM, MMM, MMF, MFM, MFF

Chapter 10 Review *(pp. 586–588)*

1. $\frac{2}{9}$; $\frac{7}{9}$ **2.** 20 points

3. **a.** 0.15625 **b.** about 0.1667

You are about 1.07 times more likely to pick a red then a green if you do not replace the first marble.

4. **a.** about 0.0586 **b.** 0.0625

You are about 1.07 times more likely to pick a blue then a red if you do not replace the first marble.

5. **a.** 0.25 **b.** about 0.2333

You are about 1.07 times more likely to pick a green and then another green if you replace the first marble.

6. about 0.529

7.

		Gender		
		Men	Women	Total
Response	Yes	200	230	430
	No	20	40	60
	Total	220	270	490

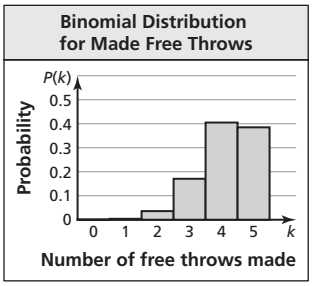
About 44.9% of responders were men, about 55.1% of responders were women, about 87.8% of responders thought it was impactful, about 12.2% of responders thought it was not impactful.

8. 0.68 **9.** 0.02 **10.** 5040 **11.** 1,037,836,800

12. 15 **13.** 70 **14.** $16x^4 + 32x^3y^2 + 24x^2y^4 + 8xy^6 + y^8$

15. $\frac{1}{84}$ **16.** about 0.12

17.



The most likely outcome is that 4 of the 5 free throw shots will be made.

Chapter 11

Chapter 11 Maintaining Mathematical Proficiency *(p. 593)*

1. about 77.2, 82.5, 82; median or mode; The mean is less than most of the data.
2. about 73.7, 70.5, 70; median or mode; The mean is greater than most of the data.
3. about 19.8, 16, 44; median; The mean and mode are both greater than most of the data.
4. about 3.85; The typical data value differs from the mean by about 3.85 units.
5. about 7.09; The typical data value differs from the mean by about 7.09 units.
6. 6.5; The typical data value differs from the mean by 6.5 units.
7. All the data values are the same; no; The formula for standard deviation includes taking only the positive square root.

11.1 Vocabulary and Core Concept Check *(p. 600)*

1. Find the value where row 1 and column 4 intersect.

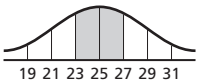
11.1 Monitoring Progress and Modeling with Mathematics *(pp. 600–602)*

3. 50% **5.** 2.5% **7.** 0.16 **9.** 0.025

11. 0.68 **13.** 0.68 **15.** 0.975 **17.** 0.84

19. **a.** 81.5% **b.** 0.15%

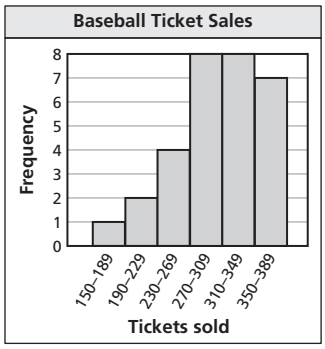
21. The values on the horizontal axis show a standard deviation of 1 instead of 2.



The probability that x is between 23 and 27 is 0.68.

23. 0.0668 **25.** no

27.



no; The histogram is skewed left, not bell-shaped.

29. **a.** about 4.52×10^{-9}

b. yes; The probability that a box contains an amount of cereal significantly less than the mean is very small.

31. one standard deviation above the mean

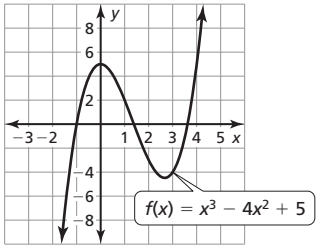
33. **a.** 88th percentile **b.** 93rd percentile

c. ACT; Your percentile on the ACT was higher than your percentile on the SAT.

35. no; When the mean is greater than the median, the distribution is skewed right.

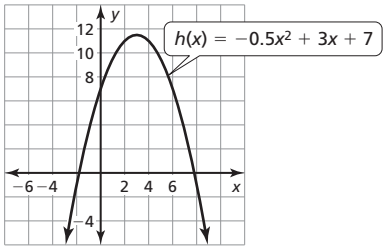
11.1 Maintaining Mathematical Proficiency *(p. 602)*

37.



x-intercepts: -1, about 1.4, and about 3.6; local maximum: (0, 5); local minimum: (2.67, -4.48); increasing when $x < 0$ and $x > 2.67$; decreasing when $0 < x < 2.67$

39.



x-intercepts: about -1.8 and about 7.8; maximum: (3, 11.5); no local minimum; increasing when $x < 3$; decreasing when $x > 3$

11.2 Vocabulary and Core Concept Check *(p. 607)*

1. sample
3. a claim about a characteristic of a population

11.2 Monitoring Progress and Modeling with Mathematics *(pp. 607–608)*

5. population; Every high school student is counted.
7. sample; The survey is given to a subset of the population of spectators.
9. population: every adult age 18 and over in the United States, sample: the 1152 adults age 18 and over who were surveyed; The sample consists of 403 adults who pretend to use their smartphone to avoid talking to someone, and 749 adults who do not.
11. population: every high school student in the district, sample: the 1300 high school students in the district who were surveyed; The sample consists of 1001 high school students who like the new healthy cafeteria food choices, and 299 high school students who do not.
13. statistic; The average annual salary of a subset of the population was calculated.
15. parameter; The percentage of every student in the school was calculated.
17. The sample number in the statement is not the size of the entire sample; The population consists of all the students in the high school. The sample consists of the 1270 students that were surveyed.
19. **a.** The maker's claim is most likely true.

    b. The maker's claim is most likely false.
21. possibly, but extremely unlikely; The result is unlikely to occur by chance. The sample size of the population is too small to make such a conclusion.

23. *Sample answer:* population: all American adults, sample: the 801 American adults surveyed; The sample consists of 606 American adults who say the world's temperature will go up over the next 100 years, 174 American adults who say it will go down, and 21 American adults who have no opinion.

25. simulation 2; Simulation 2 gives a better indication of outcomes that are not likely to occur by chance.

11.2 Maintaining Mathematical Proficiency *(p. 608)*

27. $x = 5 \pm\sqrt{29}$ or $x \approx 10.39$, $x \approx -0.39$

29. $s = -5 \pm\sqrt{17}$ or $s \approx -0.88$, $s \approx -9.12$

31. $z = \frac{1}{2}$, $z = -\frac{15}{2}$

11.3 Vocabulary and Core Concept Check *(p. 614)*

1. In a stratified sample, after the groups are formed, a random sample is selected from each group. In a cluster sample, after the groups are formed, all the members of one or more groups are randomly selected.

3. *Sample answer:* to determine how quickly an oil spill would spread through a lake

11.3 Monitoring Progress and Modeling with Mathematics *(pp. 614–616)*

5. convenience sample **7.** systematic sample

9. convenience sample; Dog owners probably have a strong opinion about an off-leash area for dogs.

11. cluster sample; Booth holders in section 5 are likely to have a different opinion than booth holders in other sections about the location of their booth.

13. Not every survey that was mailed out will be returned, so it is not a systematic sample; Because households in the neighborhood can choose whether or not to return the survey, the sample is a self-selected sample.

15. no; The sample represents the population.

17. yes; Only customers with a strong opinion about their experience are likely to complete the survey.

19. *Sample answer:* Assign each student in the school a different integer from 1 to 1225. Generate 250 unique random integers from 1 to 1225 using the random number function in a spreadsheet program. Choose the 250 students who correspond to the 250 integers generated.

21. simulation **23.** observational study

25. encourages a yes response; *Sample answer:* Reword the question, for example: Should the budget of our city be cut?

27. implies that the arsenic level is a health risk; *Sample answer:* Reword the question, for example: Do you think the government should address the issue of arsenic in tap water?

29. no; Responses to the question will accurately reflect the opinions of those being surveyed.

31. yes; Visitors are unlikely to admit to a police officer that they do not wear their seatbelt.

33. **a.** *Sample answer:* The researcher did not take into account previous heart conditions.

b. *Sample answer:* Divide the population into groups based on past heart conditions and whether or not they take fiber supplements. Select a random sample from each group.

35. self-selected sample and convenience sample; In a self-selected sample, only people with strong opinions are likely to respond. In a convenience sample, parts of the population have no chance of being selected for the survey.

37. **a.** to determine the employment rate of graduates in their field of study

b. all graduating seniors of the college

c. *Sample answer:* Are you employed? If yes, is your job in your field of study?

39. no; *Sample answer:* Some groups in the population, like the homeless, are difficult to contact.

41. **a.** self-selected sample

b. people who spend a lot of time on the Internet and visit that particular site; The survey is probably biased.

11.3 Maintaining Mathematical Proficiency *(p. 616)*

43. 9 **45.** $\frac{1}{4}$ **47.** $\frac{\sqrt[3]{18}}{18}$ **49.** 3

11.4 Vocabulary and Core Concept Check *(p. 623)*

1. replication

11.4 Monitoring Progress and Modeling with Mathematics *(pp. 623–624)*

3. The study is a randomized comparative experiment; The treatment is the drug for insomnia. The treatment group is the individuals who received the drug. The control group is the individuals who received the placebo.

5. The individuals who do not use either of the conditioners were not monitored; The control group is the individuals who use the regular conditioner.

7. observational study; *Sample answer:* Randomly choose one group of individuals who smoke. Then, randomly choose one group of individuals who do not smoke. Find the body mass index of the individuals in each group.

9. experiment; *Sample answer:* Randomly select the same number of strawberry plants to be put in each of two groups. Use the new fertilizer on the plants in one group, and use the regular fertilizer on plants in the other group. Keep all other variables constant and record the weight of the fruit produced by each plant.

11. **a.** *Sample answer:* Because the heart rates are monitored for two different types of exercise, the groups cannot be compared. Running on a treadmill may have a different effect on heart rate than lifting weights; Check the heart rates of all the athletes after the same type of exercise.

b. no potential problems

13. *Sample answer:* The sample size is not large enough to provide valid results; Increase the sample size.

15. no; Your friend would have to perform an observational study, and an observational study can show correlation, but not causality.

17. *Sample answer:* The placebo effect is response to a dummy treatment that may result from the trust in the researcher or the expectation of a cure; It can be minimized by comparing two groups so the placebo effect has the same effect on both groups.

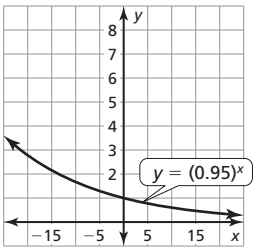
19. yes; Repetition reduces the effect of unusual results that may occur by chance.

11.4 Maintaining Mathematical Proficiency *(p. 624)*

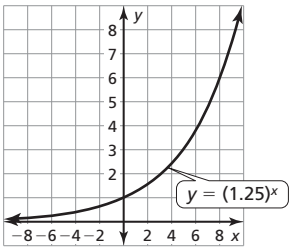
21.

skewed right

23. exponential decay



25. exponential growth



11.5 Vocabulary and Core Concept Check *(p. 630)*

1. margin of error

11.5 Monitoring Progress and Modeling with Mathematics *(pp. 630–632)*

3. 60.4 **5. a.** about 0.267 **b.** about 0.267

7. a. yes; The first 2 surveys show more than the 66.7% of votes needed to override the veto.

b. no; As the sample size increases, the percent of votes approaches 55.1%, which is not enough to override the veto.

9. a. The company's claim is probably accurate.

b. The company's claim is probably not accurate.

c. *Sample answer:* 0.42 to 0.68

11. about ±6.2% **13.** about ±2.2% **15.** about ±1.7%

17. a. about ±3.1% **b.** between 37.9% and 44.1%

19. The wrong percentage was substituted in the formula;

$\pm 0.04 = \pm\frac{1}{\sqrt{n}}$; $0.0016 = \frac{1}{n}$; $n = 625$

21. no; A sample size of 1 would have a margin of error of 100%.

23. about 453 residents

25. a. 500 voters **b.** about ±4.5%

c. candidate A: between 42.5% and 51.5%, candidate B: between 48.5% and 57.5%

d. no; 273 voters

27. more than 2500; To be confident that sports drink X is preferred, the margin of error would need to be less than 2%.

11.5 Maintaining Mathematical Proficiency *(p. 632)*

29. $y = \log_2(x + 5)$ **31.** $y = 6^{x+1}$

33. geometric; $a_n = 3(2)^{n-1}$

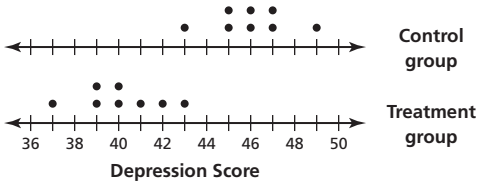
11.6 Vocabulary and Core Concept Check *(p. 637)*

1. resampling

11.6 Monitoring Progress and Modeling with Mathematics *(pp. 637–638)*

3. a. 46 **b.** 40.125 **c.** −5.875

d.



e. The music therapy may be effective in reducing depression scores of college students.

5. The order of the subtraction is reversed; $\bar{x}_{\text{treatment}} - \bar{x}_{\text{control}} = 11 - 16 = -5$; So, you can conclude the treatment decreases the score.

7. *Sample answer:* −1.75

9. The hypothesis is most likely false; Music therapy decreases depression scores.

11. The histogram in Exercise 9 has a roughly normal distribution and shows the mean differences from 200 resamplings. The histogram in Exercise 11 is random and shows the mean differences from 20 resamplings; the histogram in Exercise 9 because it uses a large number of resamplings and the roughly normal distribution suggests music therapy decreases depression scores

13. yes; As the number of samplings increase, the individual values should end up in each group approximately the same number of times, so the positive and negative differences in the means should balance out to 0.

15. 12,870; The number of combinations of 16 items in groups of 8 amounts to 12,870.

11.6 Maintaining Mathematical Proficiency *(p. 638)*

17. $(y - 2)(y^2 + 2y + 4)$ **19.** $(9w^2 + 4)(3w + 2)(3w - 2)$

21. yes; $g(x) = \frac{1}{2x} + \frac{1}{2}$ **23.** no; $y = \pm\sqrt{\frac{3}{x - 1}}$

Chapter 11 Review *(pp. 640–642)*

1. 0.0015 **2.** 0.0082

3. population: all U.S. motorists, sample: the 1000 drivers surveyed

4. statistic; The mean was calculated from a sample.

5. The host's claim is most likely false.

6. stratified sample; not biased **7.** observational study

8. It encourages a yes response; *Sample answer:* Reword the question, for example: Should the city replace the police cars it is currently using?

9. experiment; *Sample answer:* Randomly select the same number of customers to give each type of bread to. Record how many customers from each group return.

10. *Sample answer:* The volunteers may not be representative of the population; Randomly select from members of the population for the study.

11. The study is a randomized comparative experiment; The treatment is using the new design of the car wash. The treatment group is the individuals who use the new design of the car wash. The control group is the individuals who use the old design of the car wash.

12. between 58.9% and 65.1%

13. no; As the sample size increases, the percent of votes approaches 46.8%, which is not enough to win.

14. *Sample answer:* Combine the measurements from both groups and assign a number to each value. Let the numbers 1 through 10 represent the data in the original control group, and let the numbers 11 through 20 represent the data in the original treatment group. Use a random number generator. Randomly generate 20 numbers from 1 through 20 without repeating a number. Use the first 10 numbers to make the new control group, and the next 10 to make the new treatment group; Repeatedly make new control and treatment groups and see how often you get differences between the new groups that are at least as large as the one you measured.