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 $x^{\frac{3}{3}}$ Worksheet – Properties of Logarithms

Expand as much as possible using the properties of logs.

1. $\log_2(15\sqrt[3]{x^2})$

$\log_2 15 + \frac{2}{3} \log_2 x$

3. $\log_5 \frac{(xy)^2}{\sqrt{z}}$

$2\log_5 x + 2\log_5 y - \frac{1}{2} \log_5 z$

5. $\ln(xe^x)$

$\ln x + x$

7. $\log \frac{x^4}{x-2}$

$4\log x - \log(x-2)$

Write each expression as a rational number or as a single logarithm.

9. $\log 2 + \log 3 + \log 4$

$\log 2 \cdot 3 \cdot 4$
 $\log 24$

10. $\frac{1}{2} \log_6 9 + \log_6 5$

$\log_6 3 \cdot 5$
 $\log_6 15$

11. $2 \ln 6 - \ln 3$

$\ln \frac{36}{3} = \ln 12$

12. $\log M - 3 \log N$

$\log \frac{M}{N^3}$

13. $\log A - 2 \log B + 3 \log C$

$\log \frac{AC^3}{B^2}$

14. $\frac{1}{3}(2 \log_b M - \log_b N - \log_b P)$

$\log_b \frac{M^{2/3}}{NP^{1/3}}$

15. $\log \pi + 2 \log r$

$\log \pi r^2$

16. $\log_2 2 + \log_2 6 - \frac{1}{2} \log_2 9$

$\log_2 \frac{2 \cdot 6}{9} = \log_2 4 = 2$

Evaluate without using a calculator.

17. $\ln e^2$

2

18. $\ln \frac{1}{e}$

-1

19. $\ln \sqrt{e}$

$\frac{1}{2}$

20. $10^{\log 6}$

6

21. $10^{2\log 6}$

36

22. $10^{\log 3 + \log 4}$

12

23. $e^{\ln \sqrt{2}}$

$\sqrt{2}$

24. $3^{3\log_3 2 + \log_3 8}$

64

25. $\log_4 5 - \log_4 80$

$\log_4 \frac{5}{80} = -2$

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A 9-1

WORKSHEET – LOGARITHMS

Write in exponential form.

1. $\log_a 25 = x \quad a^x = 25$

2. $\log \frac{1}{10000} = -4 \quad 10^{-4} = 1/10000$

3. $\log_x M = N \quad x^N = M$

4. $\ln e = 1 \quad e^1 = e$

5. $\log_{16} 8 = \frac{3}{4} \quad 16^{\frac{3}{4}} = 8$

6. $\ln 1 = 0 \quad e^0 = 1$

Write in logarithmic form.

7. $a^3 = 64 \quad \log_a 64 = 3$

8. $x^{\pi} = e \quad \log_x e = \pi$

9. $e^{-x} = 8 \quad \ln 8 = -x$

10. $81^{\frac{1}{4}} = 3 \quad \log_{81} 3 = \frac{1}{4}$

11. $10^{\frac{1}{2}} = N \quad \log N = \frac{1}{2}$

12. $6^{-2} = \frac{1}{36} \quad \log_6 \frac{1}{36} = -2$

Evaluate each logarithm. Do not use a calculator.

13. $\log 100 = 2 \quad 14. \log 10,000 = 4 \quad 15. \log 0.01 = -2$

16. $\log_3 9 = 2 \quad 17. \log_{16} 2 = \frac{1}{4} \quad 18. \log_3 3^8 = 8$

19. $\log_{25} \frac{1}{5} = -\frac{1}{2} \quad 20. \log_4 \frac{1}{64} = -3 \quad 21. \log_4 \sqrt[4]{4} = \frac{1}{4}$

22. $\ln e = 1 \quad 23. \ln e^2 = 2 \quad 24. \ln \sqrt{e} = \frac{1}{2}$

A 9-1

25. $\log_3 \left(\frac{1}{27} \right) = -3 \quad 26. \log_{\frac{1}{2}} 9 = -2 \quad 27. \log_2 1 = 0$

28. $\ln (-10) = \text{na} \quad 29. \ln e^{4.5} = 4.5 \quad 30. \log 0 = \text{na}$

Evaluate each logarithm using the change of base formula. Round answers to three decimal places.

31. $\log_3 25 \quad \frac{\log 25}{\log 3} \approx 2.930 \quad 32. \log_8 100 \quad \frac{\log 100}{\log 8} \approx 2.215$

33. $\log_{.35} .35 \quad \frac{\ln .35}{\ln .35} \approx 0.652 \quad 34. \log_{16} 16 \quad \frac{\ln 16}{\ln 16} \approx -4$

35. $\log_8 (-36) \quad \text{not possible} \quad 36. \log_{25} 1200 \quad \frac{\log 1200}{\log 25} \approx 2.203$

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A 9-3

WORKSHEET - LOGARITHMIC EQUATIONS

Solve.

1. $\log_2(2x+1) = 3$

$$2^3 = 2^3$$

$$2x+1 = 8$$

$$2x = 7$$

$$\boxed{x = \frac{7}{2}}$$

3. $\frac{1}{2} \log_3 x = 2 \log_3 2$

$$\log_3 x^{\frac{1}{2}} = \log_3 4$$

$$(x^{\frac{1}{2}})^2 = (4)^2$$

$$\boxed{x = 16}$$

5. $3 \log_2(x-1) + \log_2 4 = 5$

$$2^3 \log_2(x-1)^3 = 2^5$$

$$4(x-1)^3 = 32$$

$$\frac{4}{4}(x-1)^3 = \frac{32}{4}$$

$$(x-1)^3 = 8$$

$$x-1 = 2$$

$$\boxed{x = 2}$$

2. $\log_3(x^2 + 1) = 2$

$$3^2 = 3^2$$

$$x^2 + 1 = 9$$

$$x^2 = 8$$

$$x = \pm \sqrt{8}$$

$$\boxed{x = \pm 2\sqrt{2}}$$

4. $2 \log_5 x = 3 \log_5 4$

$$\log_5 x^2 = \log_5 4^3$$

$$\sqrt{x^2} = \sqrt{64}$$

$$x = \pm 8$$

$$\boxed{x = 8}$$

6. $\log x + \log(x+15) = 2$

$$\log x(x+15) = 10^2$$

$$x^2 + 15x = 100$$

$$x^2 + 15x - 100 = 0$$

$$(x+20)(x-5) = 0$$

$$x = -20 \quad \boxed{x = 5}$$

7. $\log_x 4 = 2$

$$x^2 = 4$$

$$2 = x$$

$$\boxed{2 = x}$$

8. $\log_3(x-1)^2 = 2$

$$3^2 = 3^2$$

$$(x-1)^2 = 9$$

$$x-1 = \pm 3$$

$$\boxed{x = 4, -2}$$

10. $\ln x + \ln(x-2) = \ln(x+4)$

$$\ln x(x-2) = \ln(x+4)$$

$$x^2 - 2x = x+4$$

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

$$\boxed{x = 4} \quad x \neq -1$$

9. $\log_{1/2}(3x+1)^{1/3} = -2$

$$(3x+1)^{\frac{1}{3}} = 4^{\frac{3}{2}}$$

$$3x+1 = 64$$

$$3x = 63$$

$$\boxed{x = 21}$$

11. $\log_a(x-2) - \log_a(x+3) = \log_a(x-1) - \log_a(x+6)$

$$\log_a \left(\frac{x-2}{x+3} \right) = \log_a \left(\frac{x-1}{x+6} \right)$$

$$\frac{x-2}{x+3} = \frac{x-1}{x+6}$$

$$x^2 + 4x - 12 = x^2 + 2x - 3$$

$$2x = 9$$

$$\boxed{x = \frac{9}{2}}$$

Evaluate each logarithm. Do not use a calculator.

12. $\log_3 3^2$

~~$x = 9$~~
2t

13. $\log_4 \frac{1}{64}$

-3

14. $\log (-0.01)$

not
possible

15. $2^{\log_2 0.25}$

• 25

Expand as much as possible using the properties of logs.

16. $\log \frac{10}{(x^2+1)}$

$\log 10 - \log(x^2+1)$

17. $\log(m^2\sqrt{m})^4$

$8\log m + 2\log m$
 $10\log m$

Write each expression as a rational number or as a single logarithm.

18. $\log 3 - \log 12 + \log 6$

$\log \frac{3}{12} \cdot 6$

19. $\frac{1}{3}\log 8 + \log 5$

$\log \frac{3}{2}$

$\log 2 \cdot 5$

$\log 10$
1

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A 9-4

WORKSHEET – USING LOGS TO SOLVE EXPONENTIAL EQUATIONS

1. $2^{2x+1} = 4^2$

$2x+1 = 2$

$2x = 1$

$x = \frac{1}{2}$

2. $3^{x^2} = 81$

~~$3^{x^2} = 81$~~

$3^{x^2} = 3^4$

$x^2 = 4$

$x = \pm 2$

3. $2^x = 10$

$x \log 2 = \log 10$

$x = \frac{1}{\log 2} \approx 3.32$

4. $8^{-x} = 1.2$

$-x = \frac{\ln 1.2}{\ln 8}$

$x = \frac{-\ln 1.2}{\ln 8} \approx -0.087$

5. $3^{1-2x} = 4^{x+3}$

$(1-2x)\ln 3 = (x+3)\ln 4$

$\ln 3 - 2x\ln 3 = x\ln 4 + 3\ln 4$

$\ln 3 - 3\ln 4 = x\ln 4 + 2x\ln 3$

$\ln 3 - 3\ln 4 = x(\ln 4 + 2\ln 3)$

$\frac{\ln 3 - 3\ln 4}{\ln 4 + 2\ln 3} = x$

$-0.854 \approx x$

6. $\left(\frac{3}{5}\right)^x = 7^{1-x}$

$x \log \frac{3}{5} = (1-x) \log 7$

$x \log \frac{3}{5} = \log 7 - x \log 7$

$x \log \frac{3}{5} + x \log 7 = \log 7$

$x(\log \frac{3}{5} + \log 7) = \log 7$

$x = \frac{\log 7}{\log \frac{3}{5} + \log 7} \approx 1.356$

7. $1.2^x = (0.5)^{-x}$

$x \ln 1.2 = -x \ln 0.5$

$x \ln 1.2 + x \ln 0.5 = 0$

$x = 0$

8. $\pi^{1-x} = e^x$

$(1-x)\cancel{\pi} = x \ln e$

$\cancel{\pi} - x\cancel{\pi} = x$

$\cancel{\pi} = x + \pi x$

$\cancel{\pi} = x(1+\pi)$

$\boxed{\frac{\cancel{\pi}}{1+\cancel{\pi}} = x}$

9. $400e^{0.02x} = 600$

$e^{0.02x} = \frac{600}{400}$

$\ln e^{0.02x} = \ln \frac{3}{2}$

$0.02x = \ln \frac{3}{2}$

$x = \frac{\ln \frac{3}{2}}{0.02} \approx 20.273$

10. $5(2^{3x}) = 8$

$2^{3x} = \frac{8}{5}$

$3x \ln 2 = \ln \frac{8}{5}$

$x = \frac{\ln \frac{8}{5}}{3 \ln 2} \approx 0.226$

11. $\frac{400}{1+e^{-x}} = 350$

$\frac{1+e^{-x}}{400} = \frac{1}{350}$

$1+e^{-x} = \frac{8}{7}$

$\ln e^{-x} = \ln \frac{1}{7}$

$-x = \ln \frac{1}{7}$

$x = -\ln \frac{1}{7}$

$x \approx 1.946$

12. $-14 + 3e^x = 11$

$3e^x = 25$

$e^x = \frac{25}{3}$

$x = \ln \frac{25}{3} \approx 2.120$

13. $\ln(x-3) + \ln(x-2) = \ln(2x+24)$

14. $\log_4 x - \log_4(x-1) = \frac{1}{2}$

$\ln(x-3)(x-2) = \ln(2x+24)$

$x^2 - 5x + 6 = 2x + 24$

$\frac{x}{x-1} = 2$

$x = 2$

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EXPONENTIAL AND LOGARITHMIC APPLICATIONS

1. The number of bacteria in a petri dish culture grows continuously and doubles every 3 hours. Initially there are 2500 bacteria present.

a) Write an equation to model the continuous growth of the bacteria.

$$5000 = 2500 e^{kt}$$

$$2 = e^{3k}$$

$$0.231 \approx \frac{\ln 2}{3} = \frac{0.693}{3}$$

b) When will the number of bacteria be 100,000?

$$100000 = 2500 e^{0.231t}$$

$$40 = e^{0.231t}$$

~~$$\ln 40 = \frac{0.231t}{0.231}$$~~

$$15.97 = t$$

2. a) The population of a small town in the year 1890 was 6250. Assuming it increased at the rate of 3.75% per year, what was the population in 1915? (Round to the nearest person.)

$$A(t) = 6250(1 + 0.0375)^t \quad t = 25$$

$$A(25) = 6250(1 + 0.0375)^{25} = 15689$$

b) In 1940?

$$A(50) = 6250(1 + 0.0375)^{50} = 39381$$

c) What year would the population be expected to double according to the model?

$$12500 = 6250(1.0375)^t$$

$$2 = (1.0375)^t$$

$$\ln 2 = t \ln 1.0375$$

$$\frac{\ln 2}{\ln 1.0375} = t$$

$$18.8 = t$$

$$\begin{array}{r} 1890 \\ + 18 \\ \hline 1908 \end{array}$$

3. The half-life of a certain radioactive substance is 14 days and there are 6.58 g present initially. When will there be 1 gram remaining?

4. Find the principal needed to have a balance of \$1000 after 2 years at 6% compounded monthly.

5. When will an investment of \$1000 be worth \$1500 if the balance is compounded continuously at 5.25%?

6. You will be buying a new car for \$15,000 in 3 years. How much money should you invest now at 5% compounded continuously, so that you will have enough to buy the car?

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A 9-5

WORKSHEET – MORE PRACTICE SOLVING LOG EQUATIONS

1. $5^x = 125^{2+x}$

$5^x = 5^{6+3x}$

$x = 6+3x$

$-2x = 6$

$\boxed{x = -3}$

2. $\log_4 x = 3 - \log_4 (20-x)$

$\log_4 x + \log_4 (20-x) = 3$

$4 \log_4 x(20-x) = 3$

$x(20-x) = 64$

$20x - x^2 = 64$

$x^2 - 20x + 64 = 0$

$(x-16)(x-4) = 0$

3. $2\log_2(4x) - \log_2 x = 3$

$2 \log_2 \frac{(4x)^2}{x} = 3$

$(4x)^2 = 8$

$x = \frac{1}{2}$

$\boxed{x = \frac{1}{2}}$

$5. (x^{\frac{3}{2}})^{\frac{2}{3}} = (17)^{\frac{2}{3}}$

$x = 17^{\frac{2}{3}}$

$\boxed{x \approx 6.611}$

4. $\log_3 |x| = 2$

$3^2 = 3^1$

$|x| = 9$

$\boxed{x = \pm 9}$

$6. \frac{6^{2-x}}{7} = 15$

$\ln 6^{2-x} = \ln 105$

$(2-x)\ln 6 = \ln 105$

$2-x = \frac{\ln 105}{\ln 6}$

$x = \frac{\ln 105}{\ln 6} - 2$

$x = 2 - \frac{\ln 105}{\ln 6}$

$\boxed{x \approx -0.597}$

7. $\log_{10} |2x-1| = 1$

$10^1 = 10^{\frac{1}{2}}$

$|2x-1| = \frac{1}{2}$

$2x = \frac{3}{2}$

$\boxed{x = \frac{3}{4}}$

$9^{\ln e^{x+3}} = 10^x$

$x+3 = x \ln 3$

$x - x \ln 3 = -3$

$x(1-\ln 3) = -3$

$\boxed{x = \frac{-3}{1-\ln 3}}$

$x + 3.0422$

$10. e^{2x} - 3e^x = 10$

$(e^x - 5)(e^x + 2) = 0$

$e^x = 5$

$e^x = -2$

$x = \ln 5$

$\boxed{x \approx 1.609}$

$12. x^2 e^{2x} - e^{2x} = 0$

$e^{2x}(x^2 - 1) = 0$

$e^{2x} \neq 0$

$x^2 - 1 = 0$

$x = \pm 1$

$\boxed{x = \pm 1}$

$14. x \ln x = x$

$x \ln x - x = 0$

$x(\ln x - 1) = 0$

$x \neq 0$

$\ln x - 1 = 0$

$\ln x = 1$

$e^{\ln x} = e^1$

$\boxed{x = e}$

A 9-5

8. $60 \log_4 2 - x \log_4 2 = x$

$\log_4 2^{\frac{60}{x}} - \log_4 2^x = x$

$\log_4 \frac{2^{\frac{60}{x}}}{2^x} = x$

$\log_4 2^{\frac{60-x}{x}} = x$

$2^{\frac{60-x}{x}} = 2^x$

$60-x = x$

$60 = 2x$

$20 = x$

ANSWERS: 1. -3 2. 16.4 3. $\frac{1}{2}$ 4. 9 -9 5. 6.611 6. -597 7. $\frac{1}{4}$
 8. 20 9. 30.422 10. 1.609 11. -.693 1.099 12. 1. -1 13. -1.566 14. e

7. Strontium-90 is a radioactive material that decays according to the equation
 $A = A_0 e^{-0.0244t}$, where t is in terms of years. What is the half-life of strontium-90?

8. The US population, in millions, is growing according to the formula $P = 263e^{0.009t}$ where t is years since 1995. In what year would the population be expected to reach 300 million?

9. A fossilized leaf is found to contain 10 micrograms of carbon-14 whereas a leaf of this type normally contains about 13.5 micrograms of carbon-14. Estimate the age of the fossilized leaf. (The half-life of carbon 14 is 5730 years. Round to the nearest year.)

Answers: 1. a) $A(t) = 2500e^{-231t}$ b) 15.969 hours 2. a) 15,689 people b) 39,381 people c) 1908
3. 38.053 days 4. \$887.19 5. 7.723 years 6. \$12,910.62 7. 28.408 years 8. 2009 9. 2481 years

