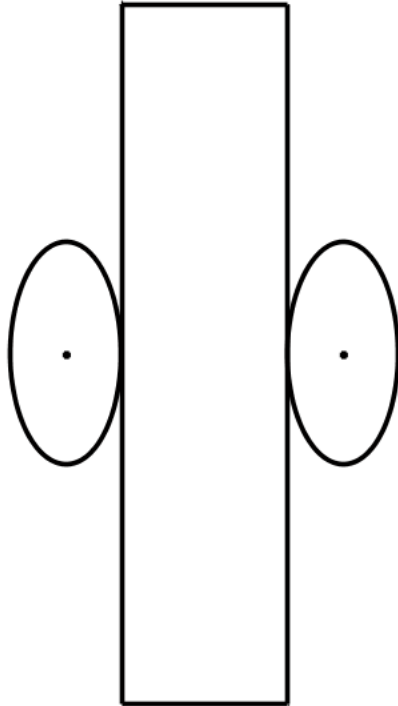


Maxi Vol of can given SA = 1800



You are trying to design a can that maximizes the volume of a right cylinder a.k.a. can.

You are allowed 1800 square cm of sheet metal to create this can.

Things you need to commit to memory

$$SA = CH + 2B = 2 \cdot \pi \cdot r \cdot h + 2 \cdot \pi \cdot r^2$$

SA = surface area of cylinder

C = circumference of a circle

r = radius of a circle

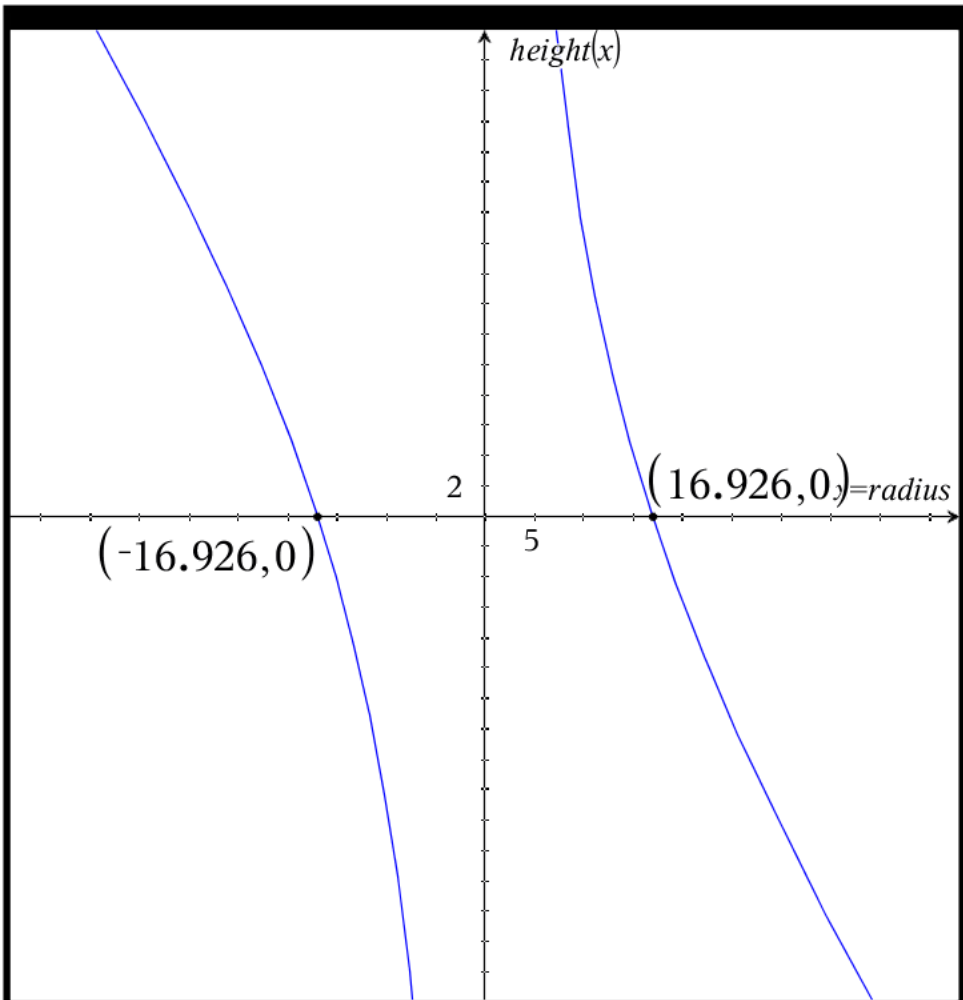
H = height of cylinder

B = Base = circle's area

$$V = BH = 2 \cdot \pi \cdot r^2 \cdot h$$

V = volume of a cylinder

B, H, same as above



$$1800 = CH + 2B = 2 \cdot \pi \cdot r \cdot h + 2 \cdot \pi \cdot r^2$$

$$1800 = 2 \cdot \pi \cdot x \cdot h + 2 \cdot \pi \cdot x^2$$

$$1800 - 2 \cdot \pi \cdot x^2 = 2 \cdot \pi \cdot x \cdot h$$

$$h(x) = \frac{1800 - 2\pi \cdot x^2}{2\pi \cdot x} = \frac{1800}{2\pi \cdot x} - \frac{2\pi \cdot x^2}{2\pi \cdot x}$$

$$h(x) = \frac{900.}{\pi \cdot x} - x$$

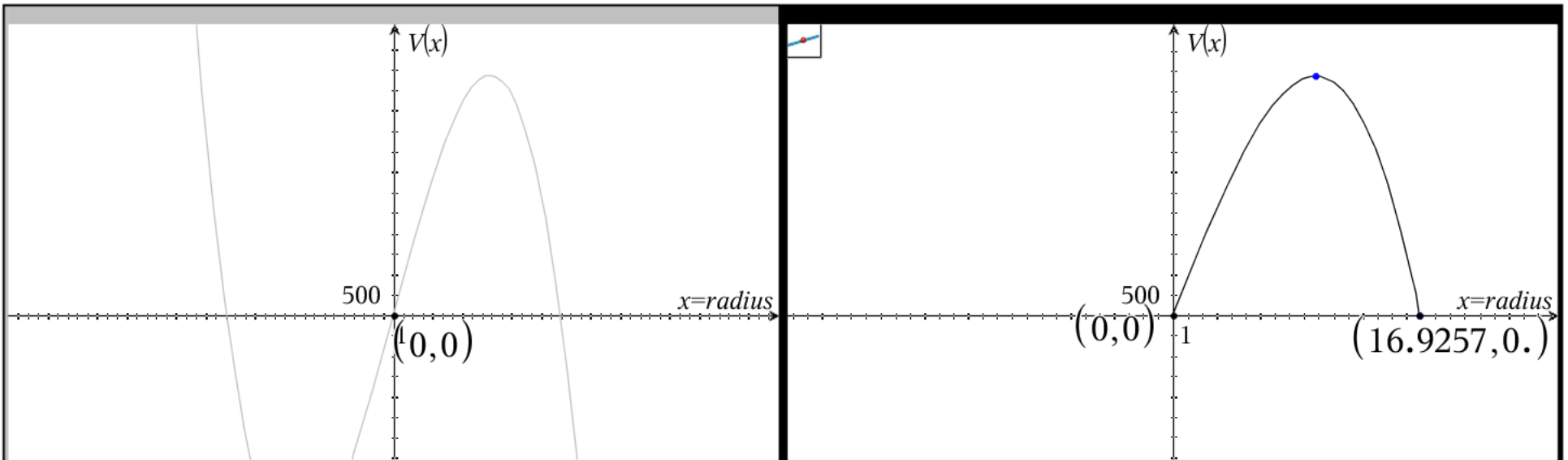
Feasibility of h

$$0 < h < \sqrt{\frac{900.}{\pi}}$$

$$0 < h < 16.926$$

Step 1: Let $x = \text{radius}$ find h in terms of SA and x

Step 2: Determine where h is feasible a.k.a, a positive



$$\text{Volume Function} = \pi \cdot r^2 \cdot h$$

$$V(x) = \pi \cdot x^2 \cdot \left(\frac{900.}{\pi \cdot x} - x \right)$$

$$V(x) = 900. \cdot x - \pi \cdot x^3$$

$$V(x) = x \left(900. - \pi \cdot x^2 \right)$$

Feasible Volume function

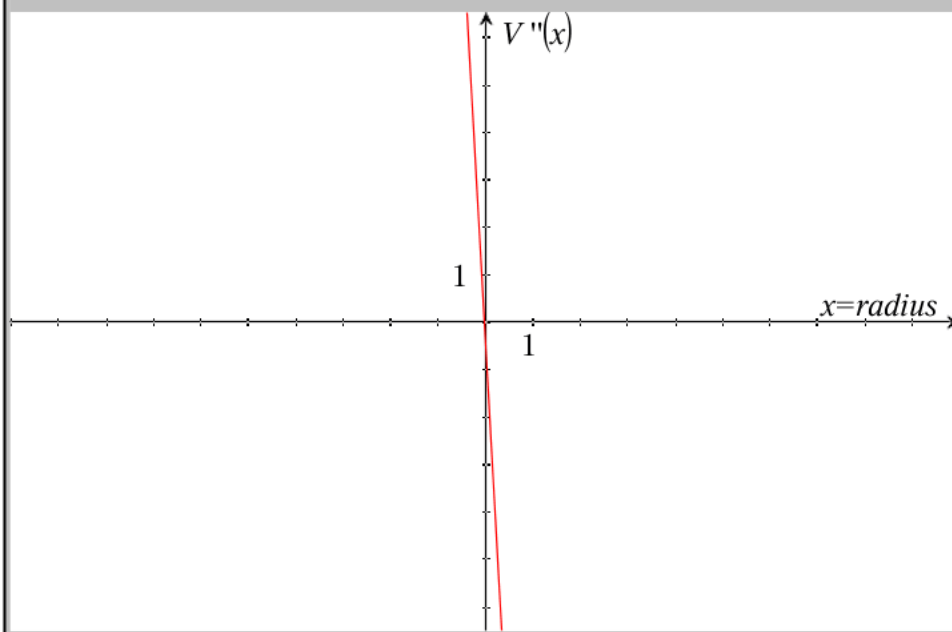
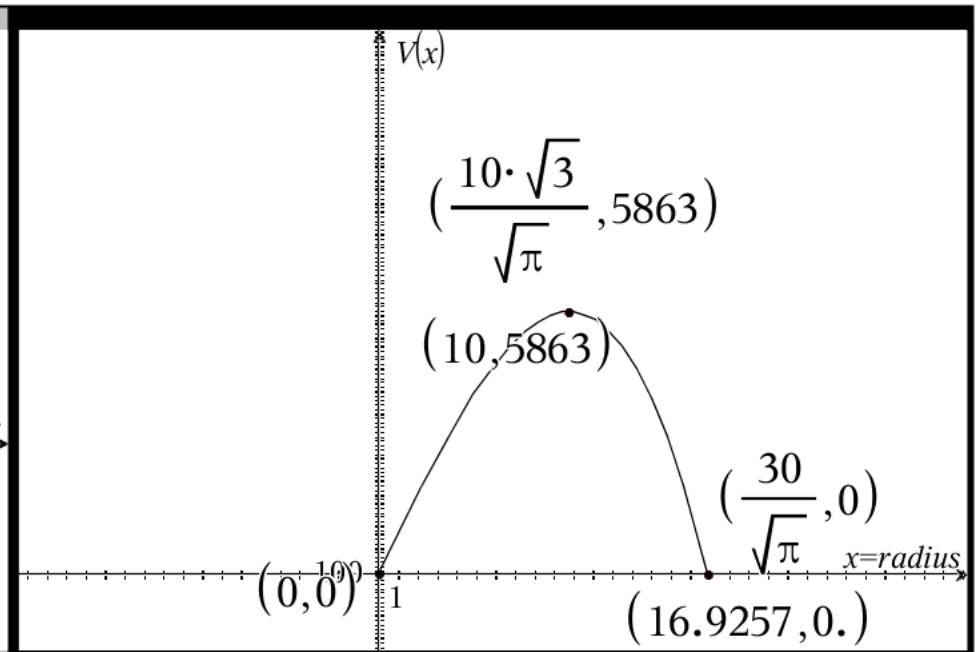
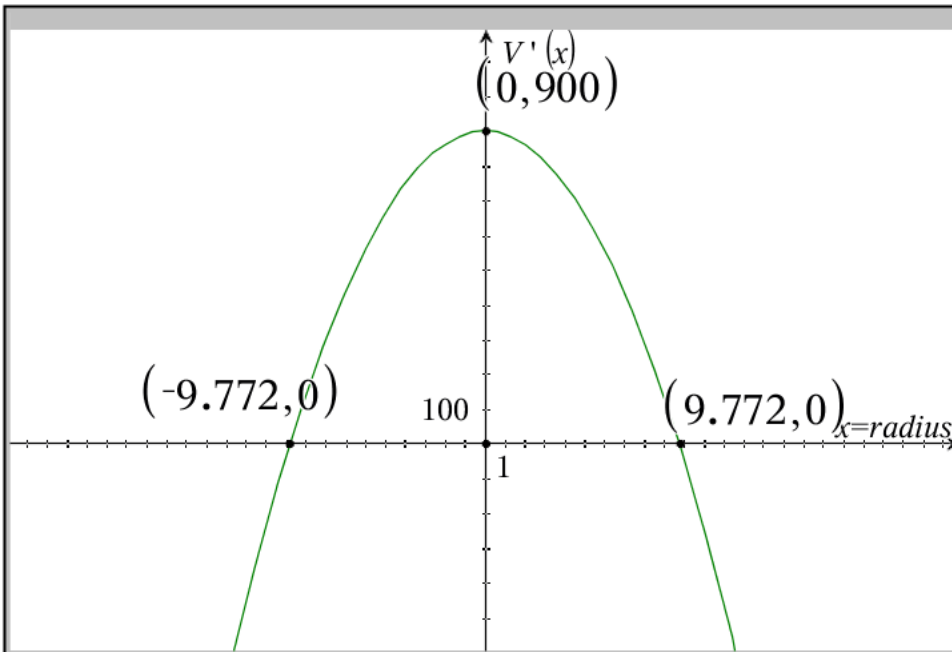
$$V(x) = 900. \cdot x - \pi \cdot x^3 \text{ for } 0 < x < 16.926$$

Feasible roots of Volume Function

$$x = 0 \quad \text{or} \quad x = \frac{30}{\sqrt{\pi}} \approx 16.926$$

NOT feasible root of Volume function

$$x = \frac{-30}{\sqrt{\pi}} \approx -16.926$$



Volume Function = $\pi \cdot r^2 \cdot h$

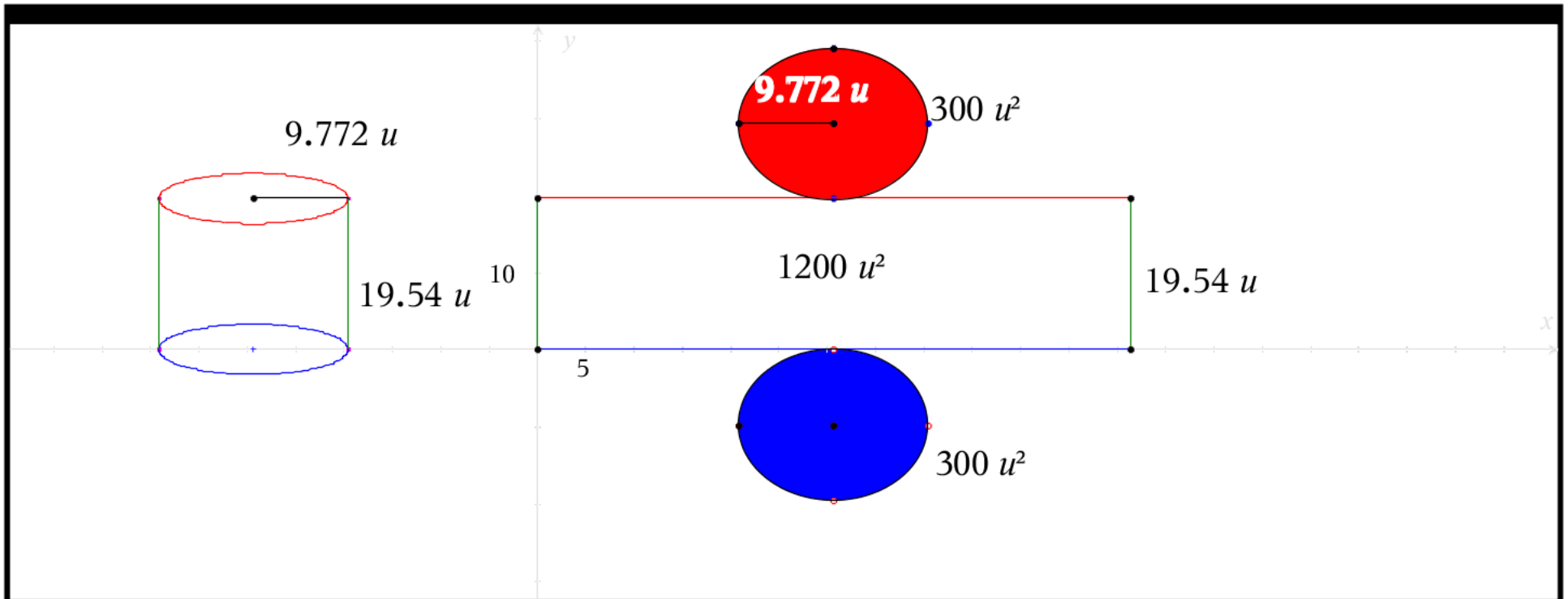
$V(x) = 900 \cdot x - \pi \cdot x^3$

Derivative of Volume function

$V'(x) = 900 - 3\pi \cdot x^2$

2nd Derivative of Volume Function'

$V''(x) = -6\pi \cdot x$

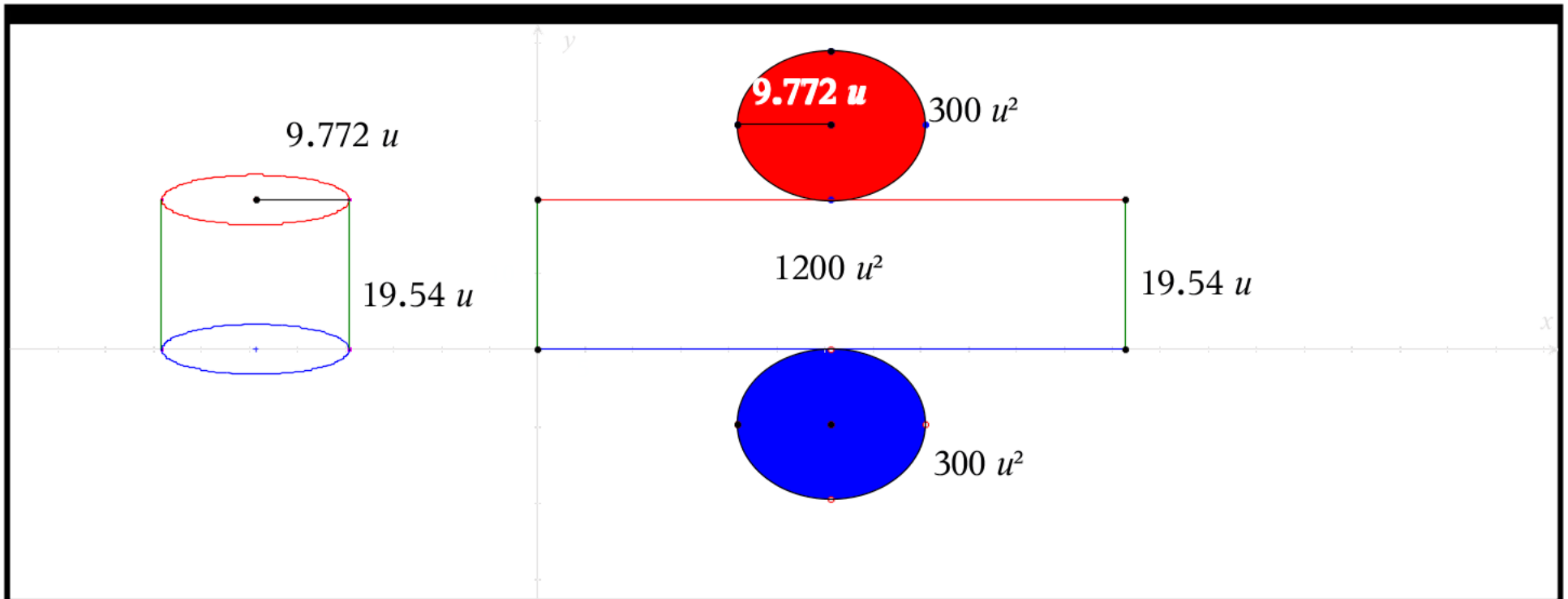


What we found!

The dimensions of the can that maximize volume of a can with surface area = 1800

$$r = x = \sqrt{\frac{1800}{6\pi}} = \frac{10 \cdot \sqrt{3}}{\sqrt{\pi}} \approx 9.772$$

$$h = \frac{1800}{2 \cdot \pi} \cdot \frac{1}{\sqrt{\frac{1800}{6\pi}}} - \sqrt{\frac{1800}{6\pi}} = \frac{20 \cdot \sqrt{3}}{\sqrt{\pi}} \approx 19.54$$



What we found!

The maximum volume of the can that maximize volume of a can with surface area of 1800

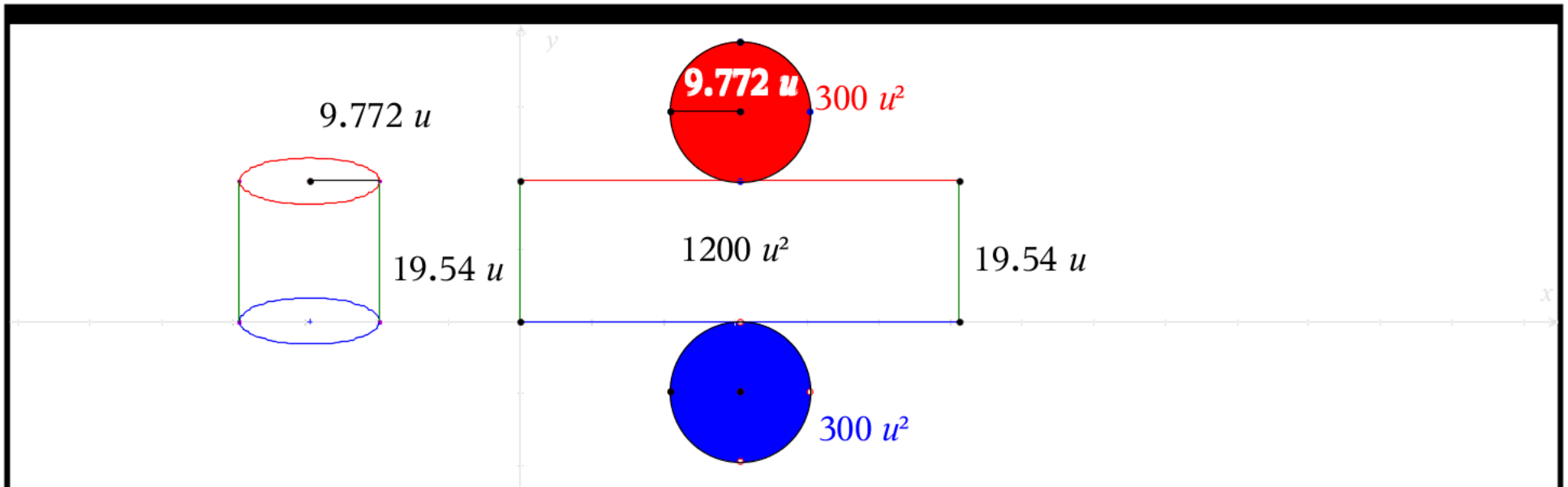
Exact Volume Approximate Volume

$$\frac{6000 \cdot \sqrt{3}}{\sqrt{\pi}}$$

5863.23

$$V = \pi \left(\frac{10 \cdot \sqrt{3}}{\sqrt{\pi}} \right)^2 \left(\frac{20 \cdot \sqrt{3}}{\sqrt{\pi}} \right)$$

$$V = 300 \cdot \frac{20 \cdot \sqrt{3}}{\sqrt{\pi}} \approx 5863.23$$



What we found! The parts of the can that maximize volume of a can with surface area of 1800

$$\begin{aligned}
 \text{radius} &= \frac{10 \cdot \sqrt{3}}{\sqrt{\pi}} \approx 9.772 & \text{Base Area} &= \pi \cdot \left(\frac{10 \cdot \sqrt{3}}{\sqrt{\pi}} \right)^2 = 300 \approx 300. \\
 \text{height} &= \frac{20 \cdot \sqrt{3}}{\sqrt{\pi}} \approx 19.54 & \text{Lateral Area} &= 2\pi \left(\frac{10 \cdot \sqrt{3}}{\sqrt{\pi}} \right) \cdot \left(\frac{20 \cdot \sqrt{3}}{\sqrt{\pi}} \right) = 1200 \approx 1200. \\
 & & \text{total surface area} &= LA + 2B = 1200 + 2(300) = 1800
 \end{aligned}$$