A metal box (WITHOUT A TOP) is to be constructed from a square sheet of metal that is 60 cm on a side by cutting square pieces of the same size from the corners of the sheet and then folding up the sides. Find the dimensions of the box with the largest volume that can be constructed in this manner

sheet =1.

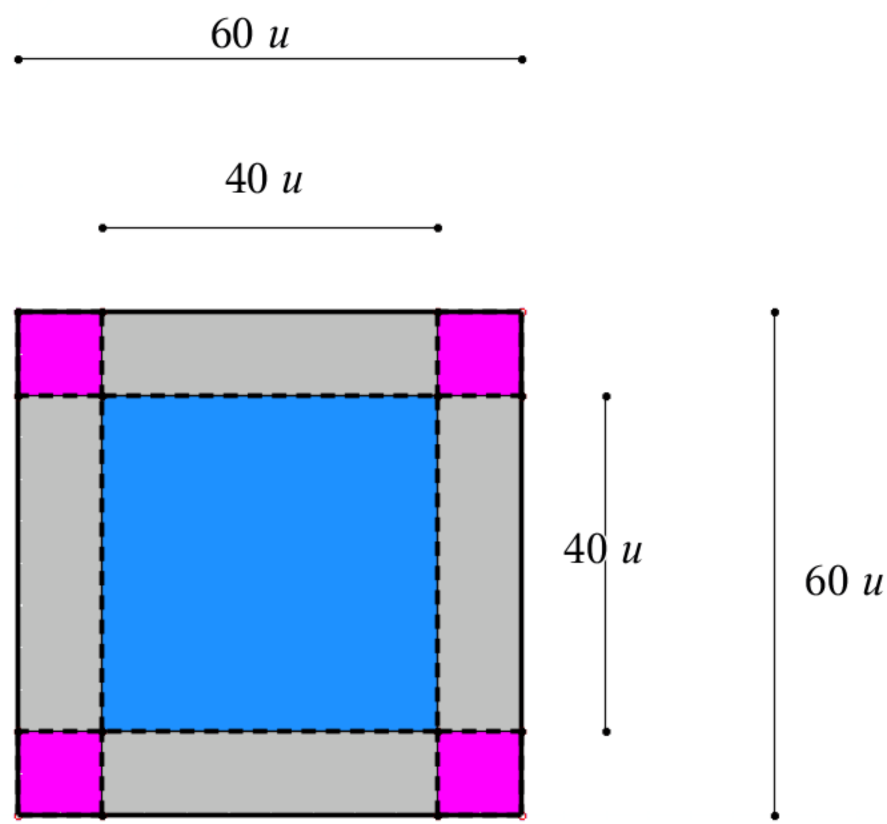
cuts =1.

folds =1.

lateral_area =1.

base_area =1.

surface_notop =0.



A metal box (WITHOUT A TOP) is to be constructed from a square sheet of metal that is 60 cm on a side by cutting square pieces of the same size from the corners of the sheet and then folding up the sides. Find the dimensions of the box with the largest volume that can be constructed in this manner

lateral_area = 1.

base_area = 1.

surface_notop = 0.

angle_1 = 19.

length	width	height
40	40	10

lateral area	surface area
1600	3200

Base Perimeter

160

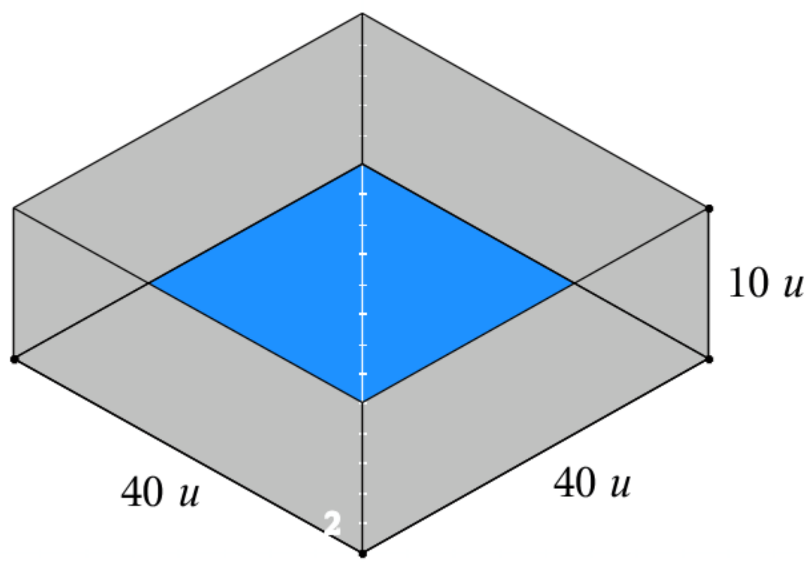
base area

1600

Volume

16000

10 *by* 40 *by* 40



$$V(x)=(side-2x)(side-2x)x=(side-2x)^2\cdot x$$

$$V(x)=\left(side^2-2\cdot side\cdot x-2\cdot side\cdot x+4x^2\right)\cdot x$$

$$V(x)=\left(4x^2-4\cdot side\cdot x+side^2\right)\cdot x$$

$$V(x)=4x^3-4\cdot side\cdot x^2+side^2\cdot x$$

$$V'(x)=\frac{dV}{dx}=3\cdot 4\cdot x^2-2\cdot 4side\cdot x+side^2$$

$$=12x^2-8side\cdot x+side^2$$

Many tried to take the derivative *of* $V(x)$ *without expanding*

Here is the correct method

$$V(x)=(side-2x)^2\cdot x \qquad V(x)=f(x)\cdot g(x)$$

$$f=(side-2x)^2 \qquad g=x$$

$$\frac{df}{dx}=2(side-2x)(-2) \qquad \frac{dg}{dx}=1$$

$$=-4(side-2x)$$

$$V(x)=f(x)\cdot g(x)\rightarrow\frac{dV}{dx}=\frac{df}{dx}\cdot g+\frac{dg}{dx}\cdot f$$

$$V'(x)=-4(side-2x)\cdot x+1\cdot(side-2x)^2$$

$$=-4side\cdot x+8x^2+\left(side^2-4side\cdot x+4x^2\right)$$

$$=12x^2-8\cdot sidex+side^2$$

$$=(6x-side)(2x-side)$$

$$V(x)=(60-2x)(60-2x)x=(60-2x)^2\cdot x$$

$$V(x)=\left((60)^2-2\cdot(60)\cdot x-2\cdot(60)\cdot x+4x^2\right)\cdot x$$

$$V(x)=\left(4x^2-4(60)\cdot x+(60)^2\right)\cdot x$$

$$V(x)=4x^3-240\cdot x^2+3600\cdot x$$

$$V'(x)=\frac{dV}{dx}=12\cdot x^2-2\cdot 240\cdot x+3600$$

$$V'(x)=\frac{dV}{dx}=12x^2-480\cdot x+3600$$

Many try *to take the* derivative *of* $V(x)$ *without expanding*

Here is the correct method

$$V(x)=(60-2x)^2\cdot x \qquad V(x)=f(x)\cdot g(x)$$

$$f=(60-2x)^2 \qquad g=x$$

$$\frac{df}{dx}=2(60-2x)(-2) \qquad \frac{dg}{dx}=1$$

$$=-4(60-2x)$$

$$V(x)=f(x)\cdot g(x)\rightarrow\frac{dV}{dx}=\frac{df}{dx}\cdot g+\frac{dg}{dx}\cdot f$$

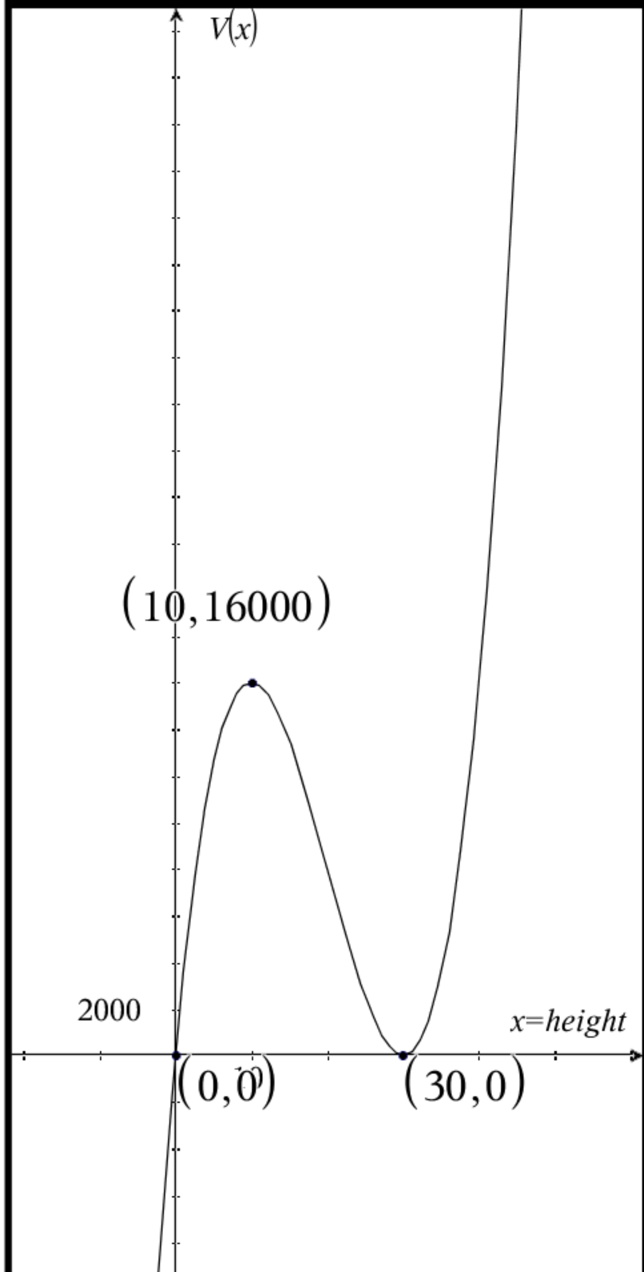
$$V'(x)=-4(60-2x)\cdot x+1\cdot(60-2x)^2$$

$$=-4(60)\cdot x+8x^2+\left((60)^2-4(60)\cdot x+4x^2\right)$$

$$=-240\cdot x+8x^2+\left(3600-240\cdot x+4x^2\right)$$

$$=12x^2-480\cdot x+3600$$

$$=(6x-60)(2x-60)$$

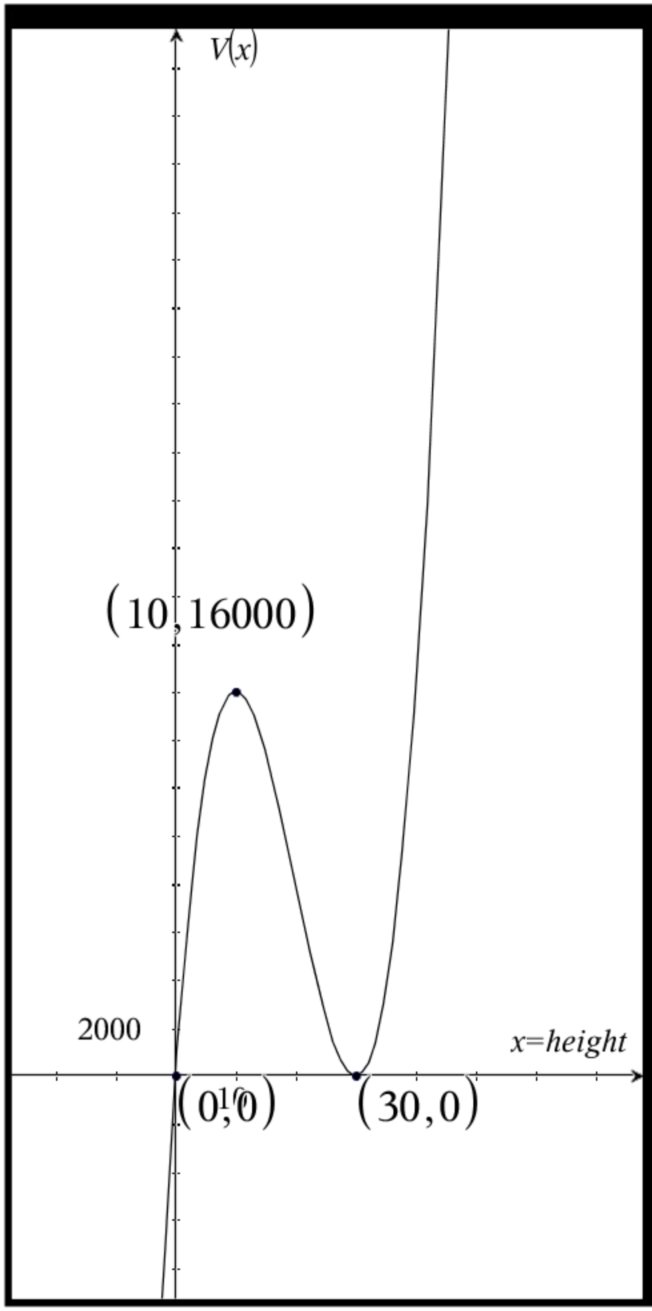


given: *Box is to be cut out of a* 60 *by* 60 *square*

Box has no top! *Box made by cutting out x by x squares*

Want: *Maximum volume* if *height = x*

general solution	*particular solution*
$height=x$	$height=x$
$width=side-2x$	$width=60-2x$
$length=side-2x$	$length=60-2x$
$V(x)=4x^3-4\cdot side\cdot x^2+side^2x$	$V(x)=4x^3-4\cdot(60)\cdot x^2+(60)^2x$
	$V(x)=4x^3-240\cdot x^2+3600x$
$V'(x)=12x^2-8\cdot side\cdot x+side^2$	$V'(x)=12x^2-8\cdot(60)\cdot x+(60)^2$
	$V'(x)=12x^2-480\cdot x+3600$



general solution

$$V(x)=4x^3-4\cdot side\cdot x^2+side^2x$$

$$V'(x)=12x^2-8\cdot side\cdot x+side^2$$

D=discriminant of quadratic

$$=b^2-4\cdot a\cdot c$$

$$=(-8\cdot side)^2-4\cdot(12)\cdot(side^2)$$

$$=64\cdot side^2-48\cdot side^2$$

$$=16\cdot side^2$$

$$x=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2a}$$

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$x=\frac{8\cdot side\pm\sqrt{16\cdot side^2}}{24}$$

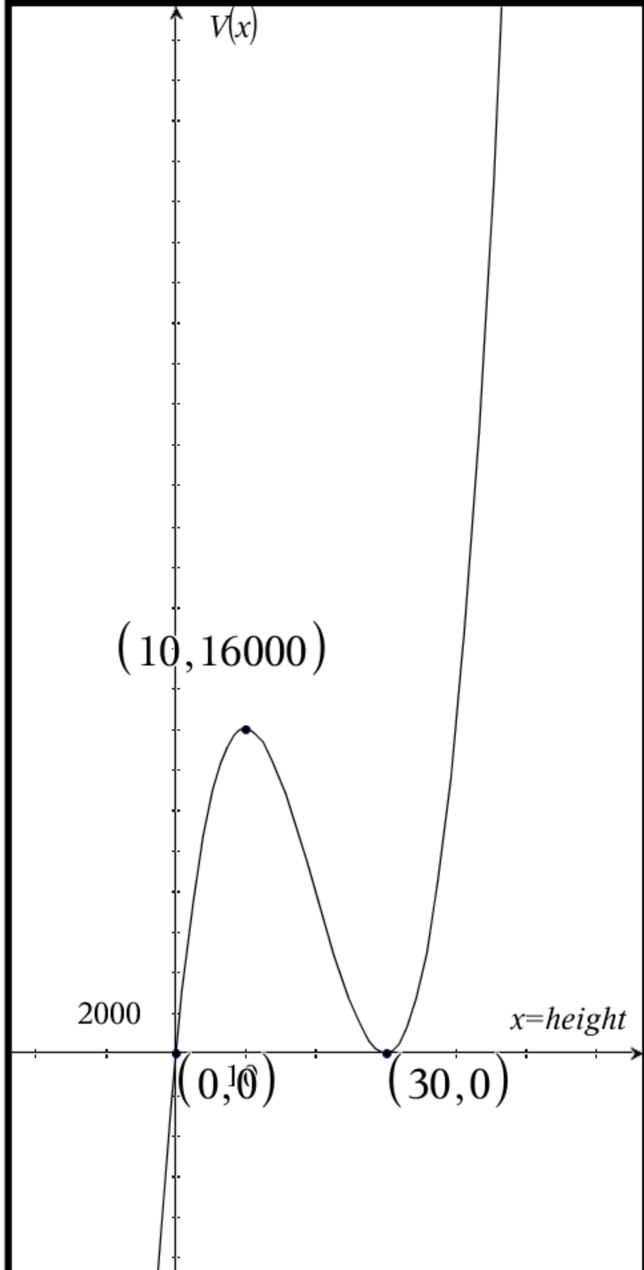
$$x=\frac{8\cdot side\pm 4\cdot side}{24}$$

$$x=\frac{8\cdot side-4\cdot side}{24}=\frac{4}{24}side=\frac{1}{6}side$$

max *volume occurs here*

$$x=\frac{8\cdot side+4\cdot side}{24}=\frac{12}{24}side=\frac{1}{2}side$$

min *volume occurs here*



particular solution

$$V(x)=4x^3-240\cdot x^2+3600x$$

$$V'(x)=12x^2-480\cdot x+3600$$

D=discriminant of quadratic

$$=b^2-4\cdot a\cdot c$$

$$=(-480)^2-4\cdot(12)\cdot(3600)$$

$$=57600$$

$$x=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2a}$$

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$x=\frac{480\pm\sqrt{57600}}{24}$$

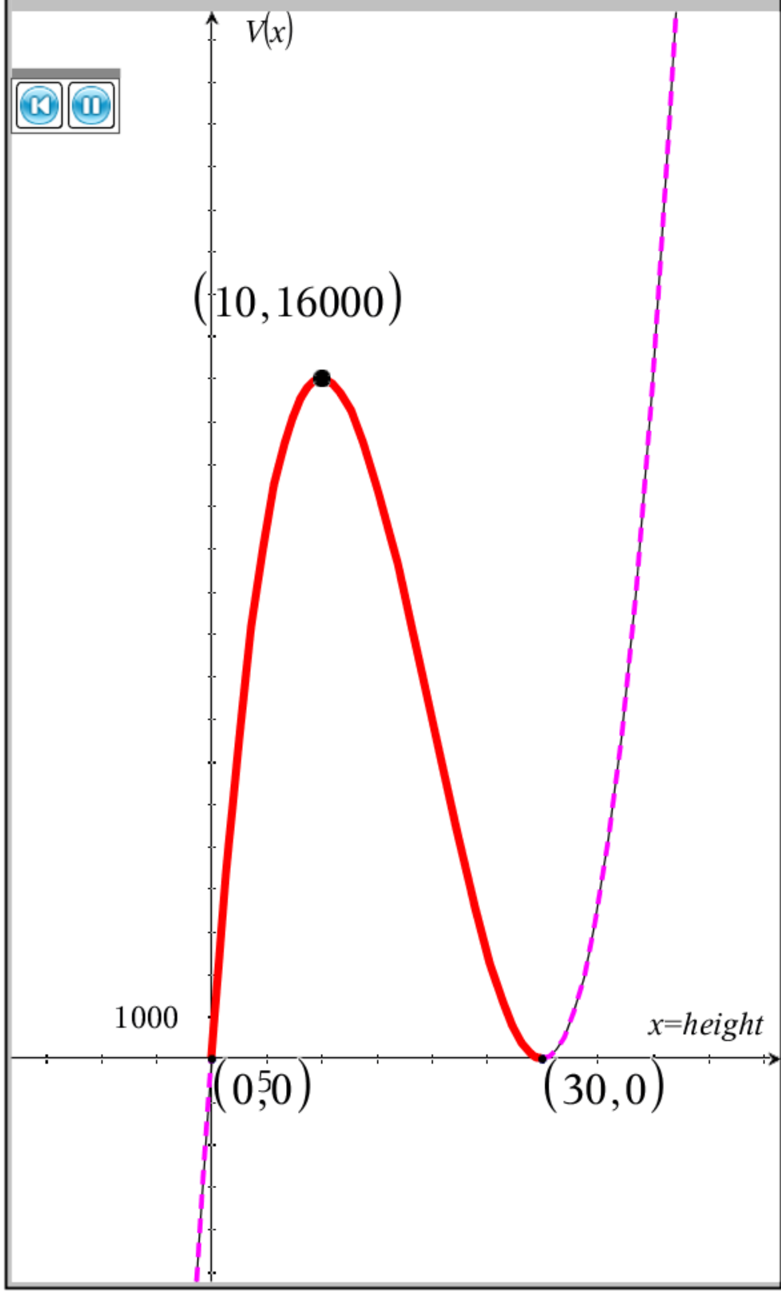
$$x=\frac{480\pm240}{24}$$

$$x=\frac{480-240}{24}=\frac{240}{24}=10$$

max *volume occurs here*

$$x=\frac{480+240}{24}=\frac{720}{24}=30$$

min *volume occurs here*



$V(x)=4x^3-240\cdot x^2+3600x$

$V'(x)=12x^2-480\cdot x+3600$

$V'(x)=0$ *at* $x=10$ or $x=30$

max *volume occurs here*

min *volume occurs here*

The feasible region of this graph

Domain $0<x<30$

why? if $x=20$ *no box*

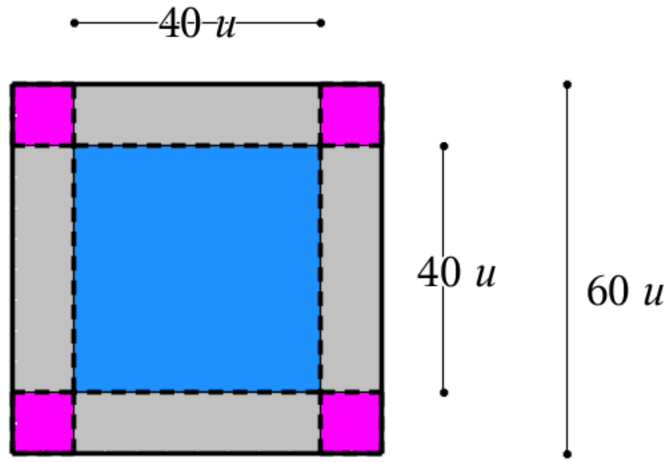
why? $30=\frac{1}{2}$*side*

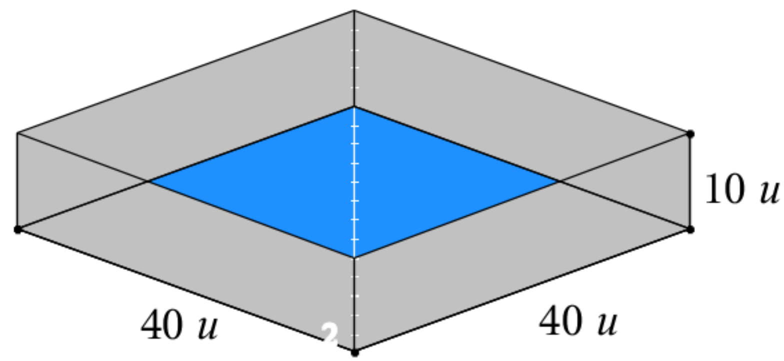
why? if $x=0$ *no box*

Range $0<V(x)<16000$

$V(0)$ or $V(30)$

$V(10)$





10 *by* 40 *by* 40 *waste* 100

$V(10) = 16000 \approx 16000$

$V'(10) = \frac{dV}{dx} = 0$

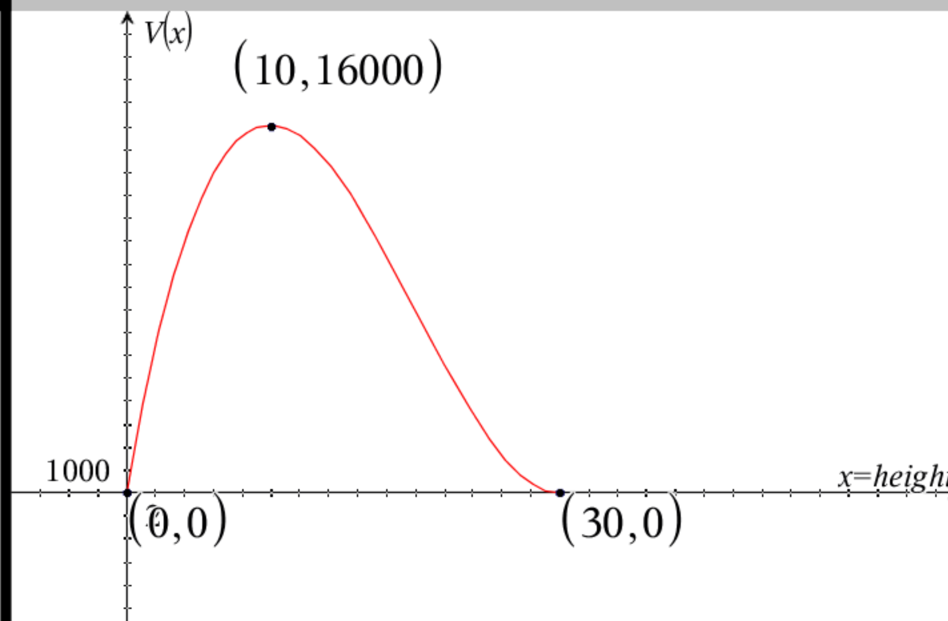
sheet =1.

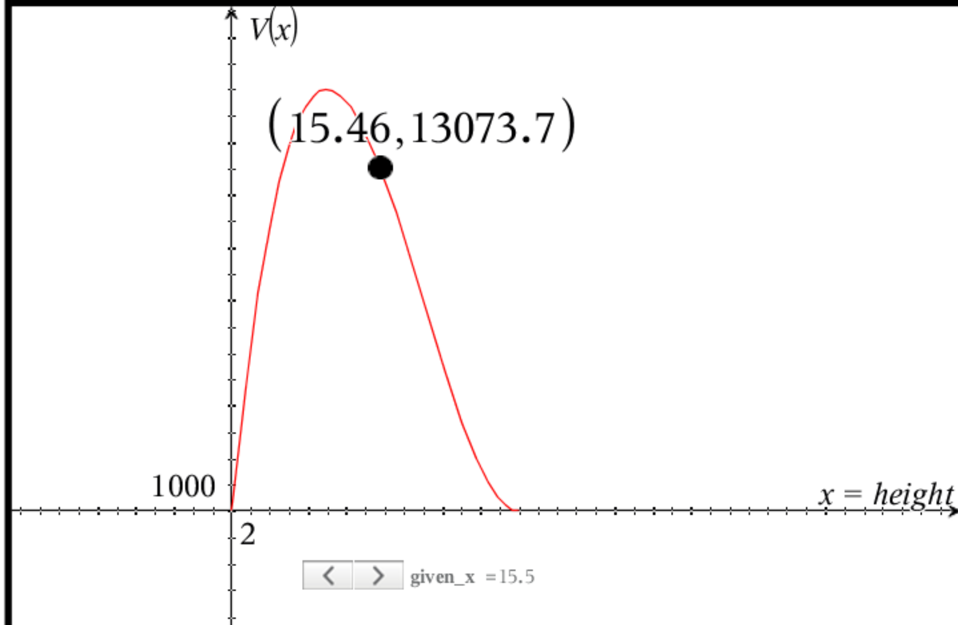
cuts =1.

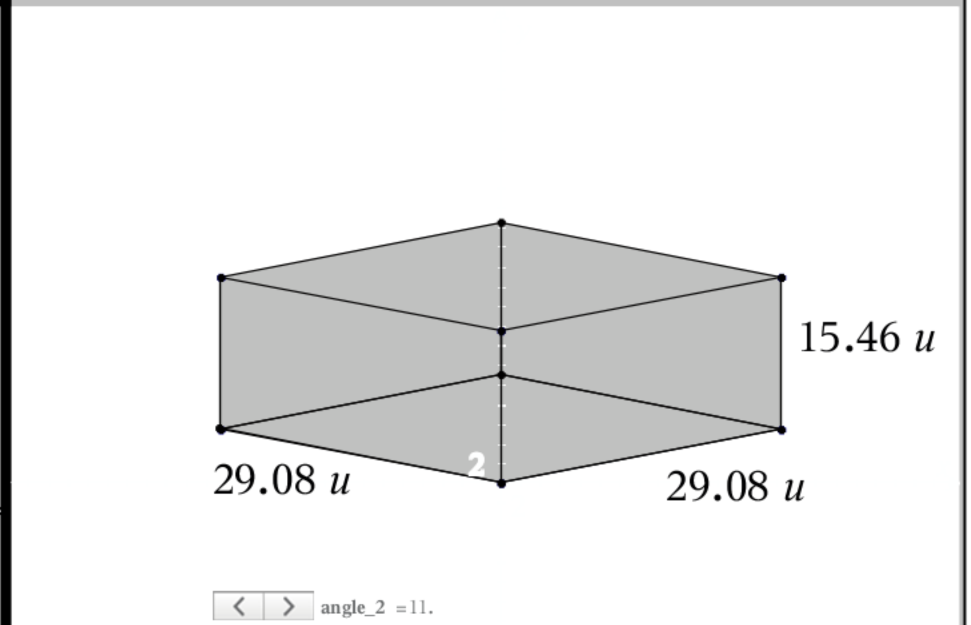
folds =1.

lateral_area =1. base_area =1. surface_notop =0.

1600 1600 3200







29.08 *by* 29.08 *by* 15.46

$V(15.46) = 13073.7$

$V'(15.46) = \frac{dV}{dx} = {}^{-}952.661$

waste

956.046

Lateral Area	*Base Area*	*Surface Area*
1798.31	845.646	2643.95