

Calculus 1 - Limits
Worksheet 3
Evaluating Limits by
Factoring, Part 1

Calculus 1 - Limits - Worksheet 3 – Evaluating Limits by Factoring, Part 1

1. Evaluate this limit.

$$\lim_{x \rightarrow 9} \frac{x - 9}{x^2 - 81}$$

2. Evaluate this limit.

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 5x + 6}$$

3. Evaluate this limit.

$$\lim_{x \rightarrow -4} \frac{x + 4}{4x + 16}$$

4. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + x - 2}$$

5. Evaluate this limit.

$$\lim_{x \rightarrow 1} \frac{35x - 35}{40x - 40}$$

6. Evaluate this limit.

$$\lim_{x \rightarrow -1} \frac{x^2 + 6x + 5}{x + 1}$$

7. Evaluate this limit.

$$\lim_{x \rightarrow 7} \frac{-x^2 + 16x - 63}{x^2 - 2x - 35}$$

8. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + 6x + 8}$$

9. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{2x^2 + 10x}{3x^2 + 15x}$$

10. Evaluate this limit.

$$\lim_{x \rightarrow -7} \frac{x + 7}{x^2 + 4x - 21}$$

11. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{x^2 - 3x - 10}{x^2 + x - 2}$$

12. Evaluate this limit.

$$\lim_{x \rightarrow 8} \frac{x - 8}{x^2 - 2x - 48}$$

13. Evaluate this limit.

$$\lim_{x \rightarrow 6} \frac{6 - x}{3x - 18}$$

14. Evaluate this limit.

$$\lim_{x \rightarrow 4} \frac{28 - 7x}{x - 4}$$

15. Evaluate this limit.

$$\lim_{x \rightarrow 7} \frac{4x - 28}{5x^3 - 35x^2}$$

Answers - Calculus 1 - Limits - Worksheet 3 – Evaluating Limits by Factoring, Part 1

1. Evaluate this limit.

$$\lim_{x \rightarrow 9} \frac{x - 9}{x^2 - 81}$$

First, attempt to evaluate the limit using direct substitution. Substitute 9 into the limit for x .

$$\lim_{x \rightarrow 9} \frac{x - 9}{x^2 - 81} = \frac{9 - 9}{9^2 - 81} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 9} \frac{x - 9}{x^2 - 81} = \lim_{x \rightarrow 9} \frac{x - 9}{(x + 9)(x - 9)} = \lim_{x \rightarrow 9} \frac{1}{x + 9}$$

Substitute 9 into the limit for x .

$$\lim_{x \rightarrow 9} \frac{1}{x + 9} = \frac{1}{9 + 9} = \frac{1}{18}$$

Answer: $\lim_{x \rightarrow 9} \frac{x - 9}{x^2 - 81} = \frac{1}{18}$

2. Evaluate this limit.

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 5x + 6}$$

First, attempt to evaluate the limit using direct substitution. Substitute 2 into the limit for x .

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 5x + 6} = \frac{(2)^2 - 3(2) + 2}{(2)^2 - 5(2) + 6} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 5x + 6} = \lim_{x \rightarrow 2} \frac{(x - 2)(x - 1)}{(x - 3)(x - 2)} = \lim_{x \rightarrow 2} \frac{x - 1}{x - 3}$$

Substitute 2 into the limit for x .

$$\lim_{x \rightarrow 2} \frac{x - 1}{x - 3} = \frac{2 - 1}{2 - 3} = \frac{1}{-1} = -1$$

Answer: $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 5x + 6} = -1$

3. Evaluate this limit.

$$\lim_{x \rightarrow -4} \frac{x + 4}{4x + 16}$$

First, attempt to evaluate the limit using direct substitution. Substitute -4 into the limit for x .

$$\lim_{x \rightarrow -4} \frac{x + 4}{4x + 16} = \frac{-4 + 4}{4(-4) + 16} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -4} \frac{x + 4}{4x + 16} = \lim_{x \rightarrow -4} \frac{x + 4}{4(x + 4)} = \lim_{x \rightarrow -4} \frac{1}{4} = \frac{1}{4}$$

Answer: $\lim_{x \rightarrow -4} \frac{x + 4}{4x + 16} = \frac{1}{4}$

4. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + x - 2}$$

First, attempt to evaluate the limit using direct substitution. Substitute -2 into the limit for x .

$$\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + x - 2} = \frac{-2 + 2}{(-2)^2 + (-2) - 2} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + x - 2} = \lim_{x \rightarrow -2} \frac{x + 2}{(x + 2)(x - 1)} = \lim_{x \rightarrow -2} \frac{1}{x - 1}$$

Substitute -2 into the limit for x .

$$\lim_{x \rightarrow -2} \frac{1}{x - 1} = \frac{1}{-2 - 1} = \frac{1}{-3} = -\frac{1}{3}$$

Answer: $\lim_{x \rightarrow -2} \frac{x+2}{x^2+x-2} = -\frac{1}{3}$

5. Evaluate this limit.

$$\lim_{x \rightarrow 1} \frac{35x - 35}{40x - 40}$$

First, attempt to evaluate the limit using direct substitution. Substitute 1 into the limit for x .

$$\lim_{x \rightarrow 1} \frac{35x - 35}{40x - 40} = \frac{35(1) - 35}{40(1) - 40} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 1} \frac{35x - 35}{40x - 40} = \lim_{x \rightarrow 1} \frac{35(x - 1)}{40(x - 1)} = \lim_{x \rightarrow 1} \frac{35}{40} = \frac{7}{8}$$

Answer: $\lim_{x \rightarrow 1} \frac{35x - 35}{40x - 40} = \frac{7}{8}$

6. Evaluate this limit.

$$\lim_{x \rightarrow -1} \frac{x^2 + 6x + 5}{x + 1}$$

First, attempt to evaluate the limit using direct substitution. Substitute -1 into the limit for x .

$$\lim_{x \rightarrow -1} \frac{x^2 + 6x + 5}{x + 1} = \frac{(-1)^2 + 6(-1) + 5}{-1 + 1} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -1} \frac{x^2 + 6x + 5}{x + 1} = \lim_{x \rightarrow -1} \frac{(x + 1)(x + 5)}{x + 1} = \lim_{x \rightarrow -1} (x + 5)$$

Substitute -1 into the limit for x .

$$\lim_{x \rightarrow -1} (x + 5) = -1 + 5 = 4$$

Answer: $\lim_{x \rightarrow -1} \frac{x^2 + 6x + 5}{x + 1} = 4$

7. Evaluate this limit.

$$\lim_{x \rightarrow 7} \frac{-x^2 + 16x - 63}{x^2 - 2x - 35}$$

First, attempt to evaluate the limit using direct substitution. Substitute 7 into the limit for x .

$$\lim_{x \rightarrow 7} \frac{-x^2 + 16x - 63}{x^2 - 2x - 35} = \frac{-7^2 + 16(7) - 63}{7^2 - 2(7) - 35} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 7} \frac{-x^2 + 16x - 63}{x^2 - 2x - 35} = \lim_{x \rightarrow 7} \frac{-(x^2 - 16x + 63)}{x^2 - 2x - 35}$$

$$\lim_{x \rightarrow 7} \frac{-(x^2 - 16x + 63)}{x^2 - 2x - 35} = \lim_{x \rightarrow 7} \frac{-(x - 7)(x - 9)}{(x + 5)(x - 7)}$$

$$\lim_{x \rightarrow 7} \frac{-(x - 9)}{x + 5}$$

Substitute 7 into the limit for x .

$$\lim_{x \rightarrow 7} \frac{-(x - 9)}{x + 5} = \frac{-(7 - 9)}{7 + 5} = \frac{2}{12} = \frac{1}{6}$$

Answer: $\lim_{x \rightarrow 7} \frac{-x^2 + 16x - 63}{x^2 - 2x - 35} = \frac{1}{6}$

8. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + 6x + 8}$$

First, attempt to evaluate the limit using direct substitution. Substitute -2 into the limit for x .

$$\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + 6x + 8} = \frac{-2 + 2}{(-2)^2 + 6(-2) + 8} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -2} \frac{x+2}{x^2+6x+8} = \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(x+4)} = \lim_{x \rightarrow -2} \frac{1}{x+4}$$

Substitute -2 into the limit for x .

$$\lim_{x \rightarrow -2} \frac{1}{x+4} = \frac{1}{-2+4} = \frac{1}{2}$$

Answer: $\lim_{x \rightarrow -2} \frac{x+2}{x^2+6x+8} = \frac{1}{2}$

9. Evaluate this limit.

$$\lim_{x \rightarrow 0} \frac{2x^2 + 10x}{3x^2 + 15x}$$

First, attempt to evaluate the limit using direct substitution. Substitute 0 into the limit for x .

$$\lim_{x \rightarrow 0} \frac{2x^2 + 10x}{3x^2 + 15x} = \frac{2(0)^2 + 10(0)}{3(0)^2 + 15(0)} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 0} \frac{2x^2 + 10x}{3x^2 + 15x} = \lim_{x \rightarrow 0} \frac{2x(x+5)}{3x(x+5)} = \frac{2}{3}$$

Answer: $\lim_{x \rightarrow 0} \frac{2x^2+10x}{3x^2+15x} = \frac{2}{3}$

10. Evaluate this limit.

$$\lim_{x \rightarrow -7} \frac{x + 7}{x^2 + 4x - 21}$$

First, attempt to evaluate the limit using direct substitution. Substitute -7 into the limit for x .

$$\lim_{x \rightarrow -7} \frac{x + 7}{x^2 + 4x - 21} = \frac{-7 + 7}{(-7)^2 + 4(-7) - 21} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -7} \frac{x + 7}{x^2 + 4x - 21} = \lim_{x \rightarrow -7} \frac{x + 7}{(x + 7)(x - 3)} = \lim_{x \rightarrow -7} \frac{1}{x - 3}$$

Substitute -7 into the limit for x .

$$\lim_{x \rightarrow -7} \frac{1}{x - 3} = \frac{1}{-7 - 3} = -\frac{1}{10}$$

Answer: $\lim_{x \rightarrow -7} \frac{x+7}{x^2+4x-21} = -\frac{1}{10}$

11. Evaluate this limit.

$$\lim_{x \rightarrow -2} \frac{x^2 - 3x - 10}{x^2 + x - 2}$$

First, attempt to evaluate the limit using direct substitution. Substitute -2 into the limit for x .

$$\lim_{x \rightarrow -2} \frac{x^2 - 3x - 10}{x^2 + x - 2} = \frac{(-2)^2 - 3(-2) - 10}{(-2)^2 + (-2) - 2} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow -2} \frac{x^2 - 3x - 10}{x^2 + x - 2} = \lim_{x \rightarrow -2} \frac{(x - 5)(x + 2)}{(x + 2)(x - 1)} = \lim_{x \rightarrow -2} \frac{x - 5}{x - 1}$$

Substitute -2 into the limit for x .

$$\lim_{x \rightarrow -2} \frac{x - 5}{x - 1} = \frac{-2 - 5}{-2 - 1} = \frac{-7}{-3} = \frac{7}{3}$$

Answer: $\lim_{x \rightarrow -2} \frac{x^2 - 3x - 10}{x^2 + x - 2} = \frac{7}{3}$

12. Evaluate this limit.

$$\lim_{x \rightarrow 8} \frac{x - 8}{x^2 - 2x - 48}$$

First, attempt to evaluate the limit using direct substitution. Substitute 8 into the limit for x .

$$\lim_{x \rightarrow 8} \frac{x - 8}{x^2 - 2x - 48} = \frac{8 - 8}{8^2 - 2(8) - 48} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 8} \frac{x - 8}{x^2 - 2x - 48} = \lim_{x \rightarrow 8} \frac{x - 8}{(x + 6)(x - 8)} = \lim_{x \rightarrow 8} \frac{1}{x + 6}$$

Substitute 8 into the limit for x .

$$\lim_{x \rightarrow 8} \frac{1}{x + 6} = \frac{1}{8 + 6} = \frac{1}{14}$$

Answer: $\lim_{x \rightarrow 8} \frac{x - 8}{x^2 - 2x - 48} = \frac{1}{14}$

13. Evaluate this limit.

$$\lim_{x \rightarrow 6} \frac{6 - x}{3x - 18}$$

First, attempt to evaluate the limit using direct substitution. Substitute 6 into the limit for x .

$$\lim_{x \rightarrow 6} \frac{6 - x}{3x - 18} = \frac{6 - 6}{3(6) - 18} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 6} \frac{6 - x}{3x - 18} = \lim_{x \rightarrow 6} \frac{-(x - 6)}{3(x - 6)} = \lim_{x \rightarrow 6} \frac{-1}{3} = -\frac{1}{3}$$

Answer: $\lim_{x \rightarrow 6} \frac{6 - x}{3x - 18} = -\frac{1}{3}$

14. Evaluate this limit.

$$\lim_{x \rightarrow 4} \frac{28 - 7x}{x - 4}$$

First, attempt to evaluate the limit using direct substitution. Substitute 4 into the limit for x .

$$\lim_{x \rightarrow 4} \frac{28 - 7x}{x - 4} = \frac{28 - 7(4)}{4 - 4} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 4} \frac{28 - 7x}{x - 4} = \lim_{x \rightarrow 4} \frac{-7(x - 4)}{x - 4} = \lim_{x \rightarrow 4} \frac{-7}{1} = -7$$

Answer: $\lim_{x \rightarrow 4} \frac{28 - 7x}{x - 4} = -7$

15. Evaluate this limit.

$$\lim_{x \rightarrow 7} \frac{4x - 28}{5x^3 - 35x^2}$$

First, attempt to evaluate the limit using direct substitution. Substitute 7 into the limit for x .

$$\lim_{x \rightarrow 7} \frac{4x - 28}{5x^3 - 35x^2} = \frac{4(7) - 28}{5(7)^3 - 35(7)^2} = \frac{28 - 28}{1715 - 1715} = \frac{0}{0}$$

The value of the limit is indeterminate using substitution. Now, factor and simplify the limit.

$$\lim_{x \rightarrow 7} \frac{4x - 28}{5x^3 - 35x^2} = \lim_{x \rightarrow 7} \frac{4(x - 7)}{5x^2(x - 7)} = \lim_{x \rightarrow 7} \frac{4}{5x^2}$$

Substitute 7 into the limit for x .

$$\lim_{x \rightarrow 7} \frac{4}{5x^2} = \frac{4}{5(7)^2} = \frac{4}{245}$$

Answer: $\lim_{x \rightarrow 7} \frac{4x - 28}{5x^3 - 35x^2} = \frac{4}{245}$