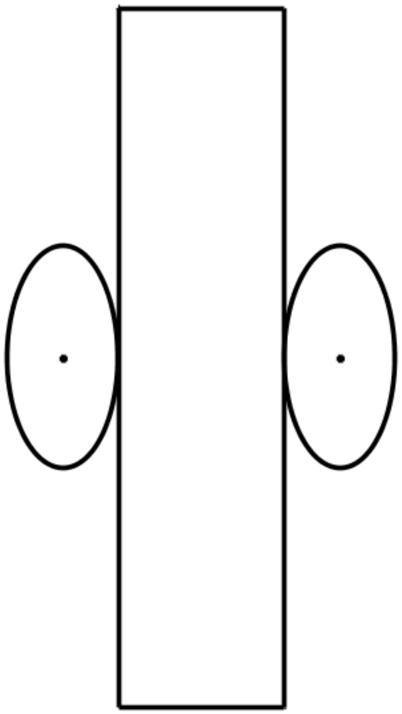


You are trying to design a can that maximizes the volume of a right cylinder a.k.a. can.

You are allowed 3200 square cm of sheet metal to create this can.

Things you need to commit to memory

SA = CH+2B = $2\cdot\pi\cdot r\cdot h+2\cdot\pi\cdot r^2$

SA = surface area of cylinder

C = circumference of a circle

r = radius of a circle

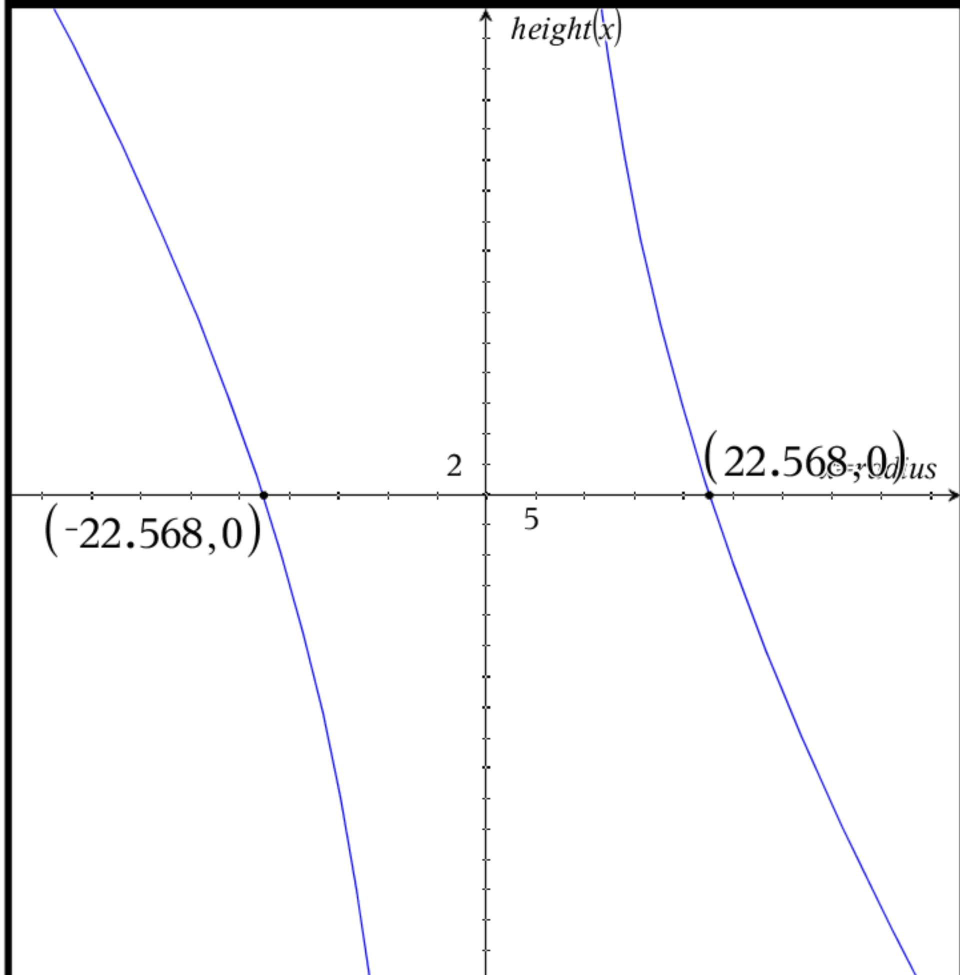
H = height of cylinder

B = Base =circle's area

V = BH = $2\cdot\pi\cdot r^2\cdot h$

V = volume of a cylinder

B, H, same as above



$$3200 = CH + 2B = 2\cdot\pi\cdot r\cdot h + 2\cdot\pi\cdot r^2$$

$$3200 = 2\cdot\pi\cdot x\cdot h + 2\cdot\pi\cdot x^2$$

$$3200 - 2\cdot\pi\cdot x^2 = 2\cdot\pi\cdot x\cdot h$$

$$h(x) = \frac{3200 - 2\pi\cdot x^2}{2\pi\cdot x} = \frac{3200}{2\pi\cdot x} - \frac{2\pi\cdot x^2}{2\pi\cdot x}$$

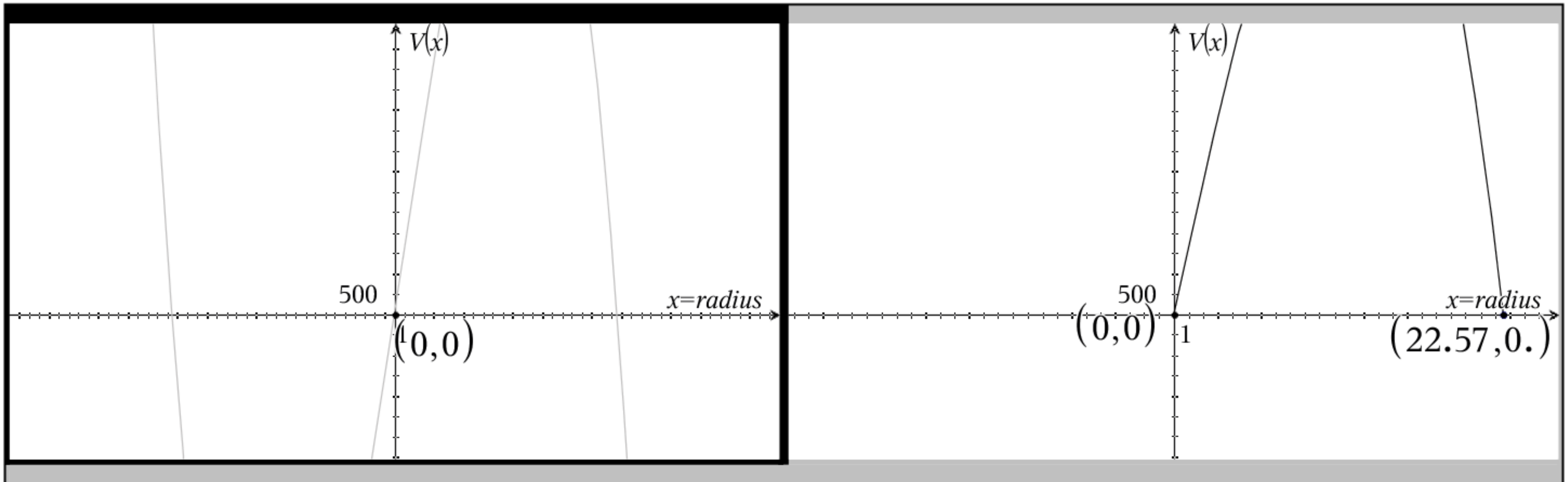
$$h(x) = \frac{1600.}{\pi\cdot x} - x$$

Feasibility of h

$$0 < h < \sqrt{\frac{1600.}{\pi}} \qquad 0 < h < 22.568$$

Step 1: Let x = radius find h in terms of SA and x

Step 2: Determine where h is feasible a.k,a positive



Volume Function $= \pi \cdot r^2 \cdot h$

$$V(x) = \pi \cdot x^2 \cdot \left(\frac{1600.}{\pi \cdot x} - x \right)$$

$$V(x) = 1600.x - \pi \cdot x^3$$

$$V(x) = x\left(1600. - \pi \cdot x^2 \right)$$

Feasible Volume function

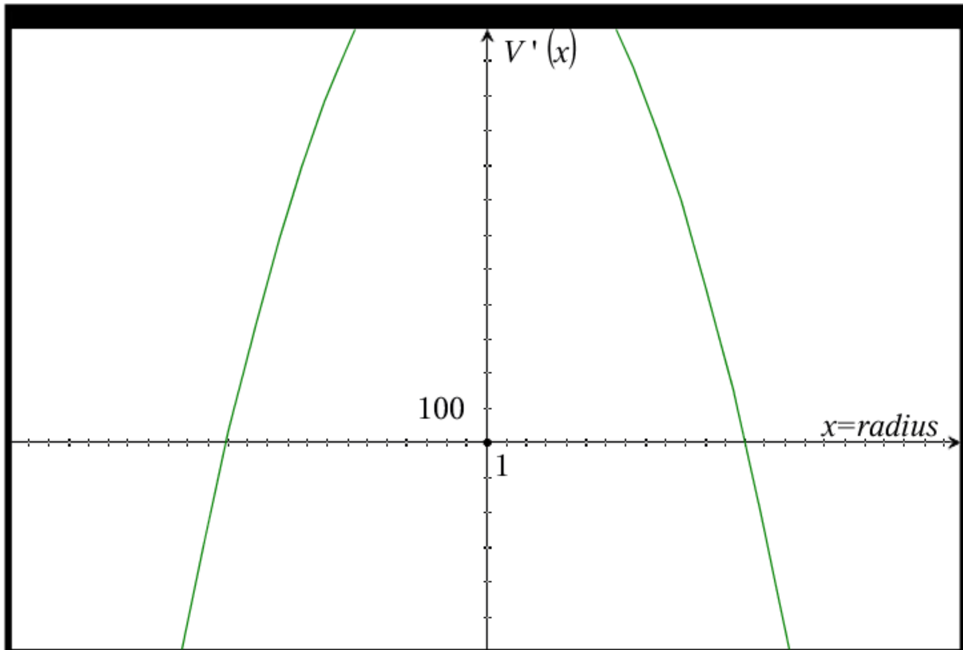
$$V(x) = 1600.x - \pi \cdot x^3 \text{ for } 0 < x < 22.568$$

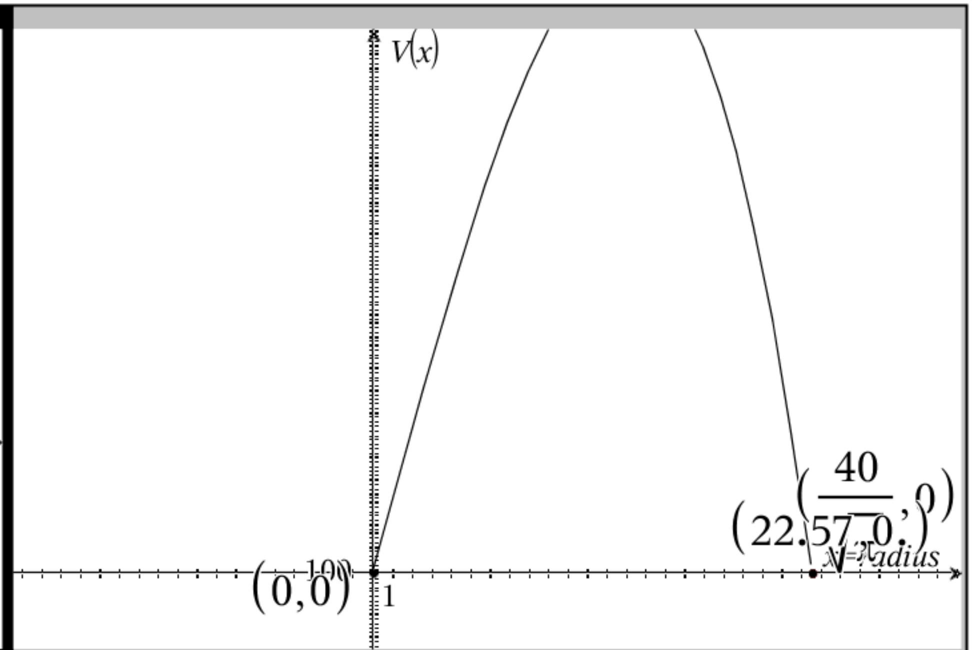
Feasible roots of Volume Function

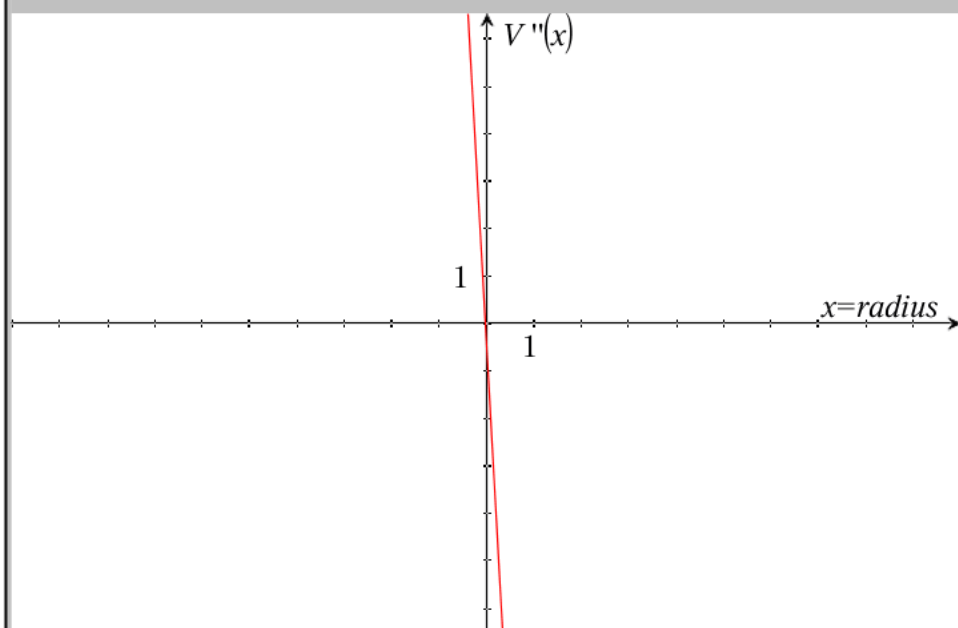
$$x = 0 \quad \text{or } x = \frac{40}{\sqrt{\pi}} \approx 22.568$$

NOT *feasible* root *of Volume function*

$$x = \frac{^-40}{\sqrt{\pi}} \approx {}^-22.568$$







$Volume\ Function = \pi \cdot r^2 \cdot h$

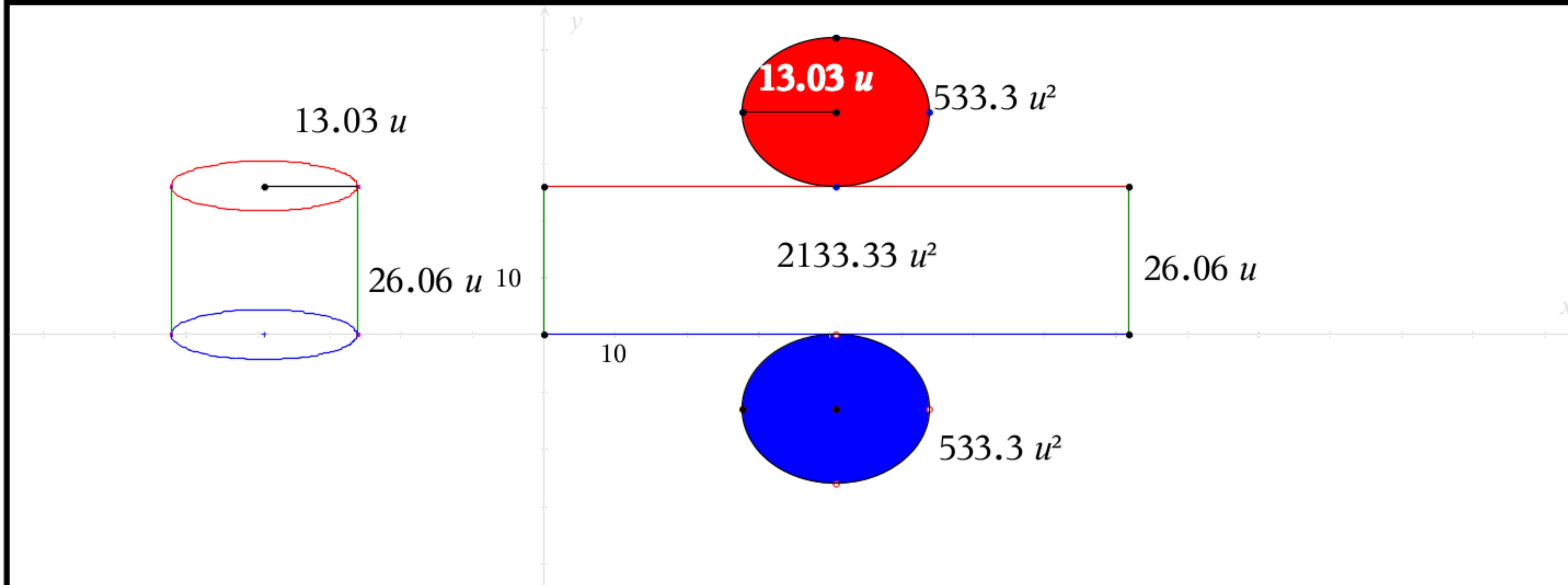
$V(x) = 1600.x - \pi \cdot x^3$

Derivative *of Volume function*

$V'(x) = 1600. - 3\pi \cdot x^2$

2*nd* Derivative *of Volume Function*'

$V''(x) = -6\pi \cdot x$

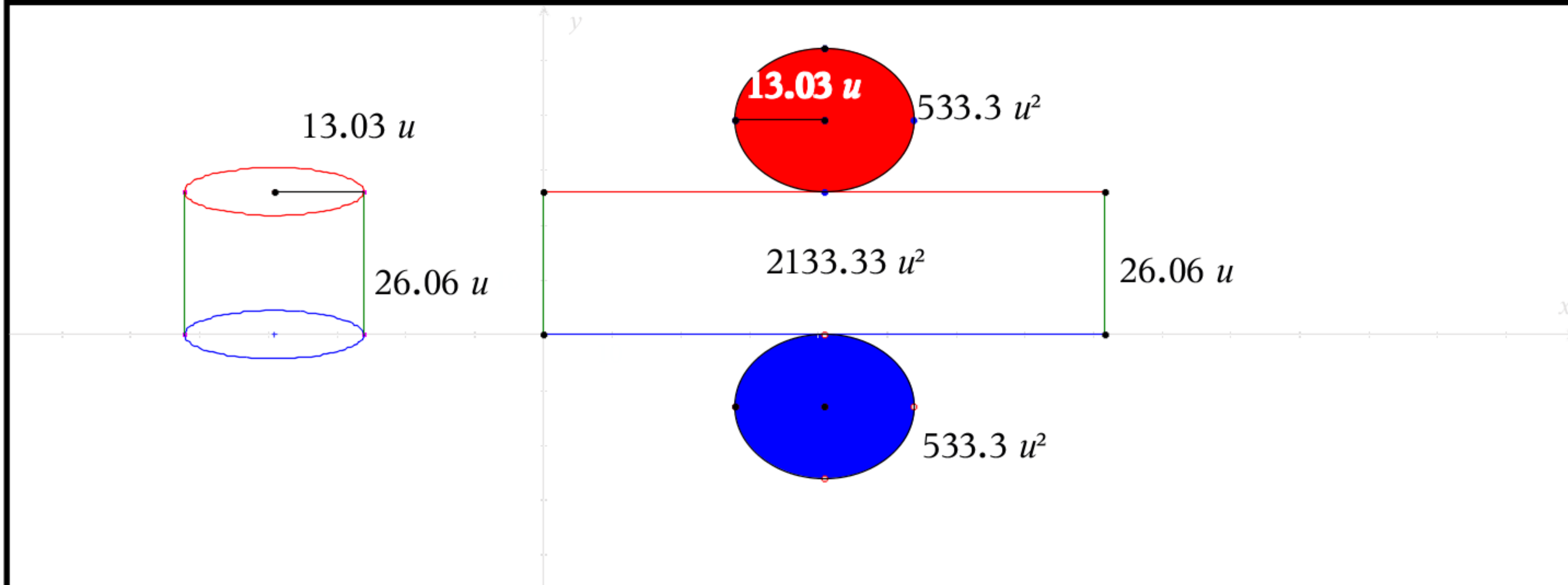


What we found!

The dimensions of the can that maximize volume of a can with surface area= 3200

$$r = x = \sqrt{\frac{3200}{6\pi}} = \frac{40\cdot\sqrt{3}}{3\cdot\sqrt{\pi}} \approx 13.03$$

$$h = \frac{3200}{2\cdot\pi}\cdot\frac{1}{\sqrt{\frac{3200}{6\pi}}} - \sqrt{\frac{3200}{6\pi}} = \frac{80\cdot\sqrt{3}}{3\cdot\sqrt{\pi}} \approx 26.06$$



What we found!

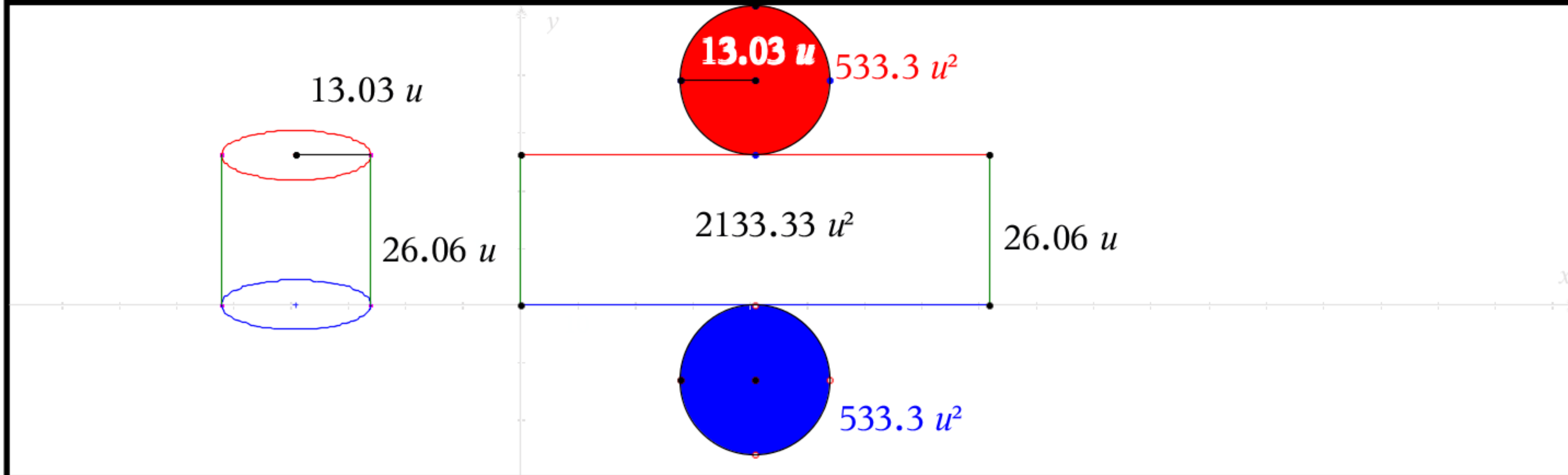
The maximum volume of the can that maximize volume of a can with surface area of 3200

Exact *Volume* *Approximate Volume*

$$\frac{128000\cdot\sqrt{3}}{9\cdot\sqrt{\pi}} \qquad 13898.027$$

$$V=\pi\left(\frac{40\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right)^2\left(\frac{80\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right)$$

$$V=\frac{1600}{3}\cdot\frac{80\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\approx 13898.027$$



What we found! *The parts of the can that maximize volume of a can with surface area of* 3200

$$\text{radius} = \frac{40\cdot\sqrt{3}}{3\cdot\sqrt{\pi}} \approx 13.03$$

$$\text{height} = \frac{80\cdot\sqrt{3}}{3\cdot\sqrt{\pi}} \approx 26.06$$

$$\text{Base Area} = \pi\cdot\left(\frac{40\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right)^2 = \frac{1600}{3} \approx 533.3$$

$$\text{Lateral Area} = 2\pi\left(\frac{40\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right)\cdot\left(\frac{80\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right) = \frac{6400}{3} \approx 2133.33$$

$$\text{total surface area} = LA + 2B = \frac{6400}{3} + 2\left(\frac{1600}{3}\right) = 3200$$