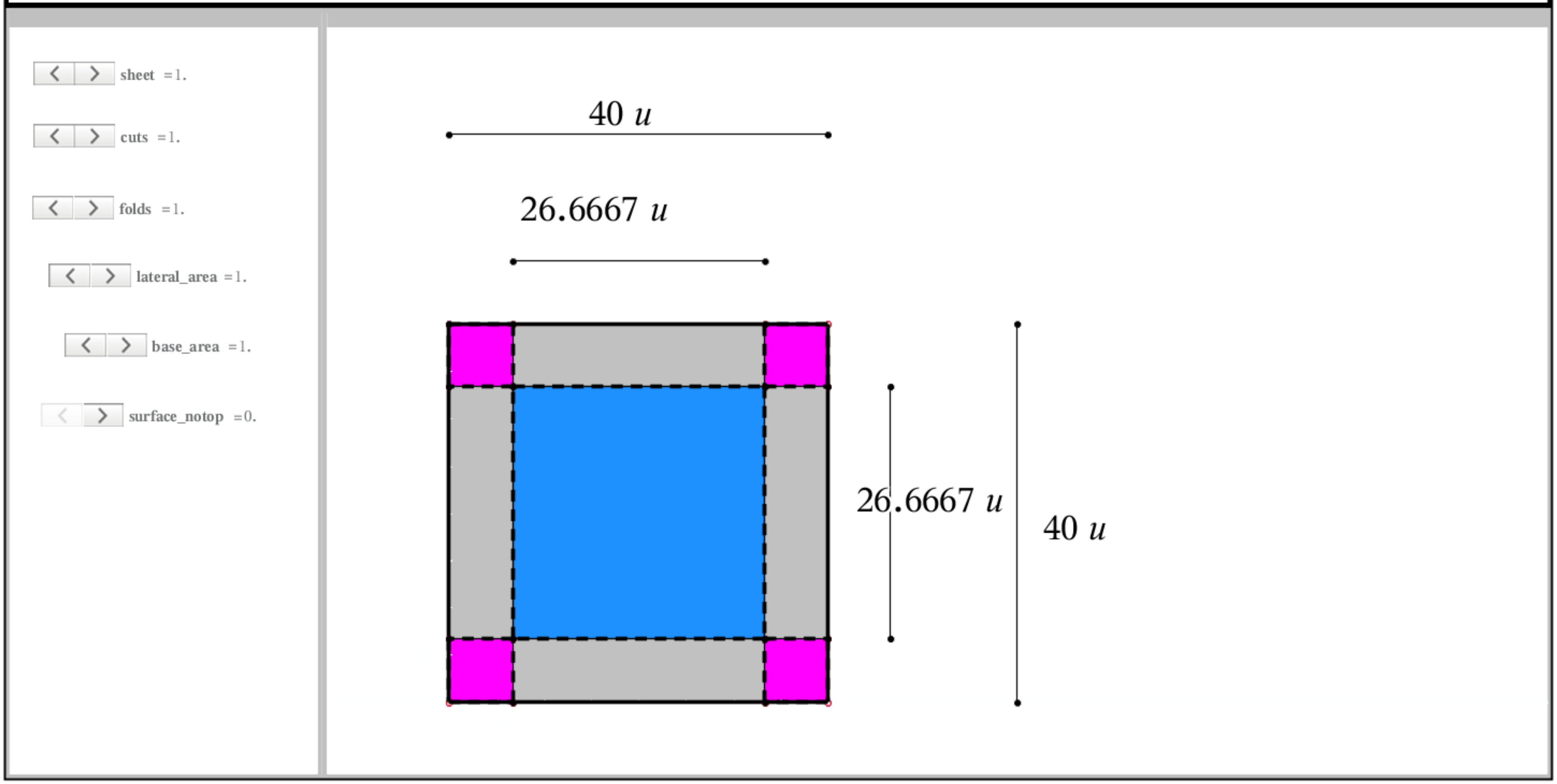
A metal box (WITHOUT A TOP) is to be constructed from a square sheet of metal that is 40 cm on a side by cutting square pieces of the same size from the corners of the sheet and then folding up the sides. Find the dimensions of the box with the largest volume that can be constructed in this manner



A metal box (WITHOUT A TOP) is to be constructed from a square sheet of metal that is **side** cm on a side by cutting square pieces of the same size from the corners of the sheet and then folding up the sides. Find the dimensions of the box with the largest volume that can be constructed in this manner

lateral_area =1.

base_area =1.

surface_notop =0.

angle_1 =19.

length	*width*	*height*
$\frac{80}{3}$	$\frac{80}{3}$	$\frac{20}{3}$

Base Perimeter

$\frac{320}{3}$

base area

$\frac{6400}{9}$

lateral area

$\frac{6400}{9}$

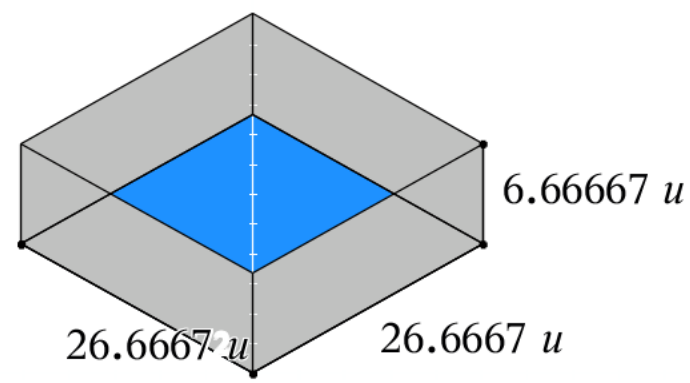
surface area

$\frac{12800}{9}$

Volume

$\frac{128000}{27}$

$\frac{20}{3}$ *by* $\frac{80}{3}$ *by* $\frac{80}{3}$



$$V(x)=(side-2x)(side-2x)x=(side-2x)^2\cdot x$$

$$V(x)=\left(side^2-2\cdot side\cdot x-2\cdot side\cdot x+4x^2\right)\cdot x$$

$$V(x)=\left(4x^2-4\cdot side\cdot x+side^2\right)\cdot x$$

$$V(x)=4x^3-4\cdot side\cdot x^2+side^2\cdot x$$

$$V'(x)=\frac{dV}{dx}=3\cdot 4\cdot x^2-2\cdot 4side\cdot x+side^2$$

$$=12x^2-8side\cdot x+side^2$$

Many tried to take the derivative *of* $V(x)$

without expanding

Here is the correct method

$V(x)=(side-2x)^2\cdot x$ $\qquad V(x)=f(x)\cdot g(x)$

$f=(side-2x)^2$ $\qquad g=x$

$\frac{df}{dx}=2(side-2x)(-2)$ $\qquad \frac{dg}{dx}=1$

$=-4(side-2x)$

$$V(x)=f(x)\cdot g(x)\rightarrow\frac{dV}{dx}=\frac{df}{dx}\cdot g+\frac{dg}{dx}\cdot f$$

$$V'(x)=-4(side-2x)\cdot x+1\cdot(side-2x)^2$$

$$=-4side\cdot x+8x^2+\left(side^2-4side\cdot x+4x^2\right)$$

$$=12x^2-8\cdot sidex+side^2$$

$$=(6x-side)(2x-side)$$

$V(x) = (40 - 2x)(40 - 2x)x = (40 - 2x)^2 \cdot x$

$V(x) = \left((40)^2 - 2 \cdot (40) \cdot x - 2 \cdot (40) \cdot x + 4x^2\right) \cdot x$

$V(x) = \left(4x^2 - 4(40) \cdot x + (40)^2\right) \cdot x$

$V(x) = 4x^3 - 160 \cdot x^2 + 1600 \cdot x$

$V'(x) = \frac{dV}{dx} = 12 \cdot x^2 - 2 \cdot 160 \cdot x + 1600$

$V'(x) = \frac{dV}{dx} = 12x^2 - 320 \cdot x + 1600$

Many try *to take the* derivative *of* $V(x)$ *without expanding*

Here is the correct method

$V(x) = (40 - 2x)^2 \cdot x$ $\quad V(x) = f(x) \cdot g(x)$

$f = (40 - 2x)^2$ $\quad g = x$

$\frac{df}{dx} = 2(40 - 2x)(-2)$ $\quad \frac{dg}{dx} = 1$

$= -4(40 - 2x)$

$V(x) = f(x) \cdot g(x) \rightarrow \frac{dV}{dx} = \frac{df}{dx} \cdot g + \frac{dg}{dx} \cdot f$

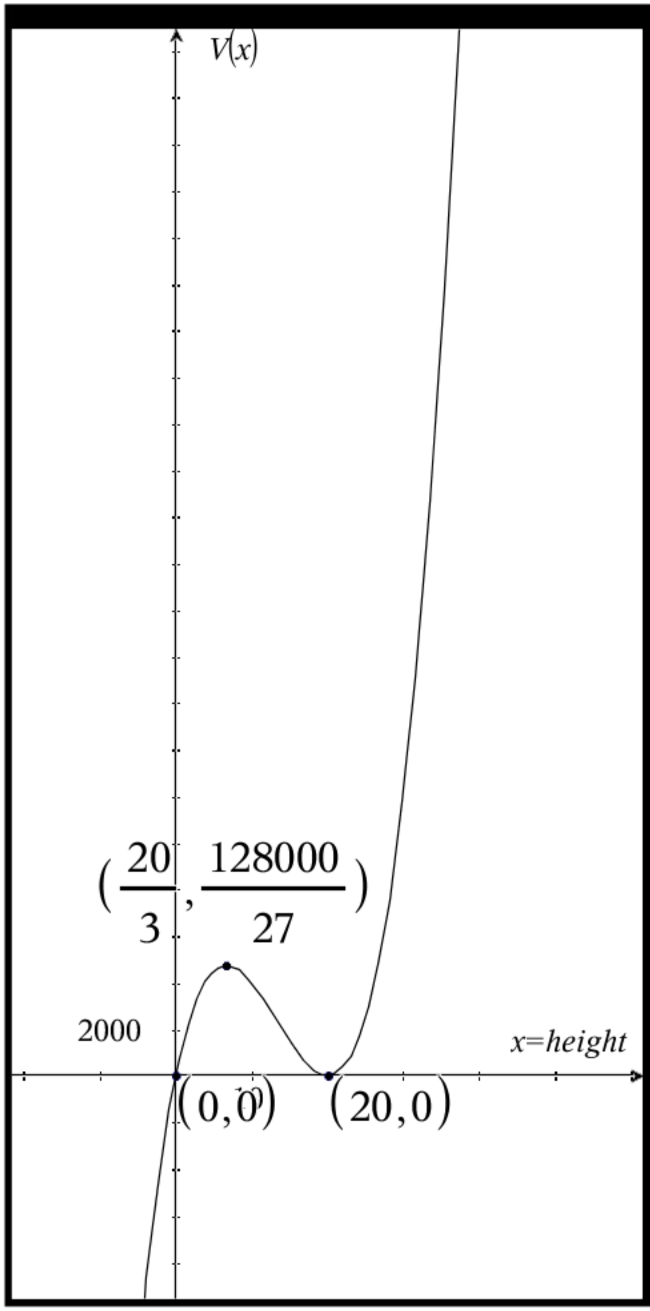
$V'(x) = -4(40 - 2x) \cdot x + 1 \cdot (40 - 2x)^2$

$= -4(40) \cdot x + 8x^2 + \left((40)^2 - 4(40) \cdot x + 4x^2\right)$

$= -160 \cdot x + 8x^2 + \left(1600 - 160 \cdot x + 4x^2\right)$

$= 12x^2 - 320 \cdot x + 1600$

$= (6x - 40)(2x - 40)$

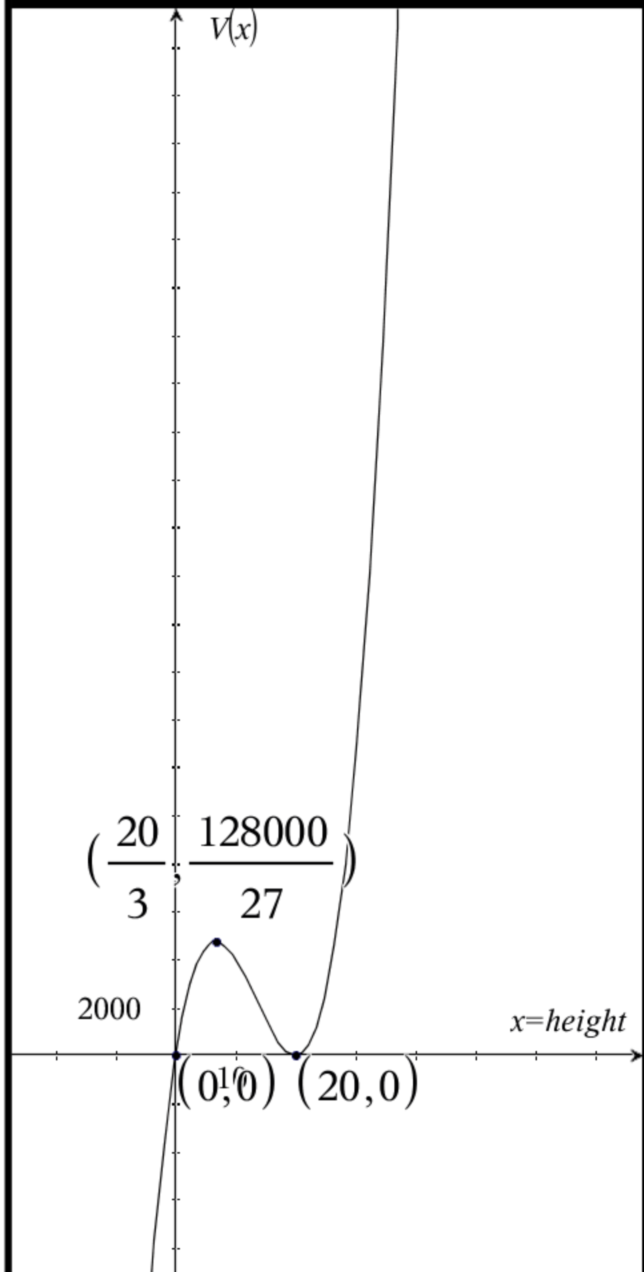


given: *Box is to be cut out of a* 40 *by* 40 *square*

Box has no top! *Box made by cutting out x by x squares*

Want: *Maximum volume* if *height = x*

general solution	*particular solution*
$height=x$	$height=x$
$width=side-2x$	$width=40-2x$
$length=side-2x$	$length=40-2x$
$V(x)=4x^3-4\cdot side\cdot x^2+side^2x$	$V(x)=4x^3-4\cdot(40)\cdot x^2+(40)^2x$
	$V(x)=4x^3-160\cdot x^2+1600x$
$V'(x)=12x^2-8\cdot side\cdot x+side^2$	$V'(x)=12x^2-8\cdot(40)\cdot x+(40)^2$
	$V'(x)=12x^2-320\cdot x+1600$



general solution

$$V(x)=4x^3-4\cdot side\cdot x^2+side^2 x$$

$$V'(x)=12x^2-8\cdot side\cdot x+side^2$$

D=discriminant of quadratic

$$=b^2-4\cdot a\cdot c$$

$$=(-8\cdot side)^2-4\cdot(12)\cdot(side^2)$$

$$=64\cdot side^2-48\cdot side^2$$

$$=16\cdot side^2$$

$$x=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2a}$$

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$x=\frac{8\cdot side\pm\sqrt{16\cdot side^2}}{24}$$

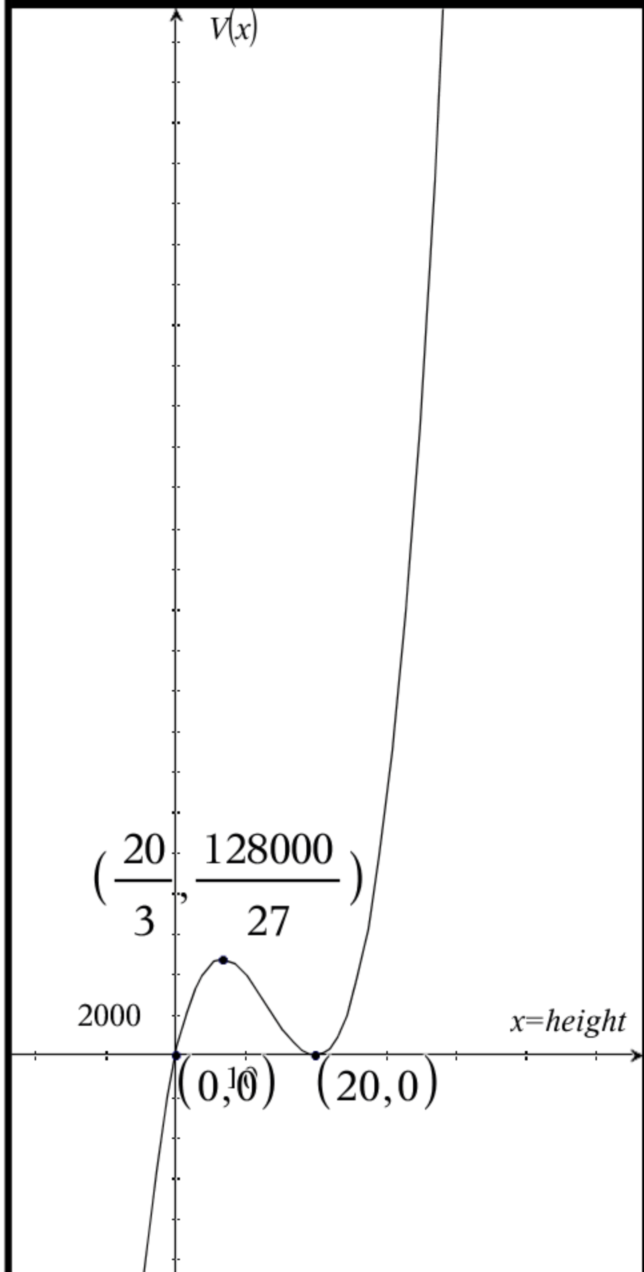
$$x=\frac{8\cdot side\pm 4\cdot side}{24}$$

$$x=\frac{8\cdot side-4\cdot side}{24}=\frac{4}{24}side=\frac{1}{6}side$$

max *volume occurs here*

$$x=\frac{8\cdot side+4\cdot side}{24}=\frac{12}{24}side=\frac{1}{2}side$$

min *volume occurs here*



particular solution

$$V(x)=4x^3-160\cdot x^2+1600x$$

$$V'(x)=12x^2-320\cdot x+1600$$

D=discriminant of quadratic

$$=b^2-4\cdot a\cdot c$$

$$=(^-320)^2-4\cdot(12)\cdot(1600)$$

$$=25600$$

$$x=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2a}$$

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

$$x=\frac{320\pm\sqrt{25600}}{24}$$

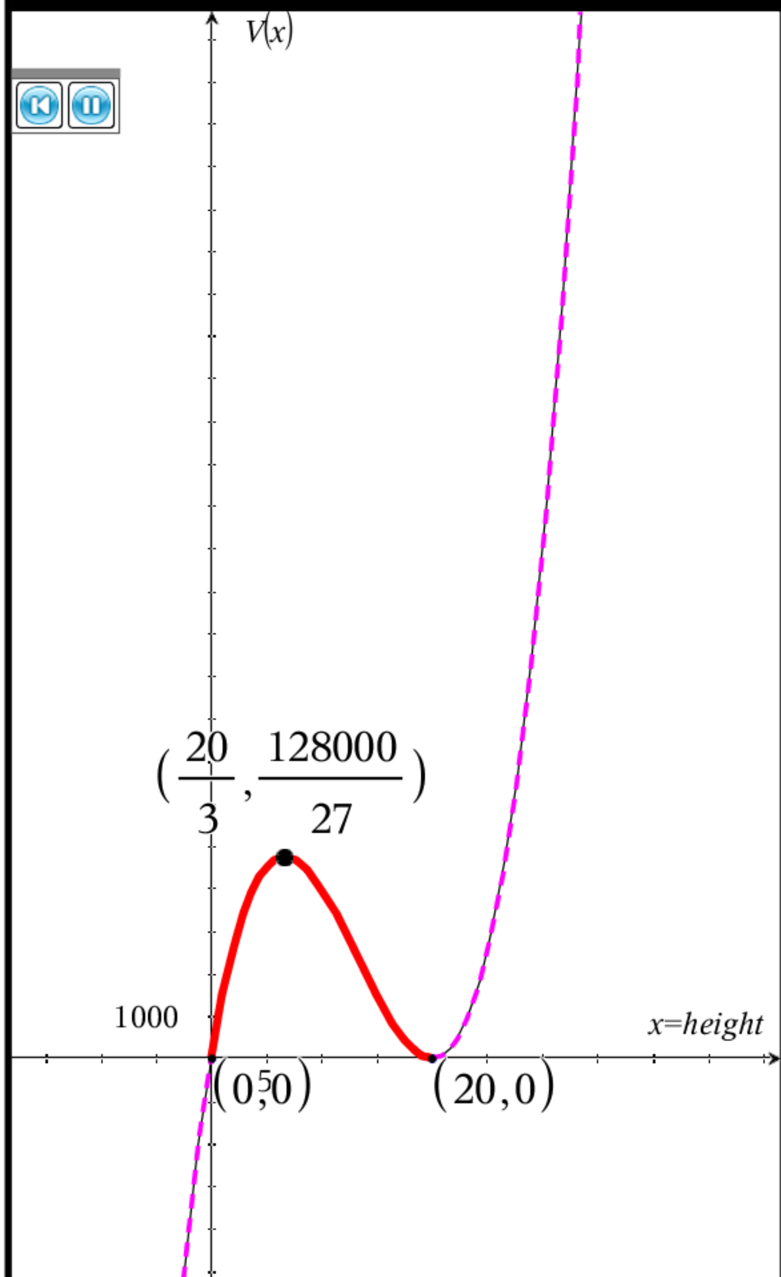
$$x=\frac{320\pm160}{24}$$

$$x=\frac{320-160}{24}=\frac{160}{24}=\frac{20}{3}$$

max *volume occurs here*

$$x=\frac{320+160}{24}=\frac{480}{24}=20$$

min *volume occurs here*



$V(x)=4x^3-160\cdot x^2+1600x$

$V'(x)=12x^2-320\cdot x+1600$

$V'(x)=0$ *at* $x=\frac{20}{3}$ or $x=20$

max *volume occurs here* (at $x=\frac{20}{3}$)

min *volume occurs here* (at $x=20$)

The feasible region of this graph

Domain $0<x<20$

why? if $x=0$ *no box*

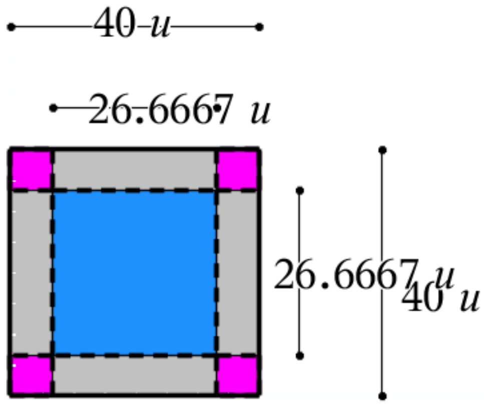
why? if $x=20$ *no box*

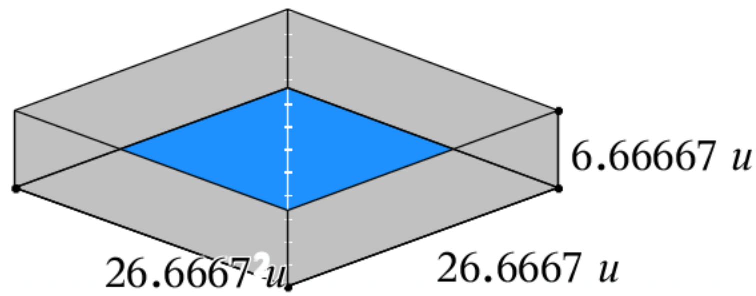
why? $20=\frac{1}{2}$*side*

Range $0<V(x)<4740.7407407$

$V(0)$ or $V(20)$

$V\left(\frac{20}{3}\right)$





$\frac{20}{3}$ *by* $\frac{80}{3}$ *by* $\frac{80}{3}$ *waste* $\frac{400}{9}$

$V\left(\frac{20}{3}\right)=\frac{128000}{27} \approx 4740.74$

sheet =1.

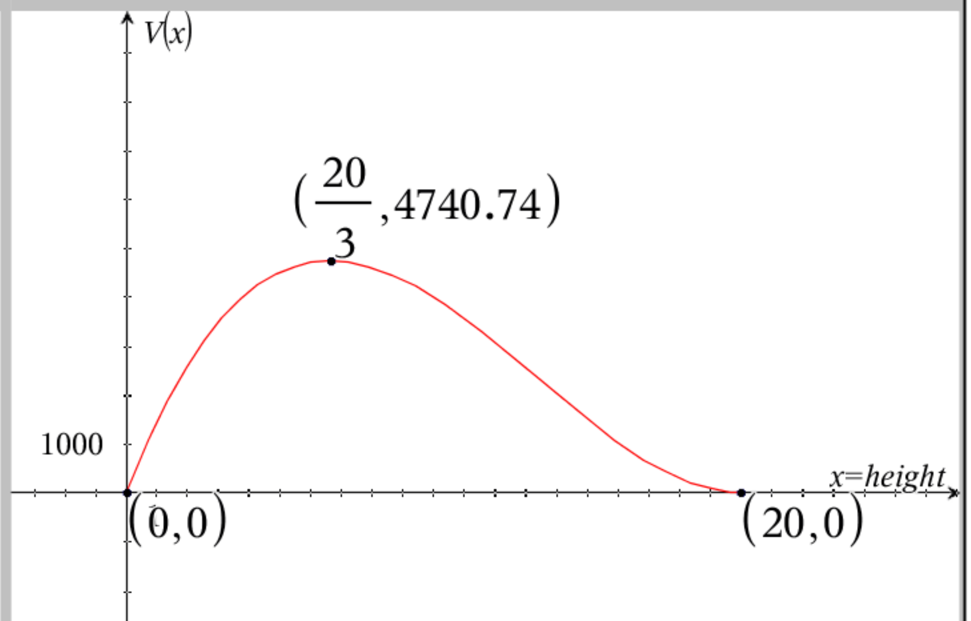
cuts =1.

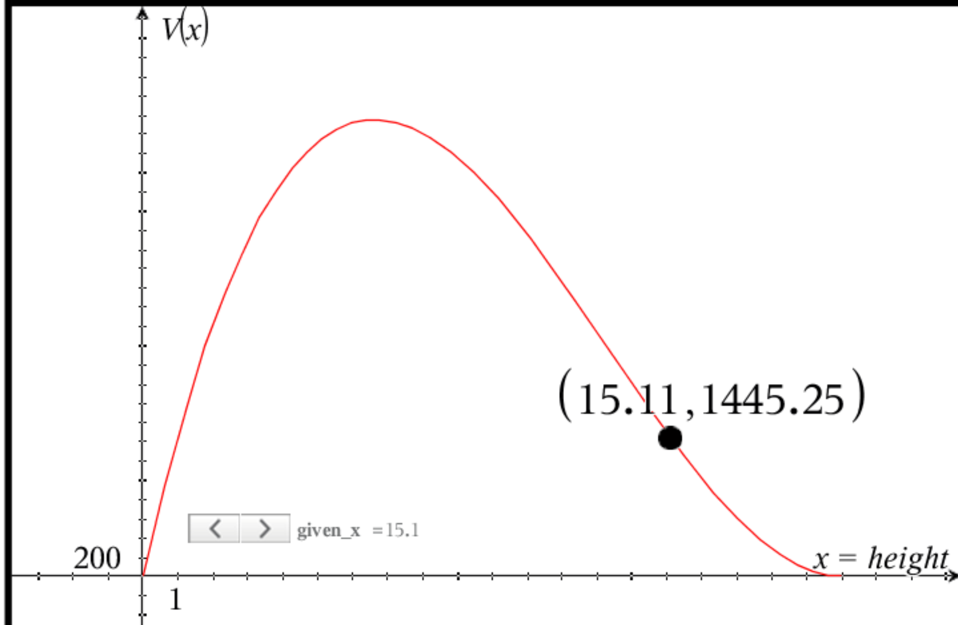
$V'\left(\frac{20}{3}\right)=\frac{dV}{dx}=0$

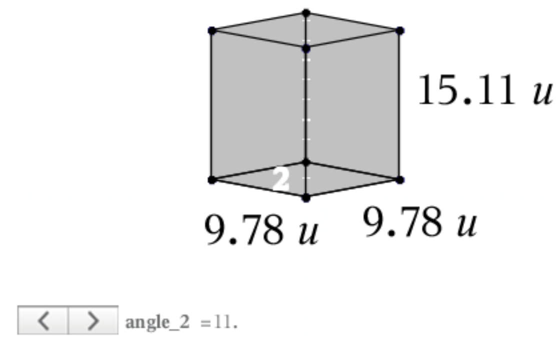
folds =1.

lateral_area =1. base_area =1. surface_notop =0.

$\frac{6400}{9}$ $\frac{6400}{9}$ 1422.22







9.78 *by* 9.78 *by* 15.11

$V(15.11)=$ 1445.25

$V'(15.11)=\frac{dV}{dx}=^{-}495.455$

waste

913.248

Lateral Area	*Base Area*	*Surface Area*
591.103	95.6484	686.752