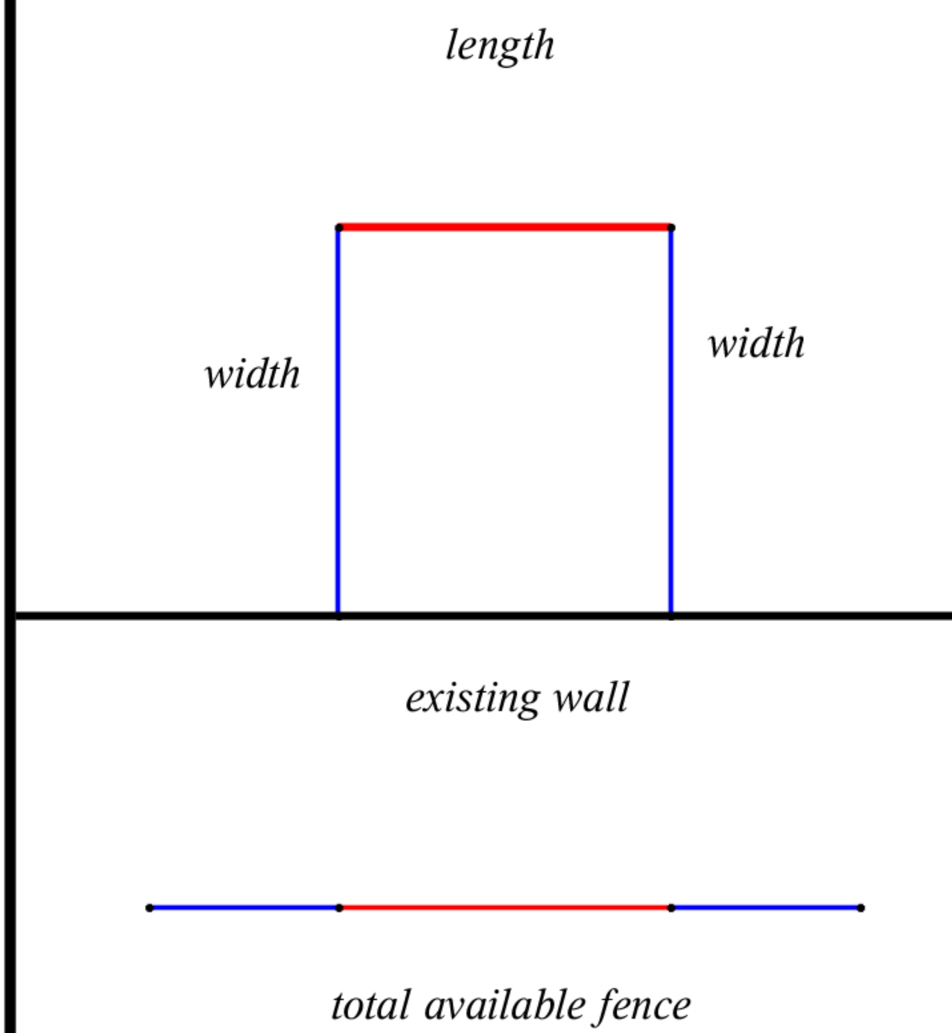
Classic rectangle area application

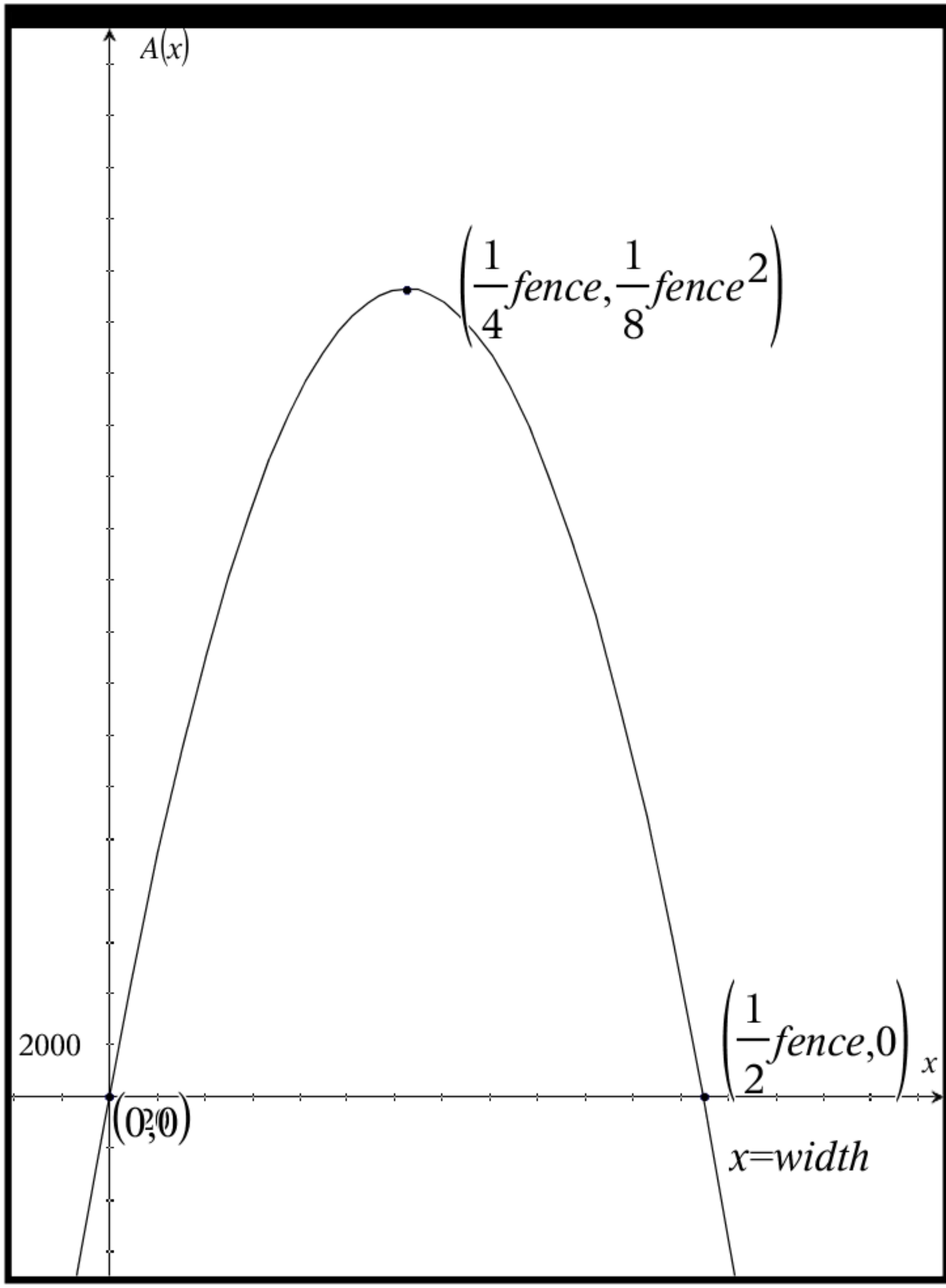
A farmer has a fixed amount of fence, say **fence** units long,

farmer wants to create a rectangle with three sides of the rectangle created by this available fence and a fourth wall of the fence being something that is already present, like an existing wall, a barn, a river, etc.

This problem can be done in mathematics classes at algebra 1, algebra 2, and calculus

How it is accomplished depends on the level of sophisication of the mathematics student





Constraints on dimensions

$0 < \text{width} < \frac{1}{2}\textbf{fence}$

$0 < \text{length} < \frac{1}{2}\textbf{fence}$

Let x = width let y = length

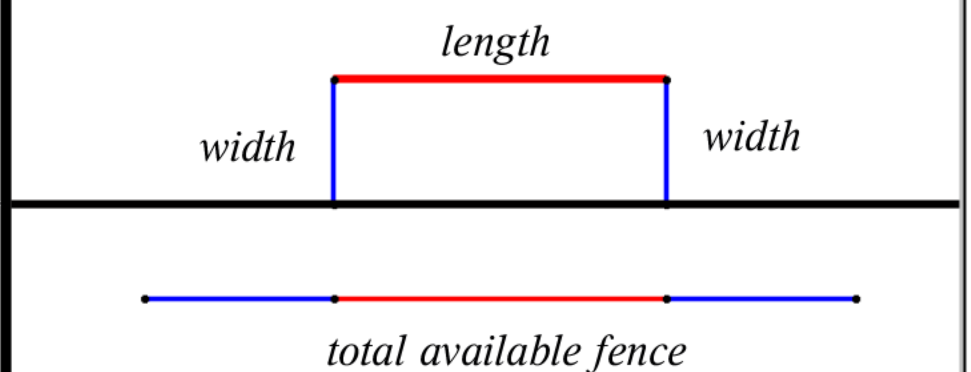
fence = 2x+y

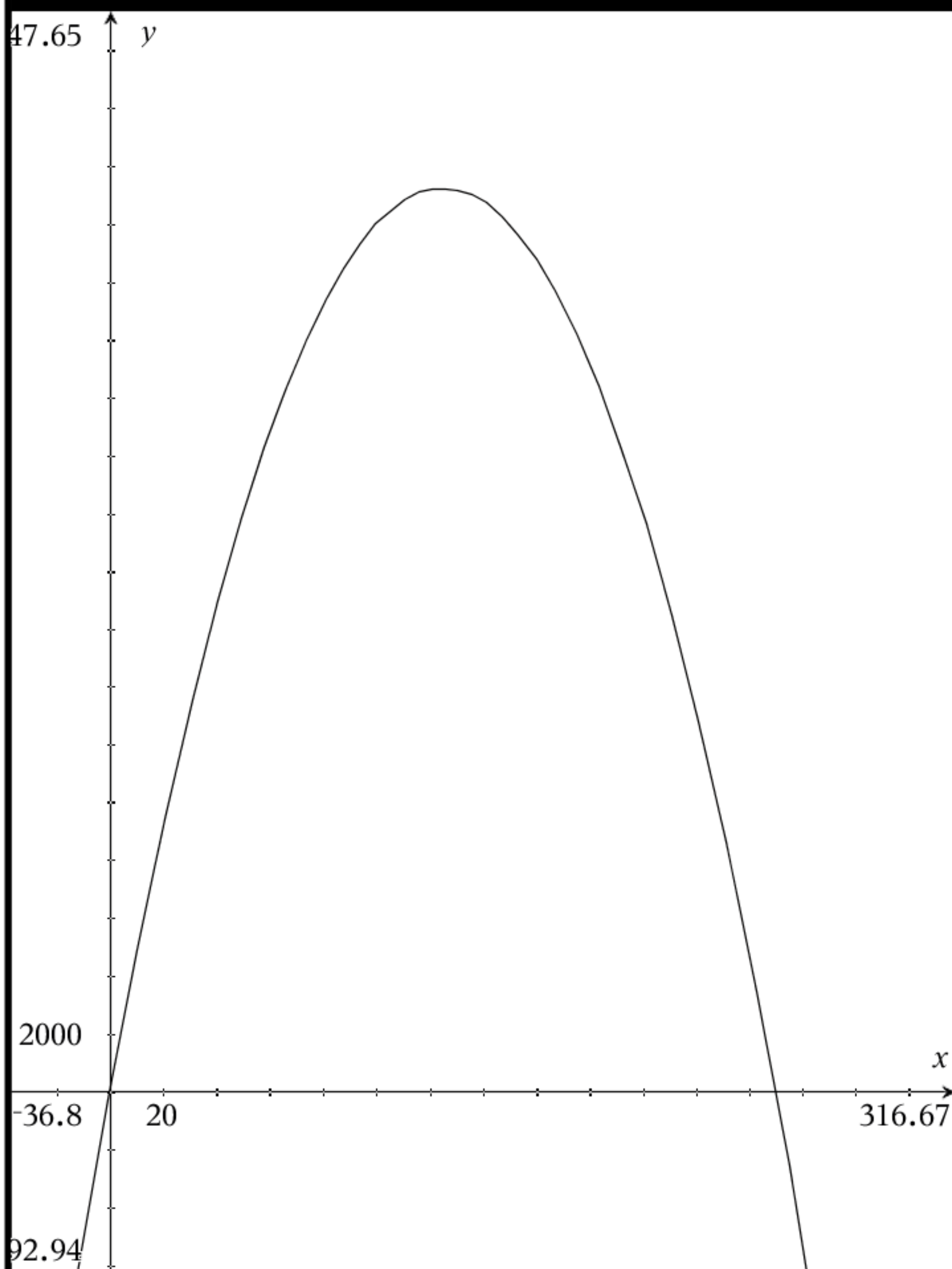
y = fence −2x

area model = (length)(width)

= (fence−2x)(x)

$= \textbf{fence}\cdot x - 2x^2$





Hints on window selection

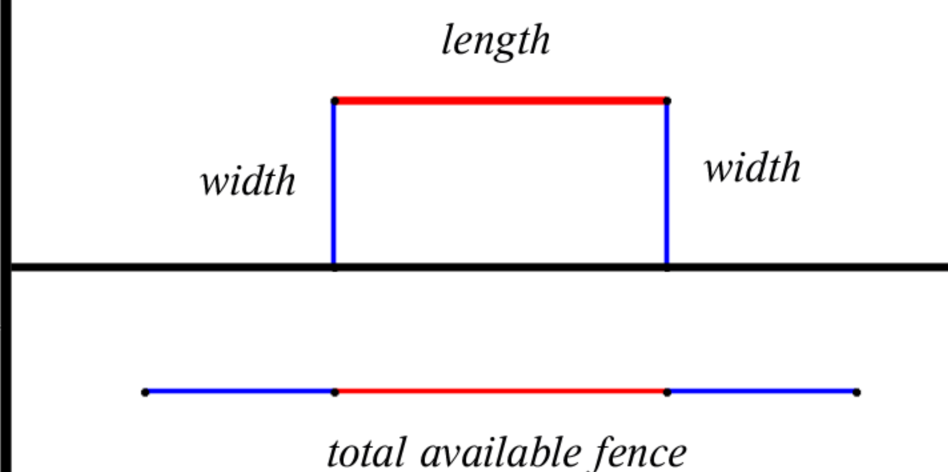
1) write area model

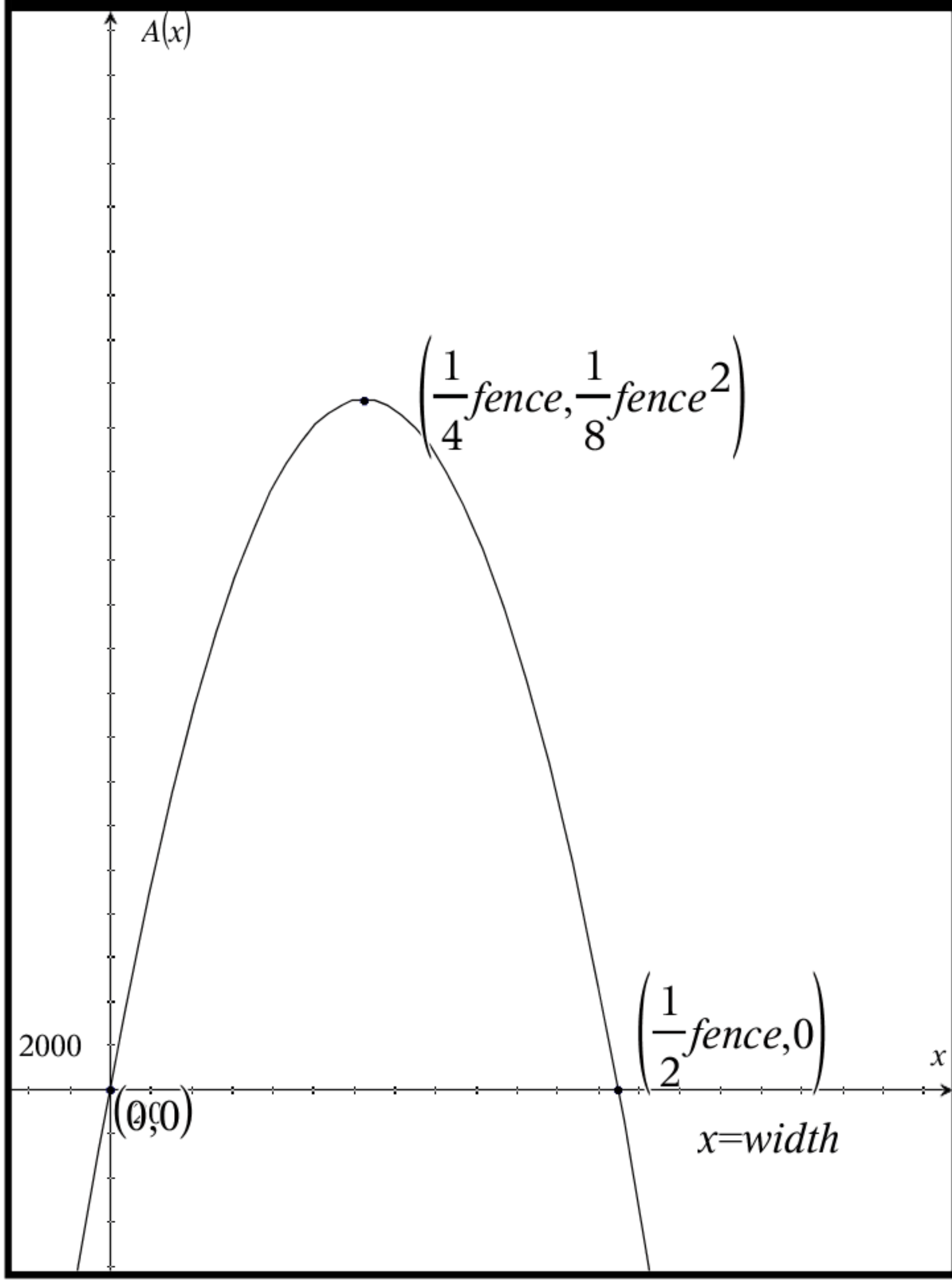
2) make x max the length of available fence

3) use zoom fit and/or zoom out to get an idea on the size of y dimensions

What are the FEASIBLE answers?

0 < width < $\frac{1}{2}$**fence**





x = width   fence = given constant

area model = (fence−2x)(x)

A(x) = **fence**$\cdot x-2x^{2}$

A'(x)=**fence**$-4x$

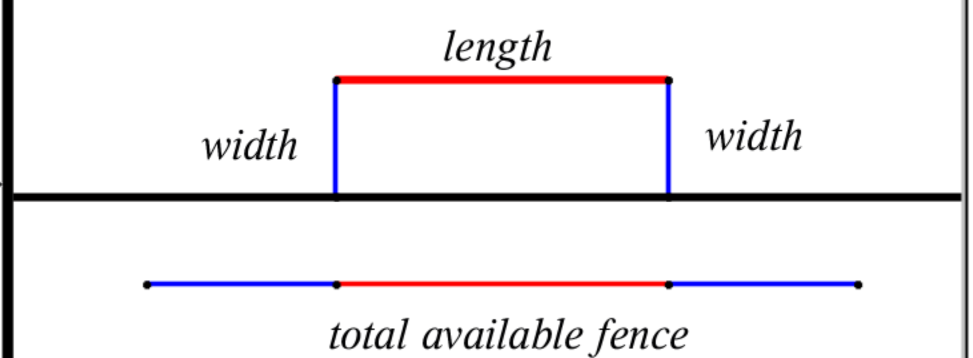
A'(x)=0 when x $=\frac{1}{4}$**fence**

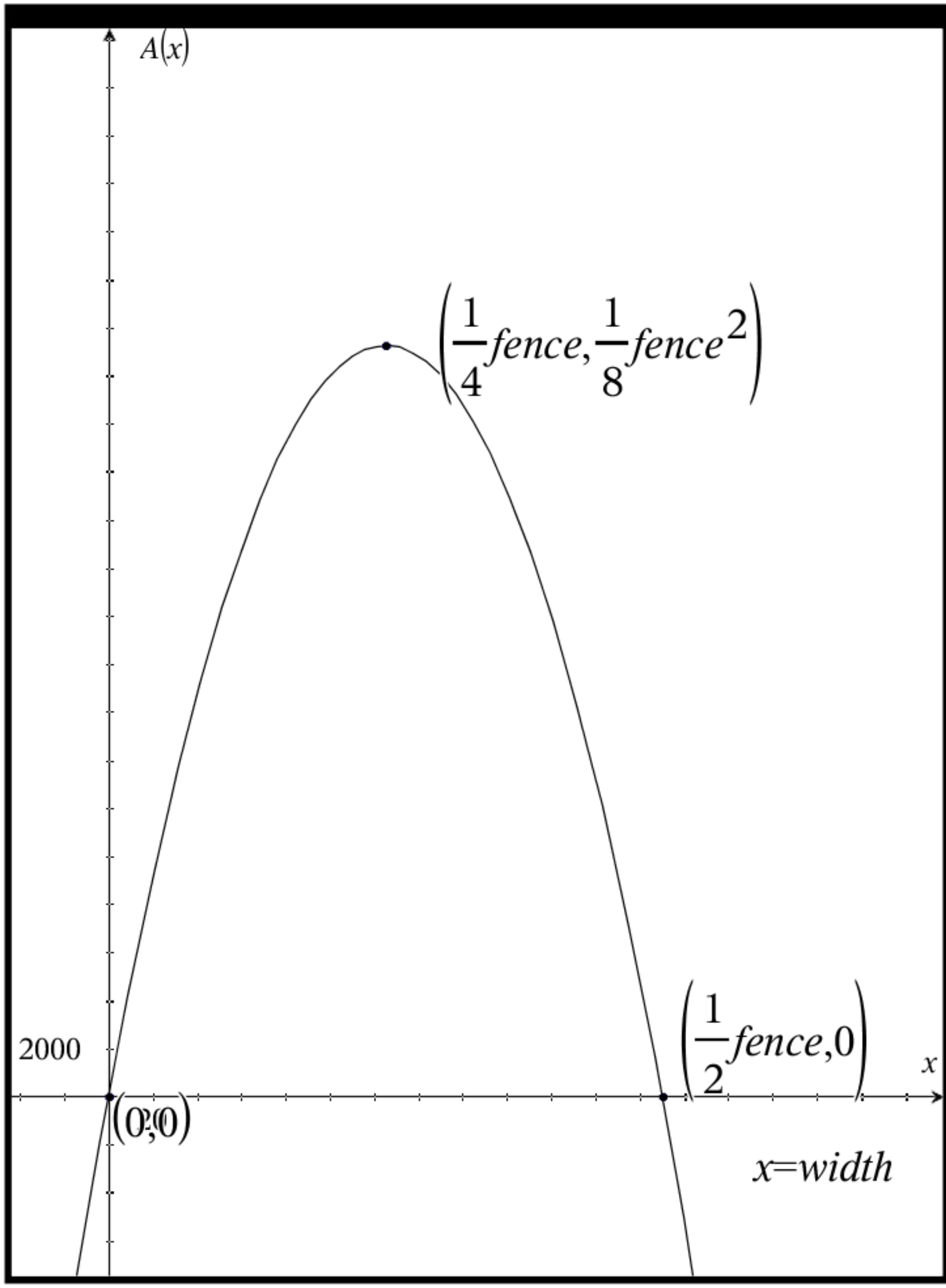
A''(x) = −4, so since A''(x) < 0

we know that there is only a maximum

this is known as the second derivative test

so the maximum exists for this model and it occurs when the width is $\frac{1}{4}$**fence**





$A(x) = \textbf{fence} \cdot x - 2x^2$

max area occurs when the width is $\frac{1}{4}\textbf{fence}$

and we know that length = fence −2width

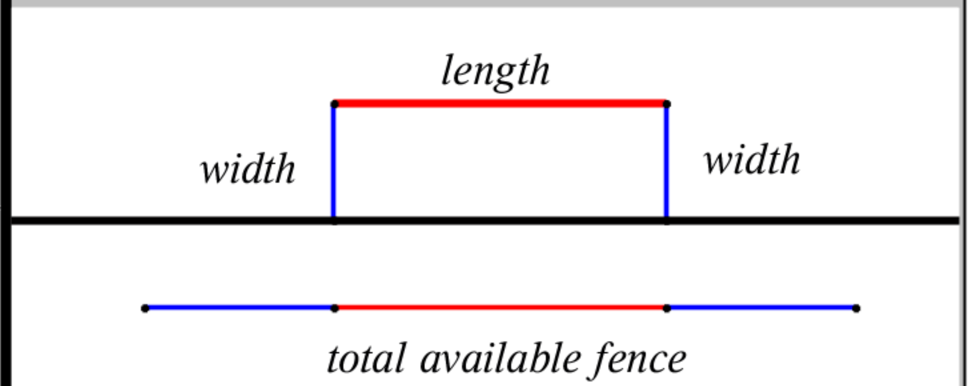
so max area occurs when

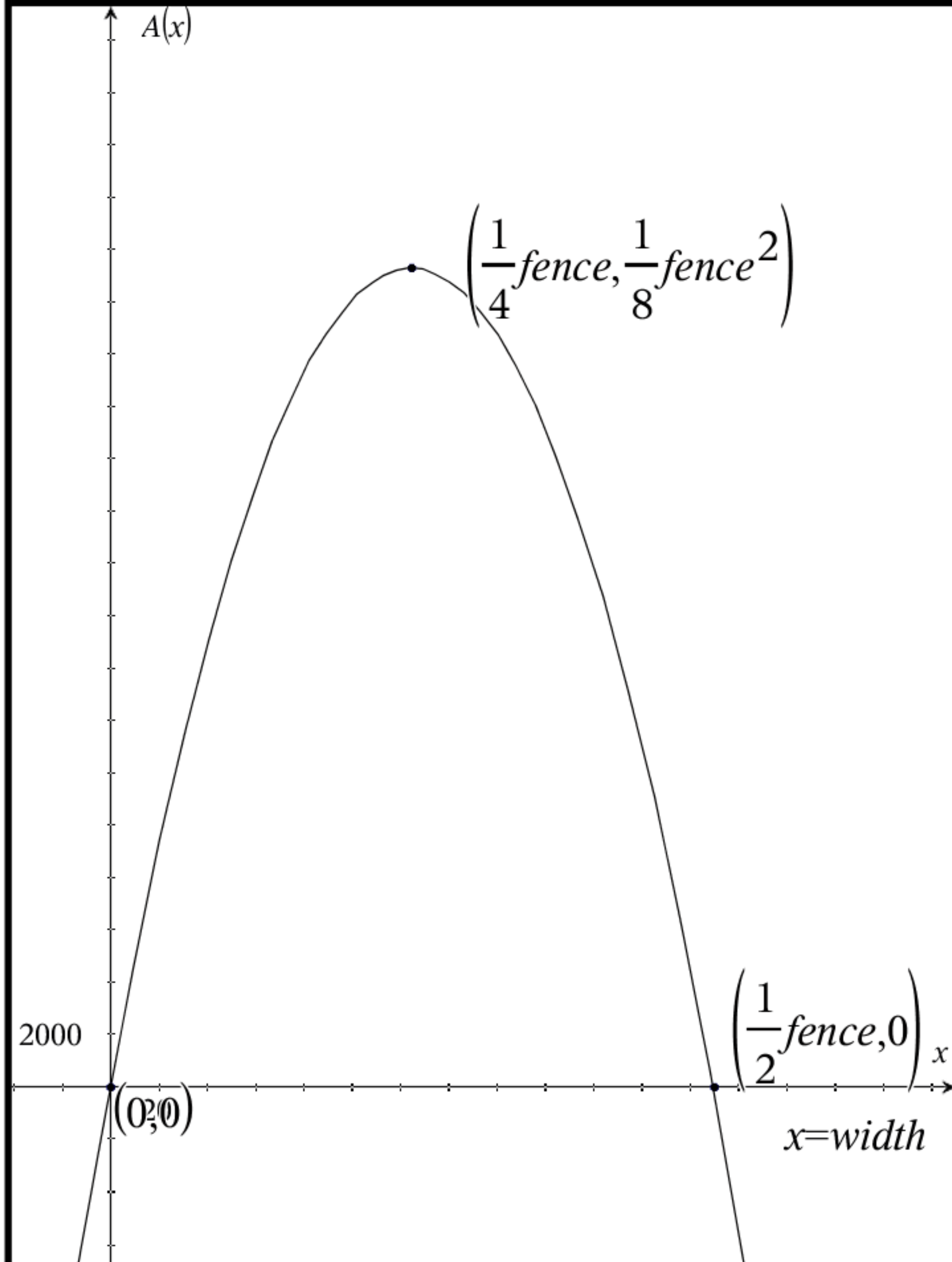
$\text{length} = \textbf{fence} - 2\left(\frac{1}{4} \cdot \textbf{fence}\right)$

$= \frac{1}{2}\textbf{fence}$

maximizing dimensions

$\frac{1}{4}\textbf{fence}$ by $\frac{1}{2}\textbf{fence}$





$A(x) = \textbf{fence} \cdot x - 2x^2$

maximizing dimensions

$\frac{1}{4}\textbf{fence}$ by $\frac{1}{2}\textbf{fence}$

Max area through geometry

$$\frac{1}{4}\textbf{fence} \cdot \frac{1}{2}\textbf{fence} = \frac{1}{8}\textbf{fence}^2$$

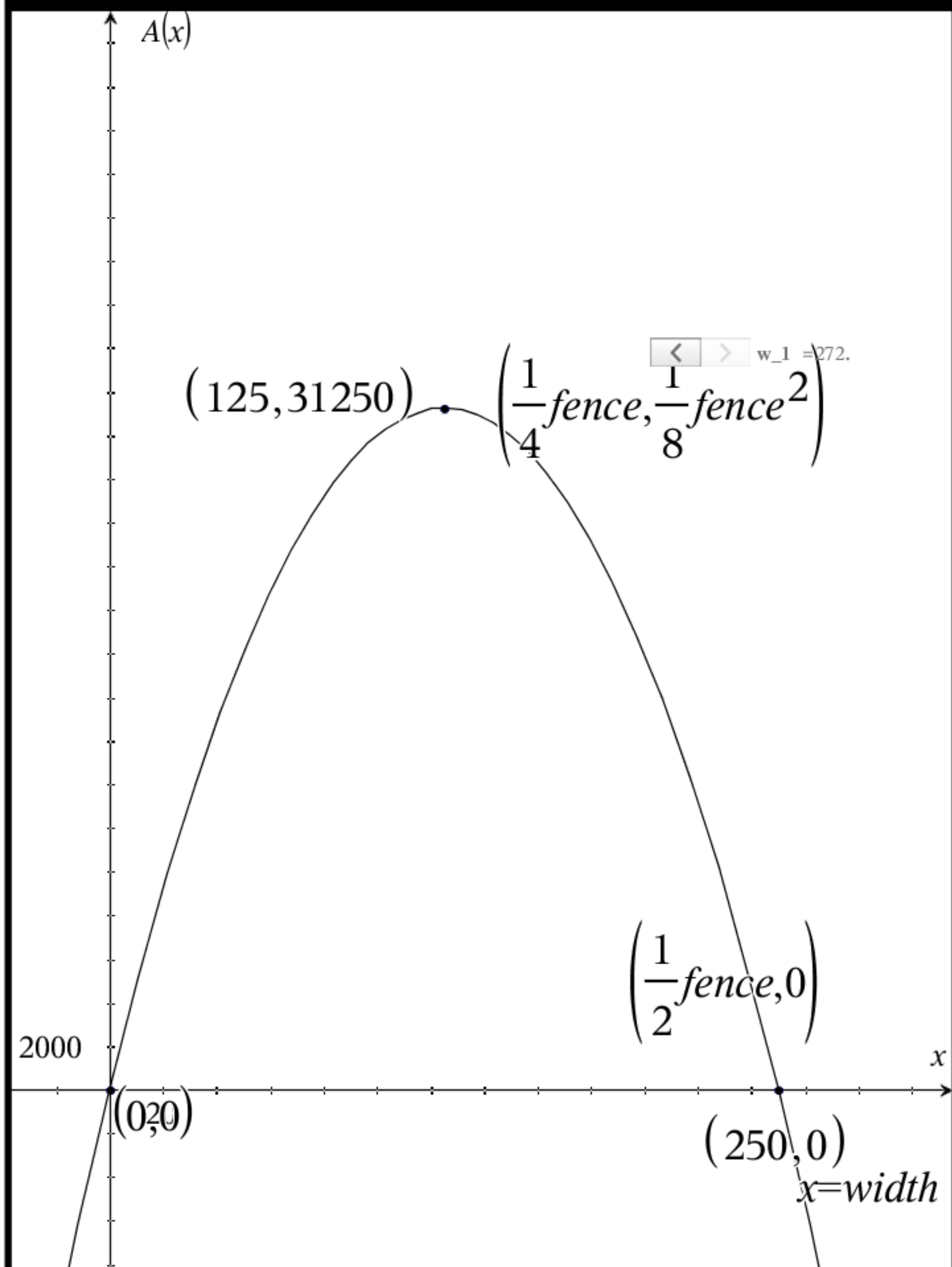
Max area through model

$$A\left(\frac{1}{4}\textbf{fence}\right) = \textbf{fence} \cdot \frac{1}{4}\textbf{fence} - 2 \cdot \left(\frac{1}{4}\textbf{fence}\right)^2$$

$$= \frac{1}{4}\textbf{fence}^2 - \frac{2}{16}\textbf{fence}^2$$

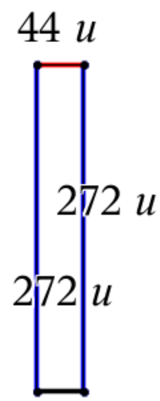
$$= \frac{1}{8}\textbf{fence}^2$$

A farmer has 500 yards of fencing and wishes to fence three sides of a rectangular field (the fourth side is along an existing stone wall that is 1200 yards long, and needs no additional fencing). Find the dimensions of the rectangular field of largest area that can be fenced.



this demonstrates what happens if we vary the width of the rectangle

area of the rectangle	*fence length*
11968 u^2	500.
rate of change in the area	*width (vertical side)*
⁻588.	272.
perimeter of rectangle	*length (horizontal side)*
632 *u*	⁻44.



Given: *Available fence* for *a three side rectangular pen with existing fourth wall*

Want: *Dimensions of rectangle that maximizes area*

Available fence 500

Let x = *width of fence*

Let y = *length of fence* = $500-2x$

Step 1: *write length in terms of fence* and *width*

general (*any fence length*)

$$2x+y=fence$$

$$y=fence-2x$$

particular (*given fence length*)

$$2x+y=500$$

$$y=500-2x$$

Step 2: *Write area model in terms of* x

general (*any fence length*)

$$A(x)=x\cdot(fence-2x)$$

$$A(x)=fence\cdot x-2x^2$$

particular (*given fence length*)

$$A(x)=x\cdot(500-2x)$$

$$A(x)=500x-2x^2$$

Step 3: *Find the* derivative *of the area model you just wrote*

general (*any fence length*)

$$A(x) = fence \cdot x - 2x^2$$

$$A'(x) = \frac{dA}{dx} = fence - 4 \cdot x$$

particular (*given fence length*)

$$A(x) = 500 \cdot x - 2x^2$$

$$A'(x) = \frac{dA}{dx} = 500 - 4 \cdot x$$

Step 4: *Find the zero of the* derivative *of the area model*

general (*any fence length*)

Set $A'(x)=0$ and solve for x

$$0 = fence - 4 \cdot x$$

$$\rightarrow x = \frac{fence}{4} = \frac{1}{4} fence$$

particular (*given fence length*)

Set $A'(x)=0$ and solve for x

$$0 = 500 - 4 \cdot x$$

$$\rightarrow x = \frac{500}{4} = \frac{1}{4}(500) = 125$$

Step 5: *Determine the missing dimension of your rectangle with the optimizing width*

general (any fence length)

$$optimizing\ width = \frac{1}{4} fence$$

$$recall\ length = fence - 2x$$

$$recall\ length = fence - 2 \cdot \frac{1}{4} fence = \frac{1}{2} fence$$

particular (given fence length)

$$optimizing\ width = \frac{1}{4} fence = 125$$

$$recall\ length = fence - 2x$$

$$recall\ length = 500 - 2(125) = 250$$

now we know the dimensions that will optimize or *maximize the area of the rectangle*

general (any fence length)

$$\frac{1}{4} \cdot fence \quad by \quad \frac{1}{2} \cdot fence$$

particular (given fence length)

$$125 \quad by \quad 250$$