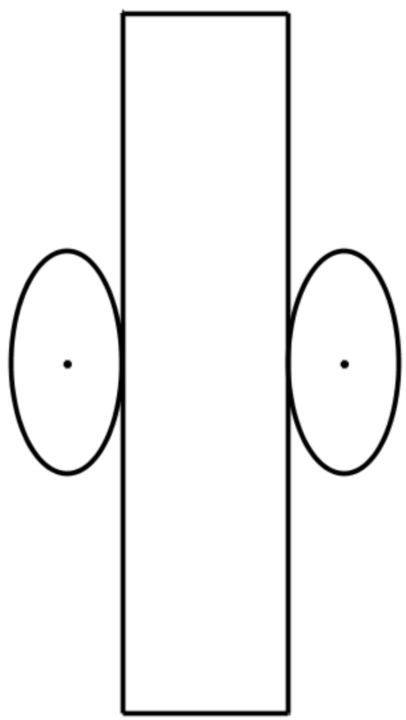


You are trying to design a can that maximizes the volume of a right cylinder a.k.a. can.

You are allowed 200 square cm of sheet metal to create this can.

Things you need to commit to memory

SA = CH+2B = $2\cdot\pi\cdot r\cdot h+2\cdot\pi\cdot r^{2}$

SA = surface area of cylinder

C = circumference of a circle

r = radius of a circle

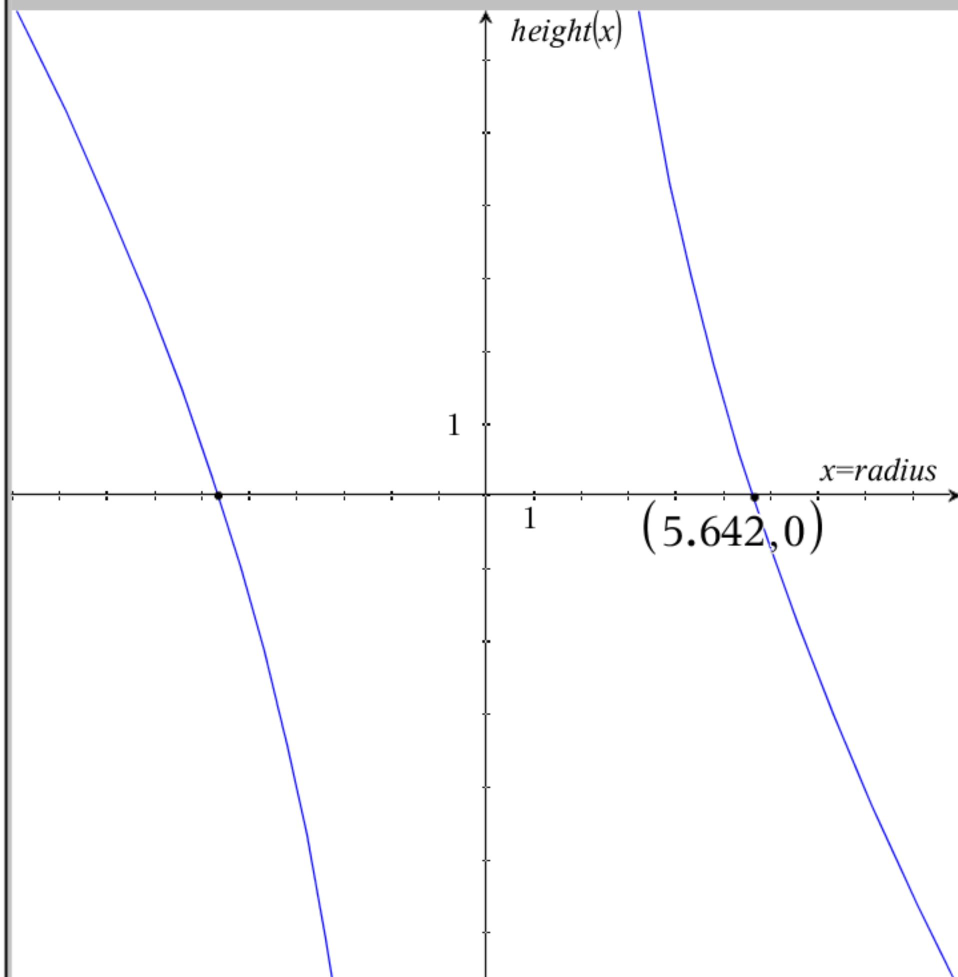
H = height of cylinder

B = Base = circle's area

V = BH = $2\cdot\pi\cdot r^{2}\cdot h$

V = volume of a cylinder

B, H, same as above



$$200 = CH + 2B = 2\cdot\pi\cdot r\cdot h + 2\cdot\pi\cdot r^2$$

$$200 = 2\cdot\pi\cdot x\cdot h + 2\cdot\pi\cdot x^2$$

$$200 - 2\cdot\pi\cdot x^2 = 2\cdot\pi\cdot x\cdot h$$

$$h(x) = \frac{200 - 2\pi\cdot x^2}{2\pi\cdot x} = \frac{200}{2\pi\cdot x} - \frac{2\pi\cdot x^2}{2\pi\cdot x}$$

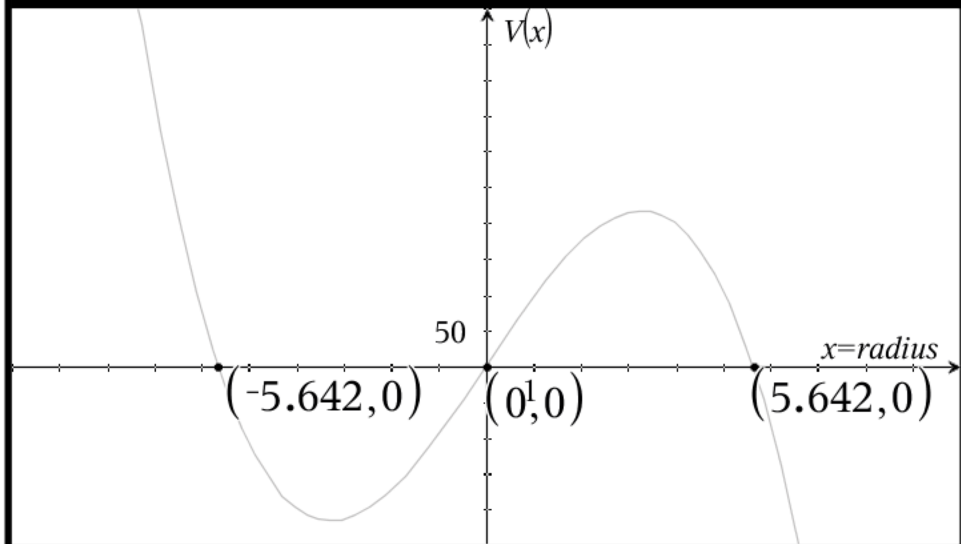
$$h(x) = \frac{100.}{\pi\cdot x} - x$$

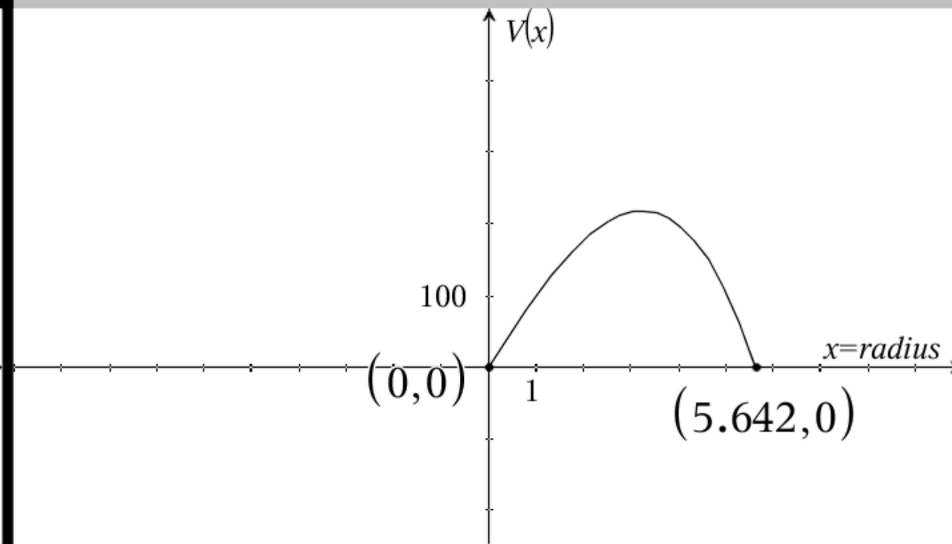
Feasibility of h

$$0 < h < \sqrt{\frac{100.}{\pi}} \qquad 0 < h < 5.642$$

Step 1: Let x = radius find h in terms of SA and x

Step 2: Determine where h is feasible a.k,a positive





Volume Function $= \pi \cdot r^2 \cdot h$

$$V(x) = \pi \cdot x^2 \cdot \left(\frac{100.}{\pi \cdot x} - x \right)$$

$$V(x) = 100.\ x - \pi \cdot x^3$$

$$V(x) = x\left(100. - \pi \cdot x^2 \right)$$

Feasible Volume function

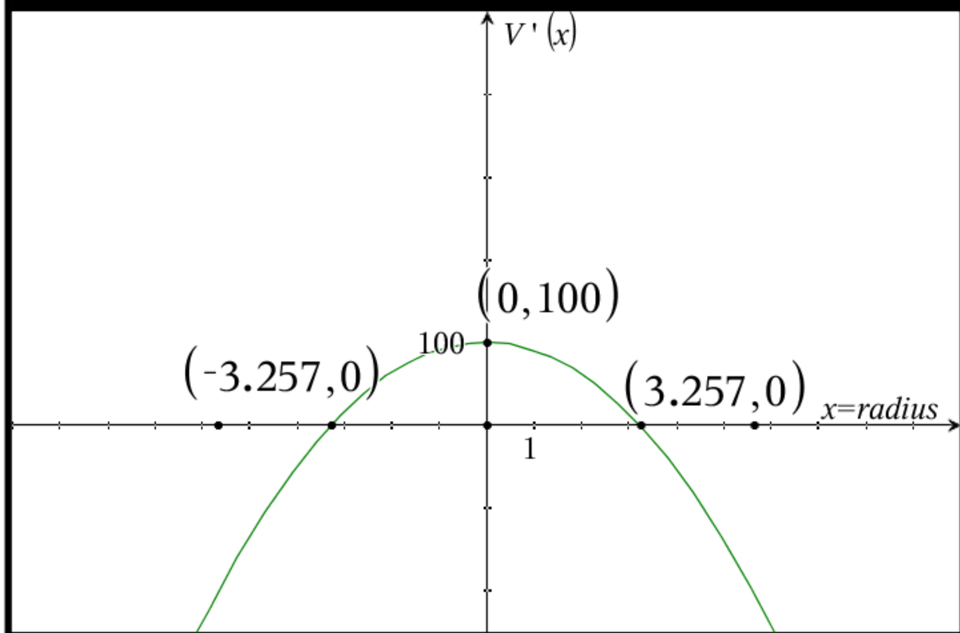
$$V(x) = 100.\ x - \pi \cdot x^3 \text{ for } 0 < x < 5.642$$

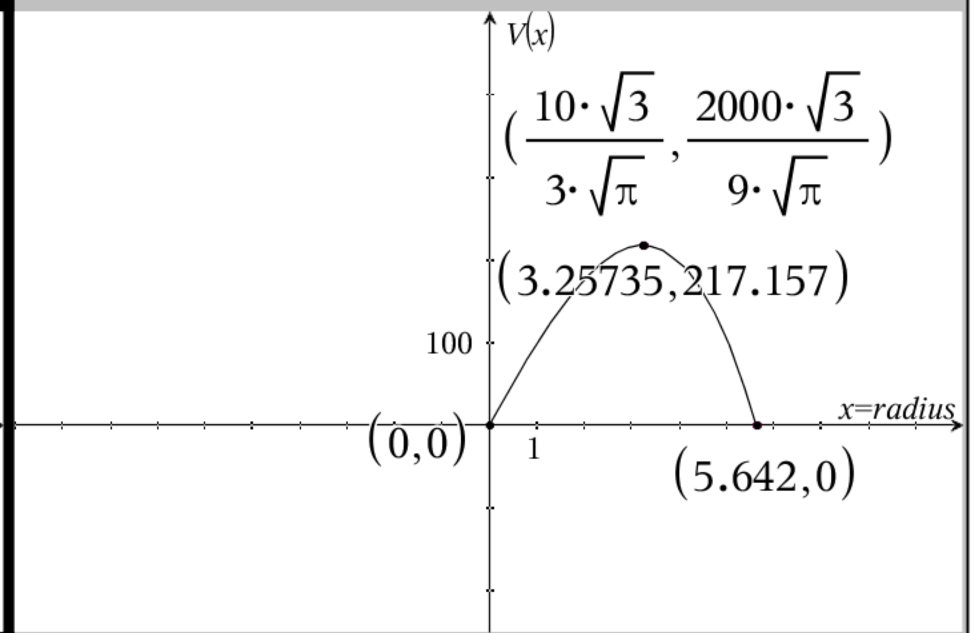
Feasible roots of Volume Function

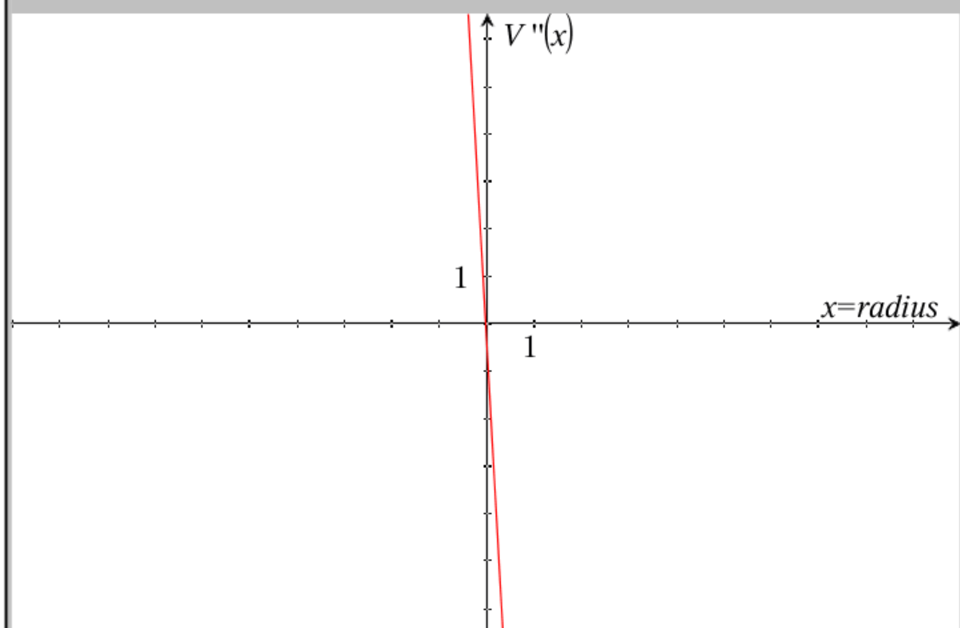
$$x = 0 \quad \text{or } x = \frac{10}{\sqrt{\pi}} \approx 5.642$$

NOT *feasible* root *of Volume function*

$$x = \frac{-10}{\sqrt{\pi}} \approx -5.642$$







Volume Function $= \pi\cdot r^2\cdot h$

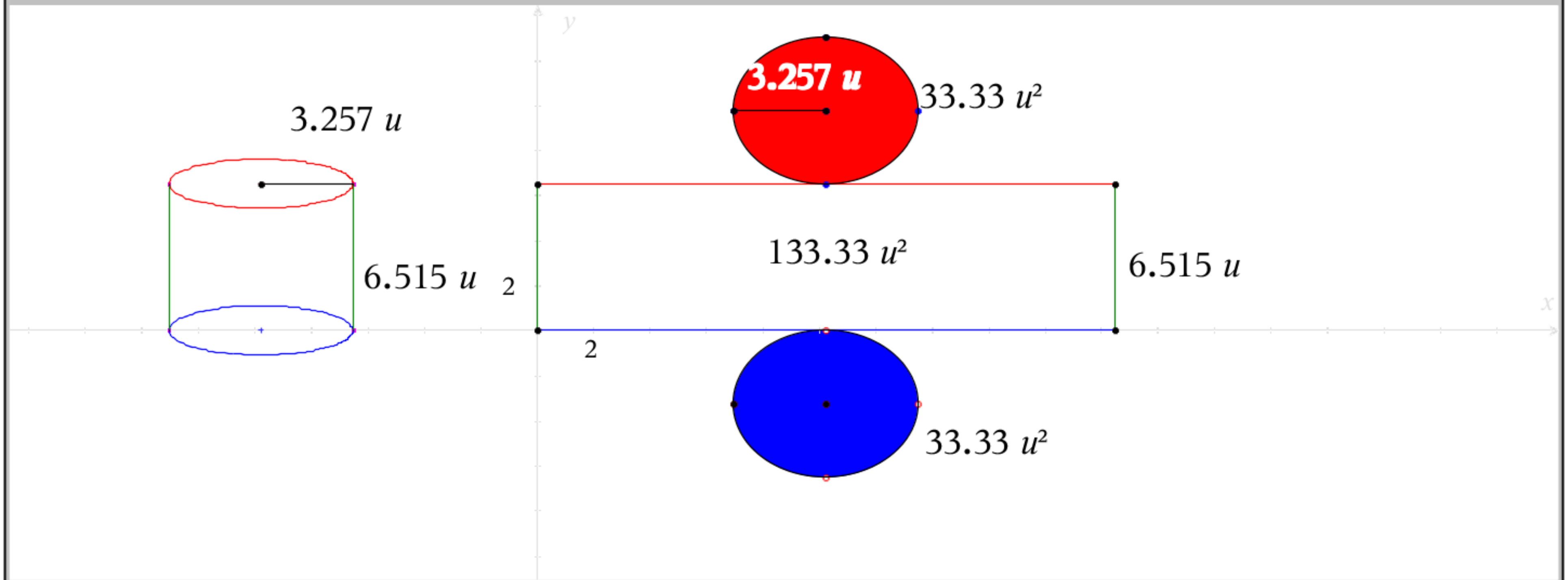
$V(x) = 100.\ x - \pi\cdot x^3$

Derivative *of Volume function*

$V'(x) = 100. - 3\pi\cdot x^2$

2nd Derivative *of Volume Function'*

$V''(x) = -6\pi\cdot x$

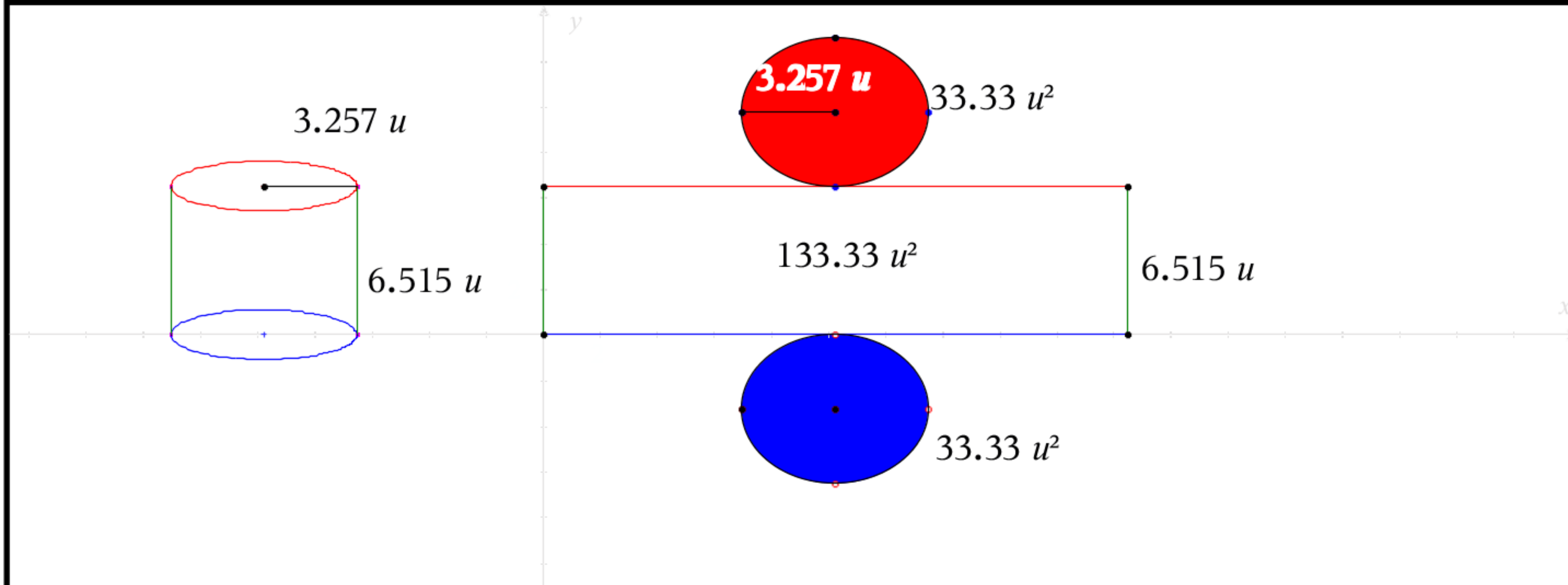


What we found!

The dimensions of the can that maximize volume of a can with surface area= 200

$$r = x = \sqrt{\frac{200}{6\pi}} = \frac{10\cdot\sqrt{3}}{3\cdot\sqrt{\pi}} \approx 3.257$$

$$h = \frac{200}{2\cdot\pi}\cdot\frac{1}{\sqrt{\frac{200}{6\pi}}} - \sqrt{\frac{200}{6\pi}} = \frac{20\cdot\sqrt{3}}{3\cdot\sqrt{\pi}} \approx 6.515$$



What we found!

The maximum volume of the can that maximize volume of a can with surface area of 200

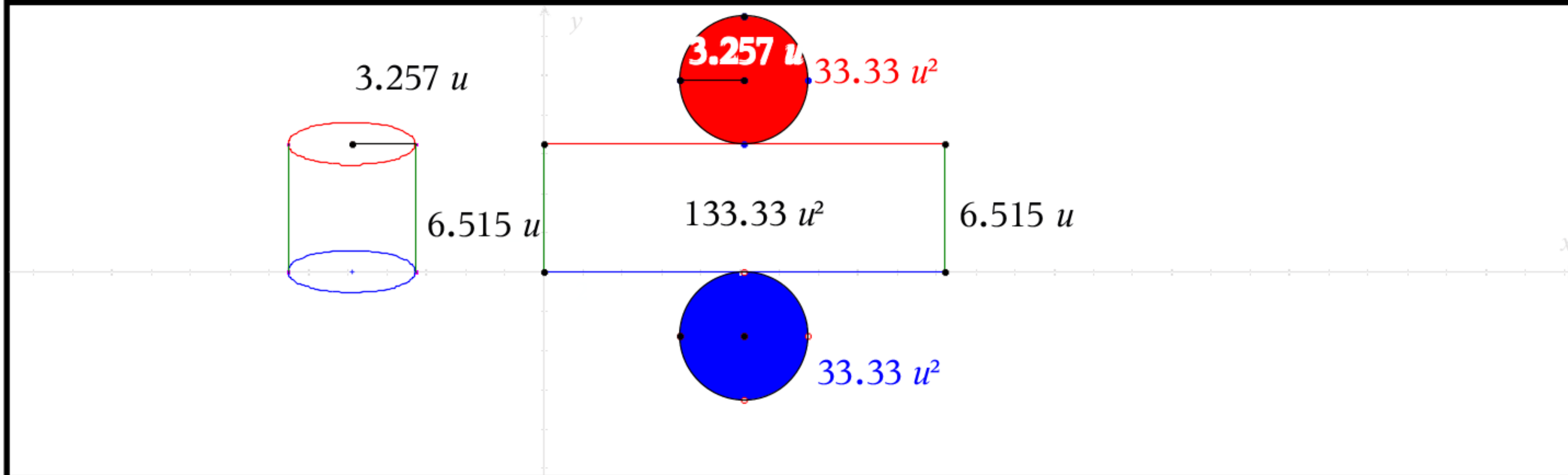
Exact *Volume* | *Approximate Volume*

$$\frac{2000\cdot\sqrt{3}}{9\cdot\sqrt{\pi}}$$

217.157

$$V=\pi\left(\frac{10\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right)^2\left(\frac{20\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right)$$

$$V=\frac{100}{3}\cdot\frac{20\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\approx 217.157$$



What we found! *The parts of the can that maximize volume of a can with surface area of* 200

$$\textit{Base Area} = \pi \cdot \left(\frac{10\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right)^2 = \frac{100}{3} \approx 33.33$$

$$\textit{radius} = \frac{10\cdot\sqrt{3}}{3\cdot\sqrt{\pi}} \approx 3.257$$

$$\textit{Lateral Area} = 2\pi\left(\frac{10\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right)\cdot\left(\frac{20\cdot\sqrt{3}}{3\cdot\sqrt{\pi}}\right) = \frac{400}{3} \approx 133.33$$

$$\textit{height} = \frac{20\cdot\sqrt{3}}{3\cdot\sqrt{\pi}} \approx 6.515$$

$$\textit{total surface area} = LA + 2B = \frac{400}{3} + 2\left(\frac{100}{3}\right) = 200$$