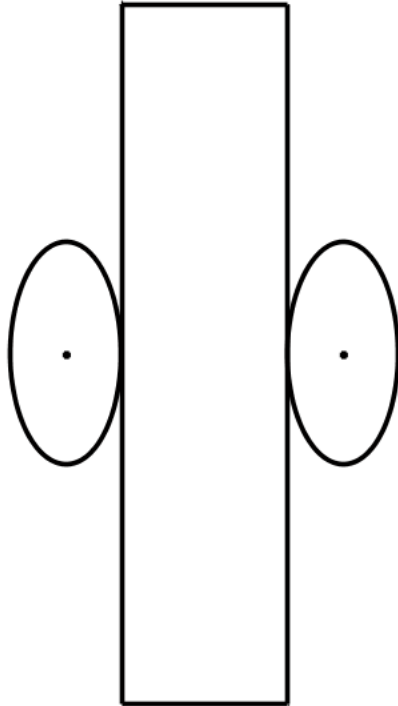


Maxi Vol of can given SA = 5600



You are trying to design a can that maximizes the volume of a right cylinder a.k.a. can.

You are allowed 5600 square cm of sheet metal to create this can.

Things you need to commit to memory

$$SA = CH + 2B = 2 \cdot \pi \cdot r \cdot h + 2 \cdot \pi \cdot r^2$$

SA = surface area of cylinder

C = circumference of a circle

r = radius of a circle

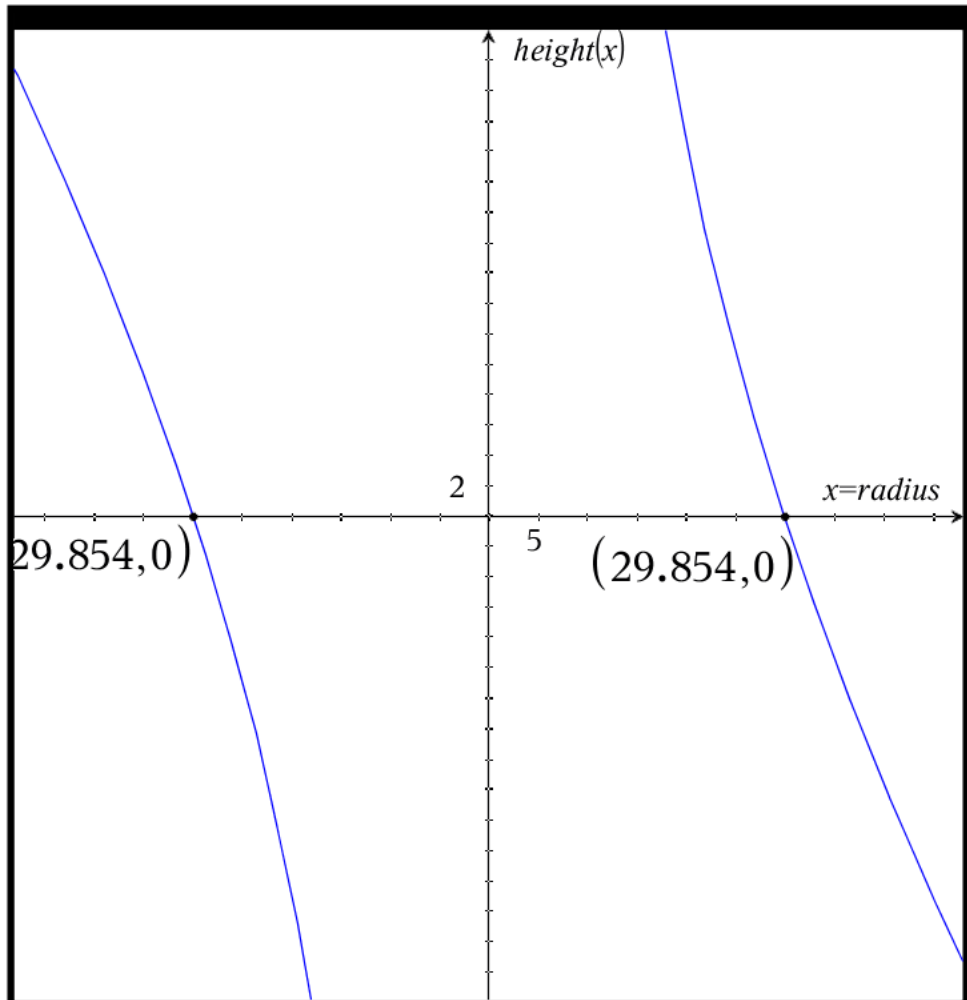
H = height of cylinder

B = Base = circle's area

$$V = BH = 2 \cdot \pi \cdot r^2 \cdot h$$

V = volume of a cylinder

B, H, same as above



$$5600 = CH + 2B = 2 \cdot \pi \cdot r \cdot h + 2 \cdot \pi \cdot r^2$$

$$5600 = 2 \cdot \pi \cdot x \cdot h + 2 \cdot \pi \cdot x^2$$

$$5600 - 2 \cdot \pi \cdot x^2 = 2 \cdot \pi \cdot x \cdot h$$

$$h(x) = \frac{5600 - 2\pi \cdot x^2}{2\pi \cdot x} = \frac{5600}{2\pi \cdot x} - \frac{2\pi \cdot x^2}{2\pi \cdot x}$$

$$h(x) = \frac{2800.}{\pi \cdot x} - x$$

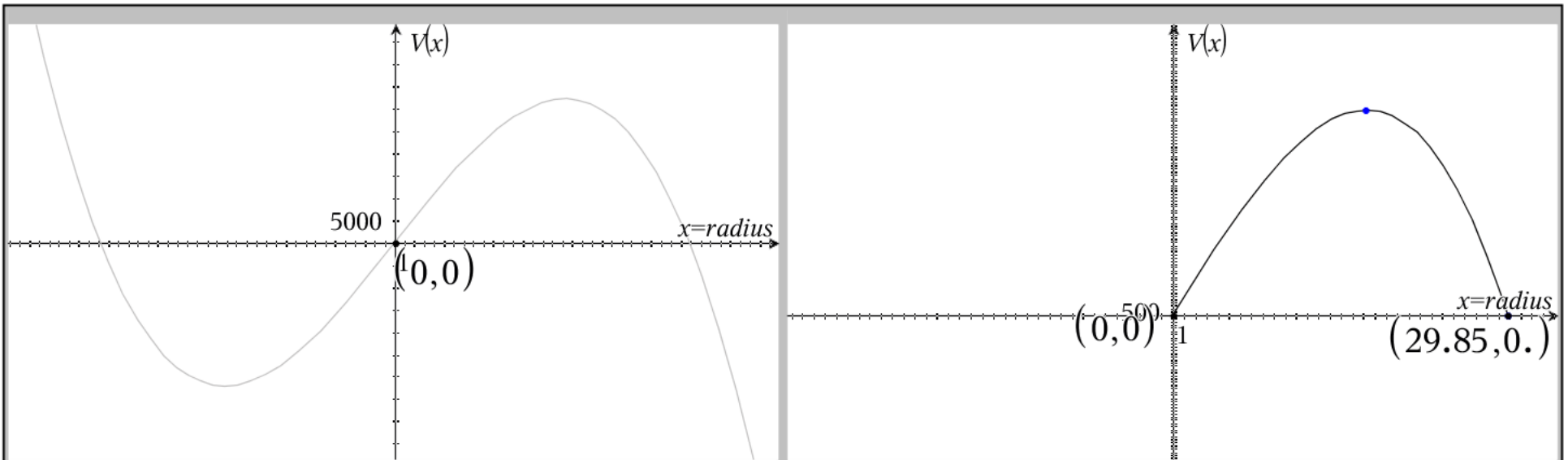
Feasibility of h

$$0 < h < \sqrt{\frac{2800.}{\pi}}$$

$$0 < h < 29.854$$

Step 1: Let $x = \text{radius}$ find h in terms of SA and x

Step 2: Determine where h is feasible a.k,a positive



$$\text{Volume Function} = \pi \cdot r^2 \cdot h$$

$$V(x) = \pi \cdot x^2 \cdot \left(\frac{2800}{\pi \cdot x} - x \right)$$

$$V(x) = 2800 \cdot x - \pi \cdot x^3$$

$$V(x) = x \left(2800 - \pi \cdot x^2 \right)$$

Feasible Volume function

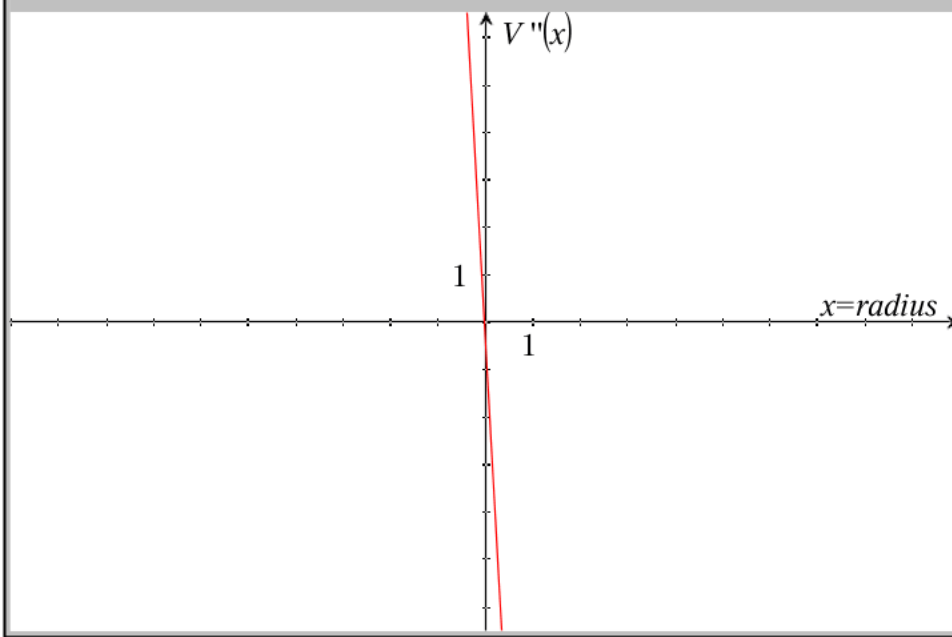
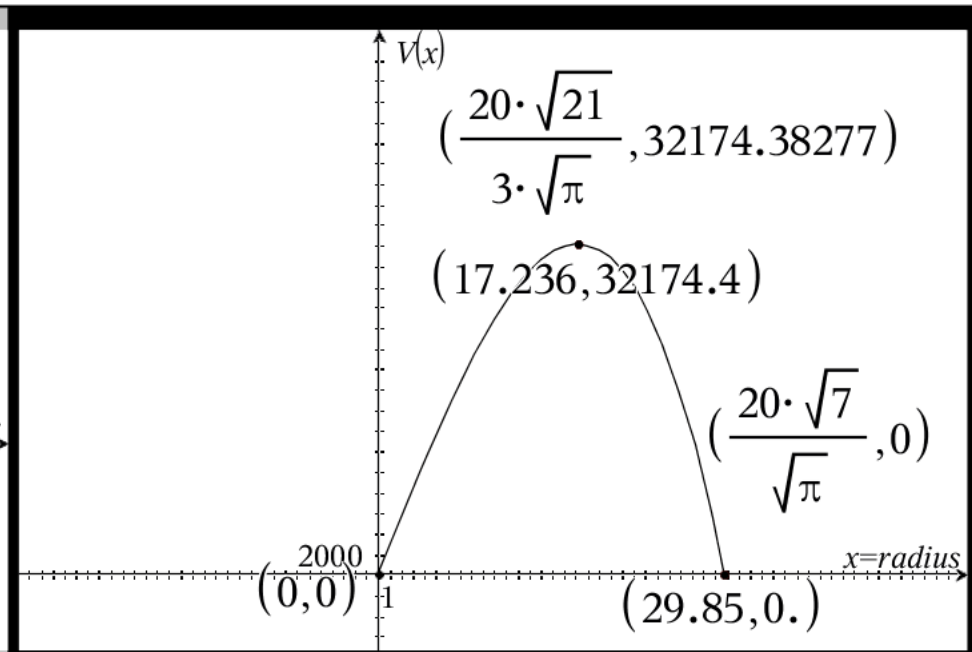
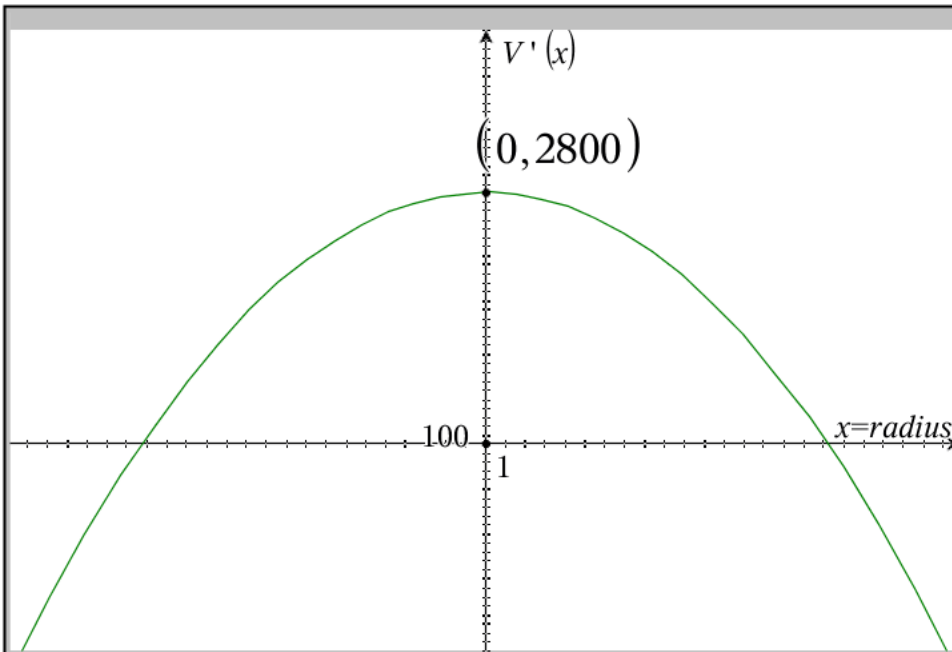
$$V(x) = 2800 \cdot x - \pi \cdot x^3 \text{ for } 0 < x < 29.854$$

Feasible roots of Volume Function

$$x = 0 \text{ or } x = \frac{20 \cdot \sqrt{7}}{\sqrt{\pi}} \approx 29.854$$

NOT feasible root of Volume function

$$x = \frac{-20 \cdot \sqrt{7}}{\sqrt{\pi}} \approx -29.854$$



Volume Function = $\pi \cdot r^2 \cdot h$

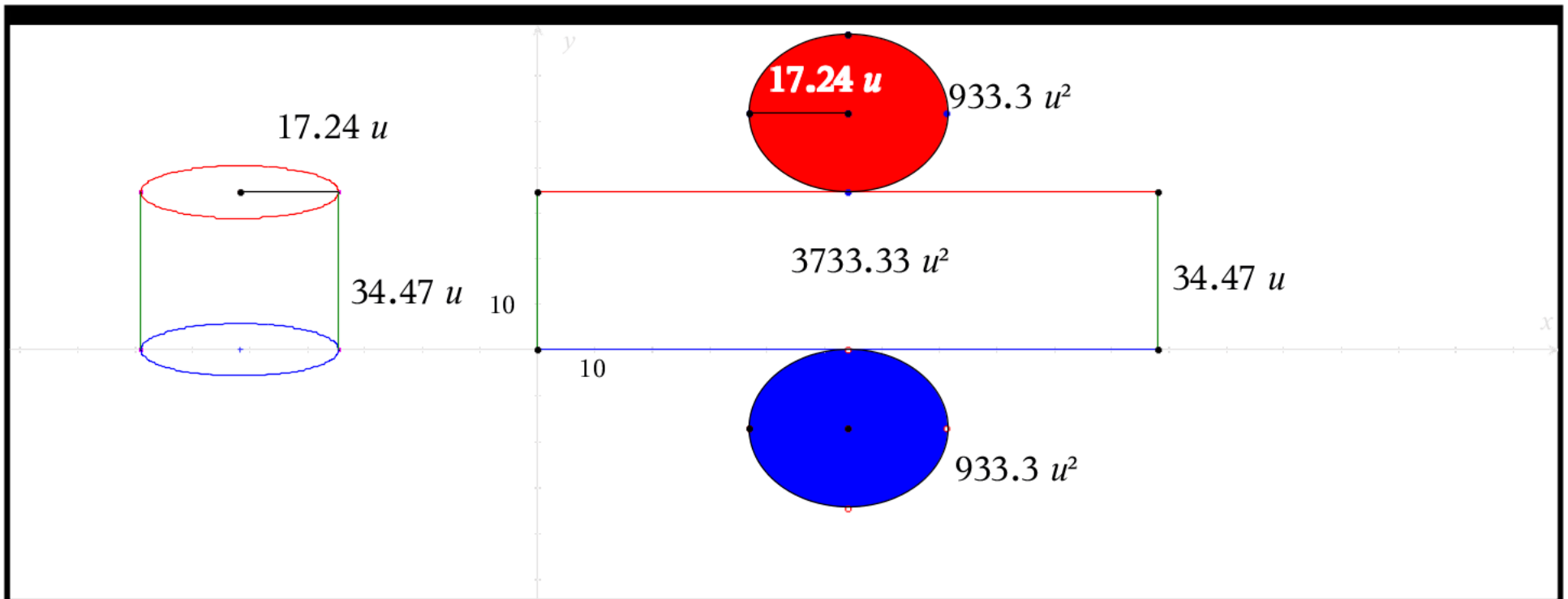
$V(x) = 2800 \cdot x - \pi \cdot x^3$

Derivative of Volume function

$V'(x) = 2800 - 3\pi \cdot x^2$

2nd Derivative of Volume Function'

$V''(x) = -6\pi \cdot x$

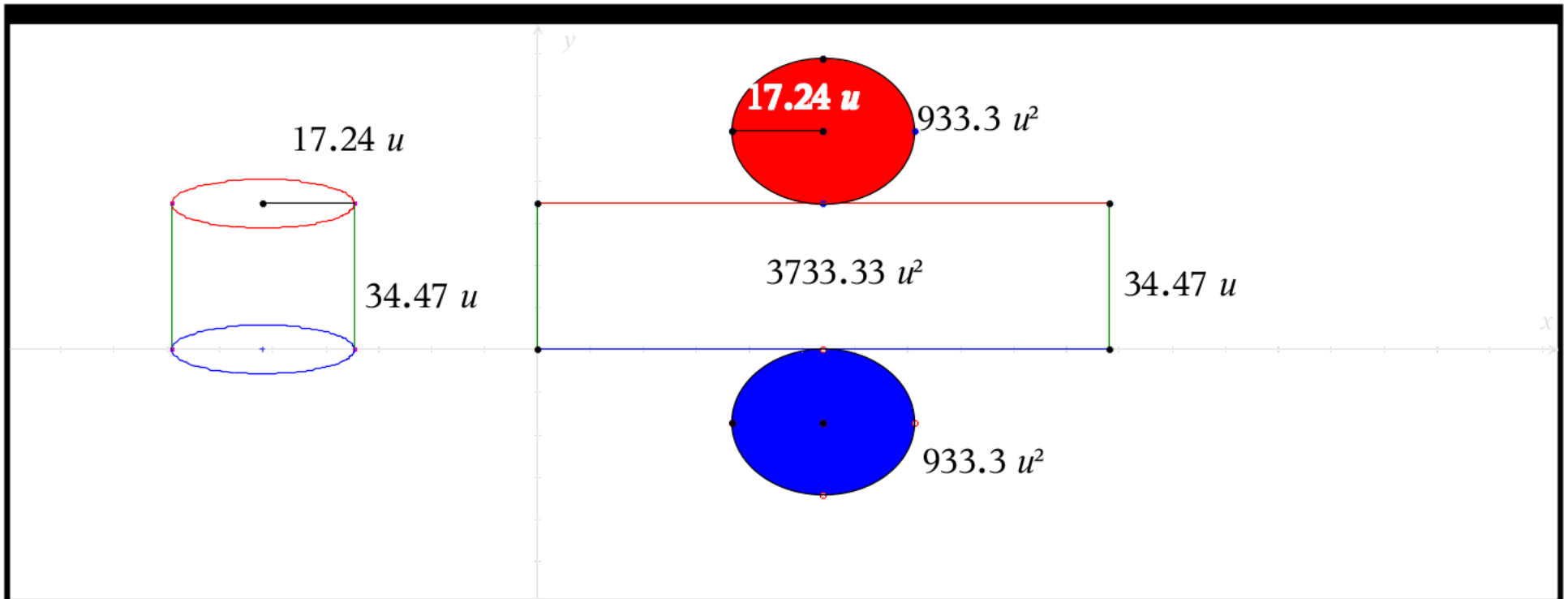


What we found!

The dimensions of the can that maximize volume of a can with surface area= 5600

$$r = x = \sqrt{\frac{5600}{6\pi}} = \frac{20 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \approx 17.24$$

$$h = \frac{5600}{2 \cdot \pi} \cdot \frac{1}{\sqrt{\frac{5600}{6\pi}}} - \sqrt{\frac{5600}{6\pi}} = \frac{40 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \approx 34.47$$



What we found!

The maximum volume of the can that maximize volume of a can with surface area of 5600

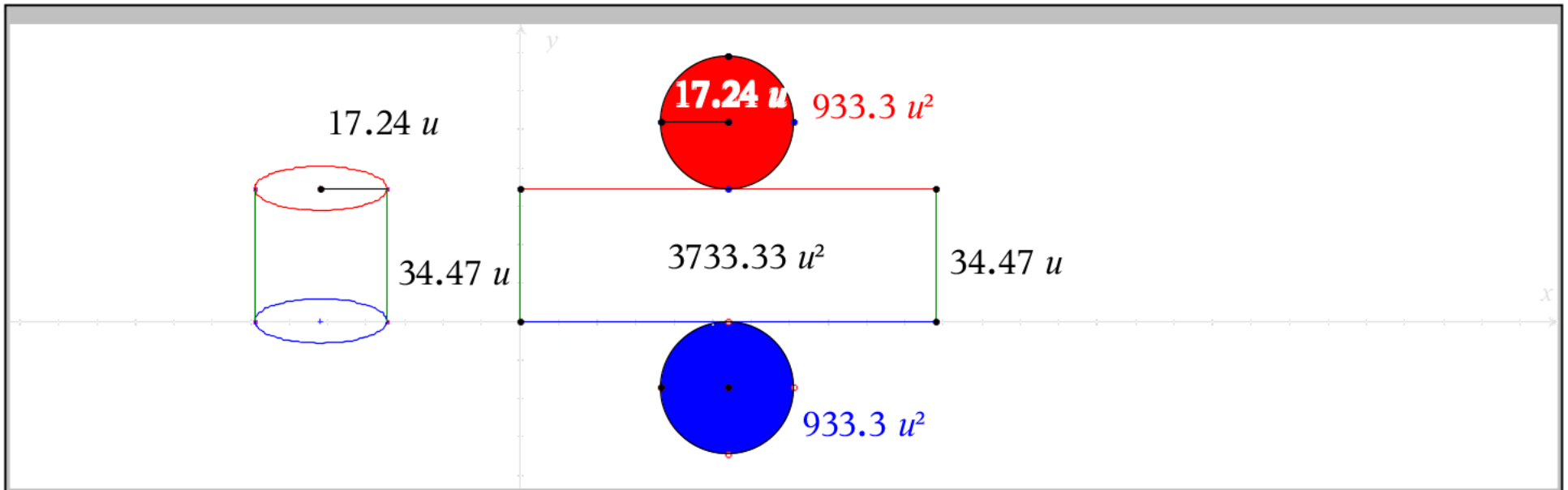
Exact Volume Approximate Volume

3.217E+4

32174.383

$$V = \pi \left(\frac{20 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \right)^2 \left(\frac{40 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \right)$$

$$V = \frac{2800}{3} \cdot \frac{40 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \approx 32174.383$$



What we found! The parts of the can that maximize volume of a can with surface area of 5600

$$\text{radius} = \frac{20 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \approx 17.24$$

$$\text{height} = \frac{40 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \approx 34.47$$

$$\text{Base Area} = \pi \cdot \left(\frac{20 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \right)^2 = \frac{2800}{3} \approx 933.3$$

$$\text{Lateral Area} = 2\pi \left(\frac{20 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \right) \cdot \left(\frac{40 \cdot \sqrt{21}}{3 \cdot \sqrt{\pi}} \right) = \frac{11200}{3} \approx 3733.33$$

$$\text{total surface area} = LA + 2B = \frac{11200}{3} + 2 \left(\frac{2800}{3} \right) = 5600$$