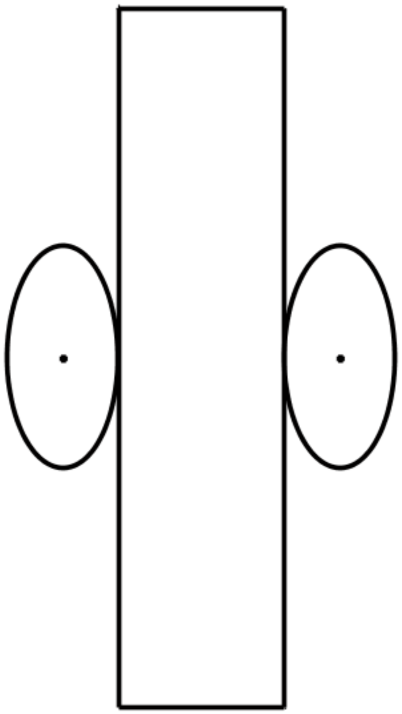


You are trying to design a can that maximizes the volume of a right cylinder a.k.a. can.

You are allowed 1200 square cm of sheet metal to create this can.

Things you need to commit to memory

SA = CH+2B = $2\cdot\pi\cdot r\cdot h+2\cdot\pi\cdot r^{2}$

SA = surface area of cylinder

C = circumference of a circle

r = radius of a circle

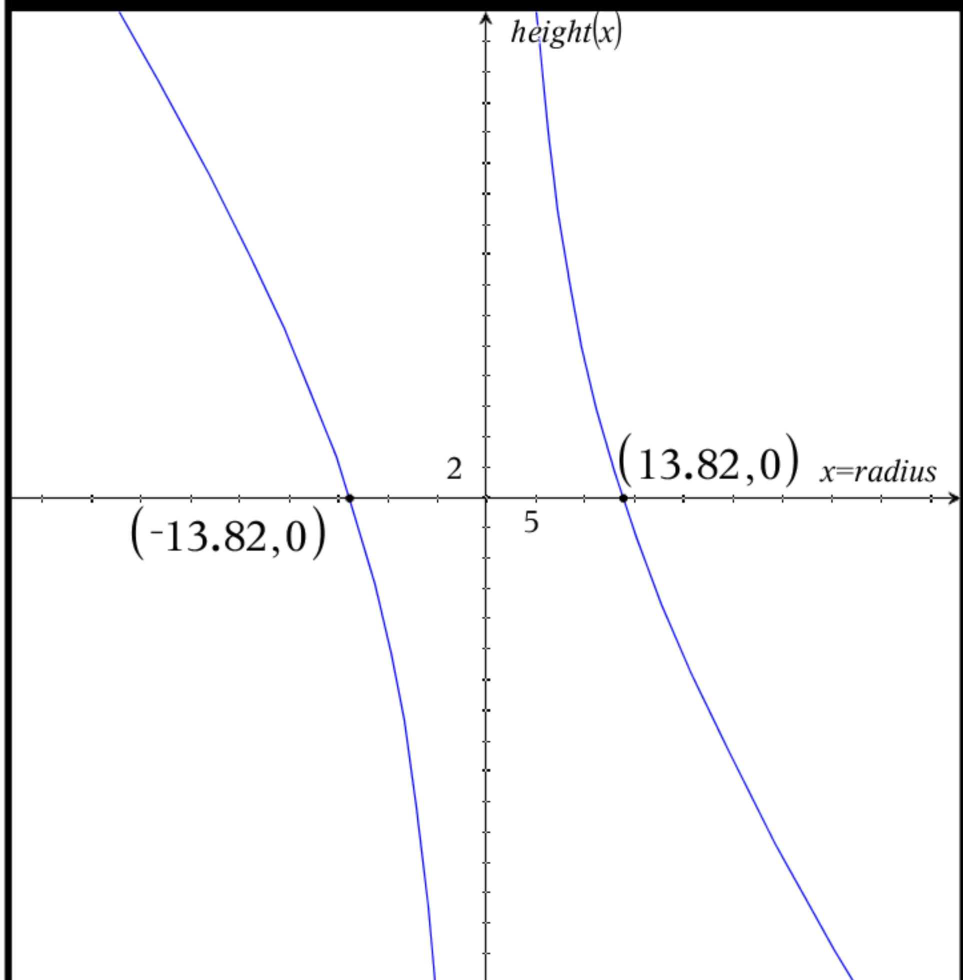
H = height of cylinder

B = Base =circle's area

V = BH = $2\cdot\pi\cdot r^{2}\cdot h$

V = volume of a cylinder

B, H, same as above




$$1200 = CH + 2B = 2\cdot\pi\cdot r\cdot h + 2\cdot\pi\cdot r^2$$

$$1200 = 2\cdot\pi\cdot x\cdot h + 2\cdot\pi\cdot x^2$$

$$1200 - 2\cdot\pi\cdot x^2 = 2\cdot\pi\cdot x\cdot h$$

$$h(x) = \frac{1200 - 2\pi\cdot x^2}{2\pi\cdot x} = \frac{1200}{2\pi\cdot x} - \frac{2\pi\cdot x^2}{2\pi\cdot x}$$

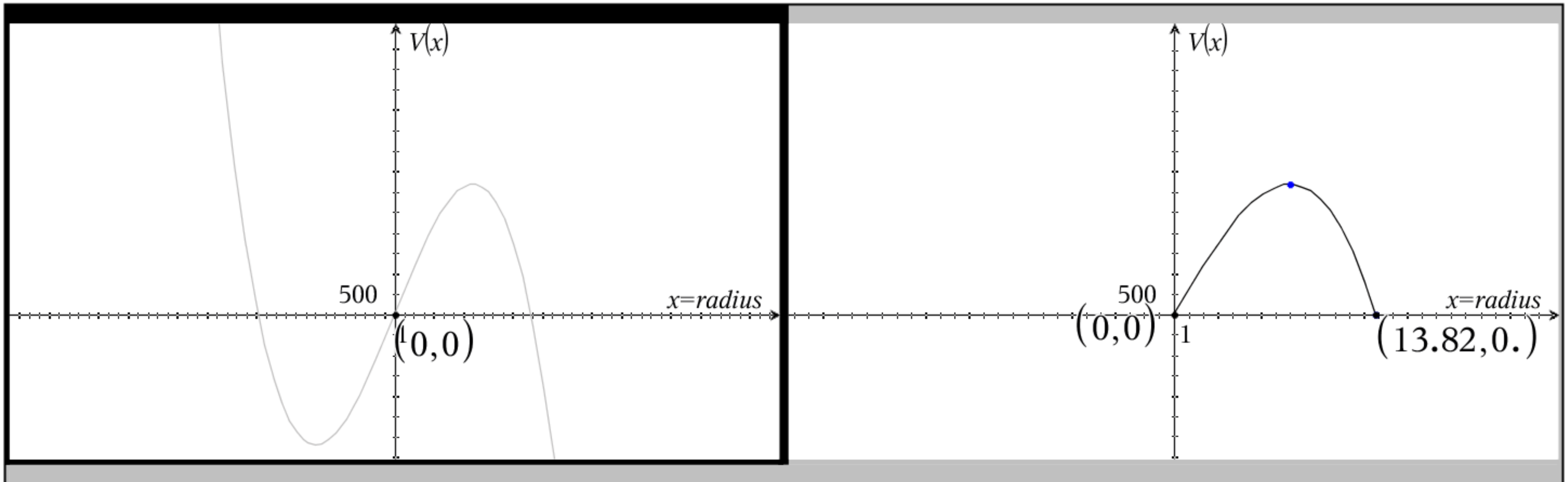
$$h(x) = \frac{600.}{\pi\cdot x} - x$$

*Feasibility of h*

$$0 < h < \sqrt{\frac{600.}{\pi}} \qquad 0 < h < 13.82$$

Step 1: Let x = radius find h in terms of SA and x

Step 2: Determine where h is feasible a.k,a positive




*Volume Function* $= \pi \cdot r^2 \cdot h$

$$V(x) = \pi \cdot x^2 \cdot \left( \frac{600.}{\pi \cdot x} - x \right)$$

$$V(x) = 600.\ x - \pi \cdot x^3$$

$$V(x) = x\left( 600. - \pi \cdot x^2 \right)$$

*Feasible Volume function*

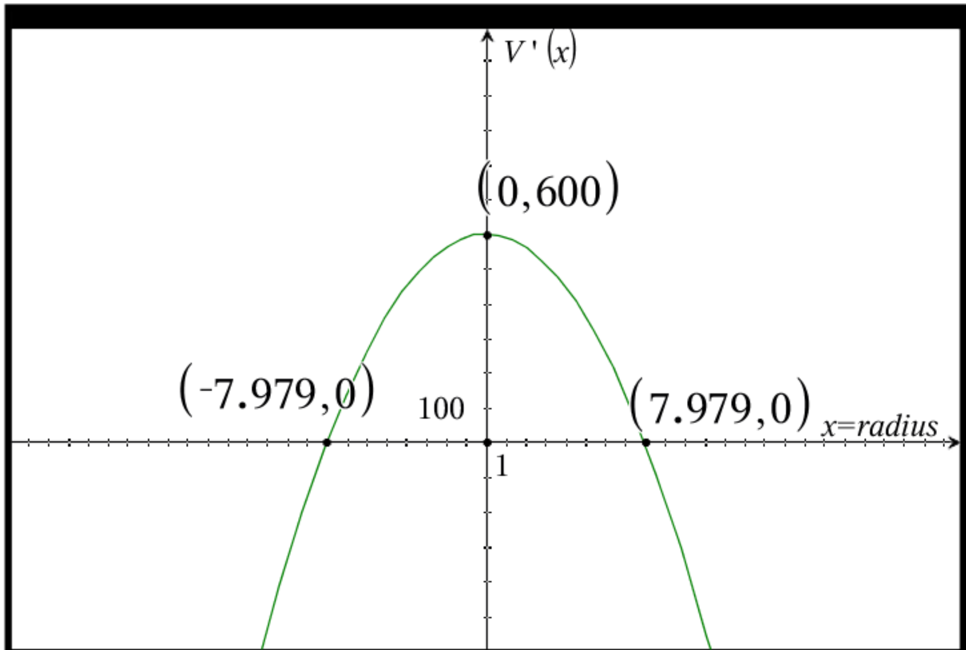
$V(x) = 600.\ x - \pi \cdot x^3$ for $0 < x < 13.82$

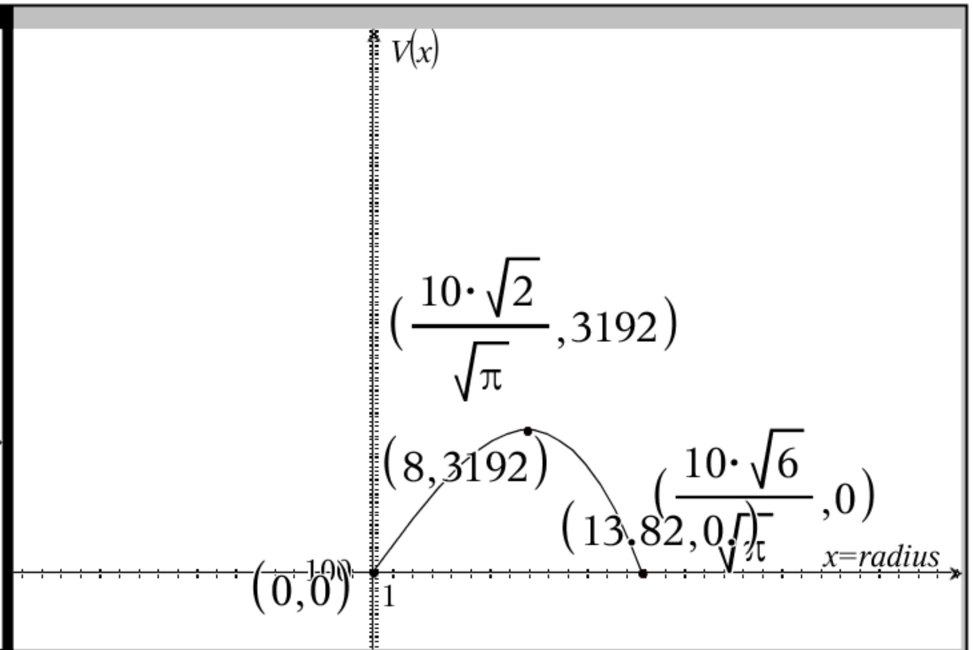
*Feasible roots of Volume Function*

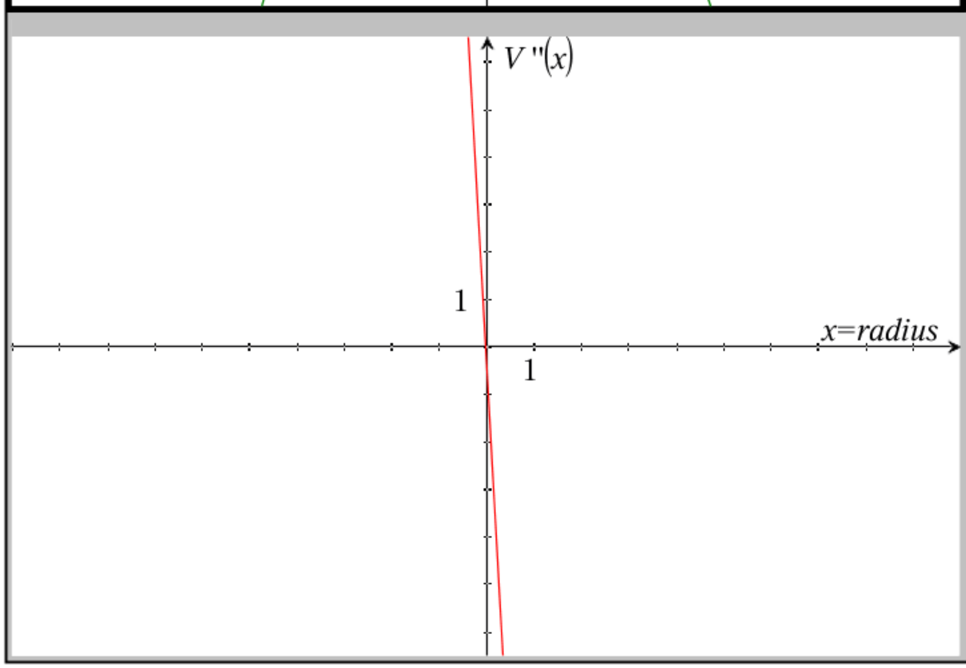
$x = 0$ or $x = \dfrac{10 \cdot \sqrt{6}}{\sqrt{\pi}} \approx 13.82$

NOT *feasible* root *of Volume function*

$x = \dfrac{-10 \cdot \sqrt{6}}{\sqrt{\pi}} \approx -13.82$




$Volume\ Function = \pi \cdot r^2 \cdot h$

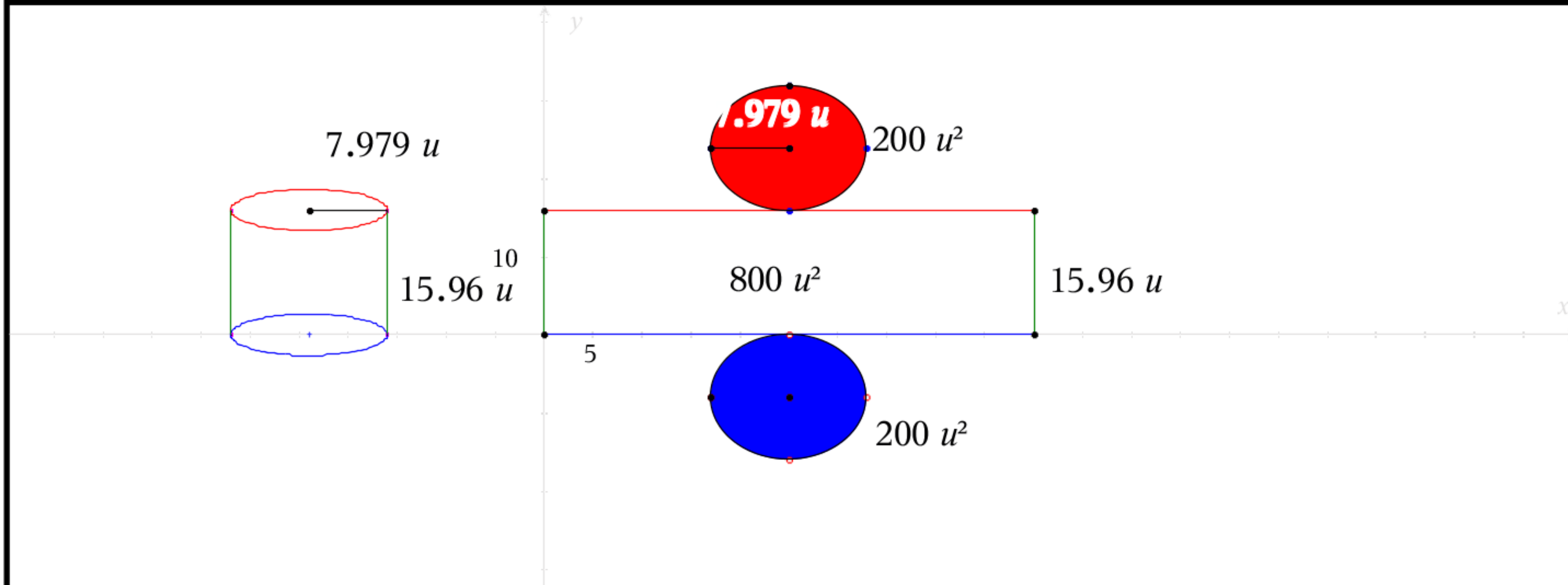
$V(x) = 600.\ x - \pi \cdot x^3$

Derivative *of Volume function*

$V\ '\ (x) = 600. - 3\pi \cdot x^2$

*2nd* Derivative *of Volume Function'*

$V\ ''\ (x) = -6\pi \cdot x$

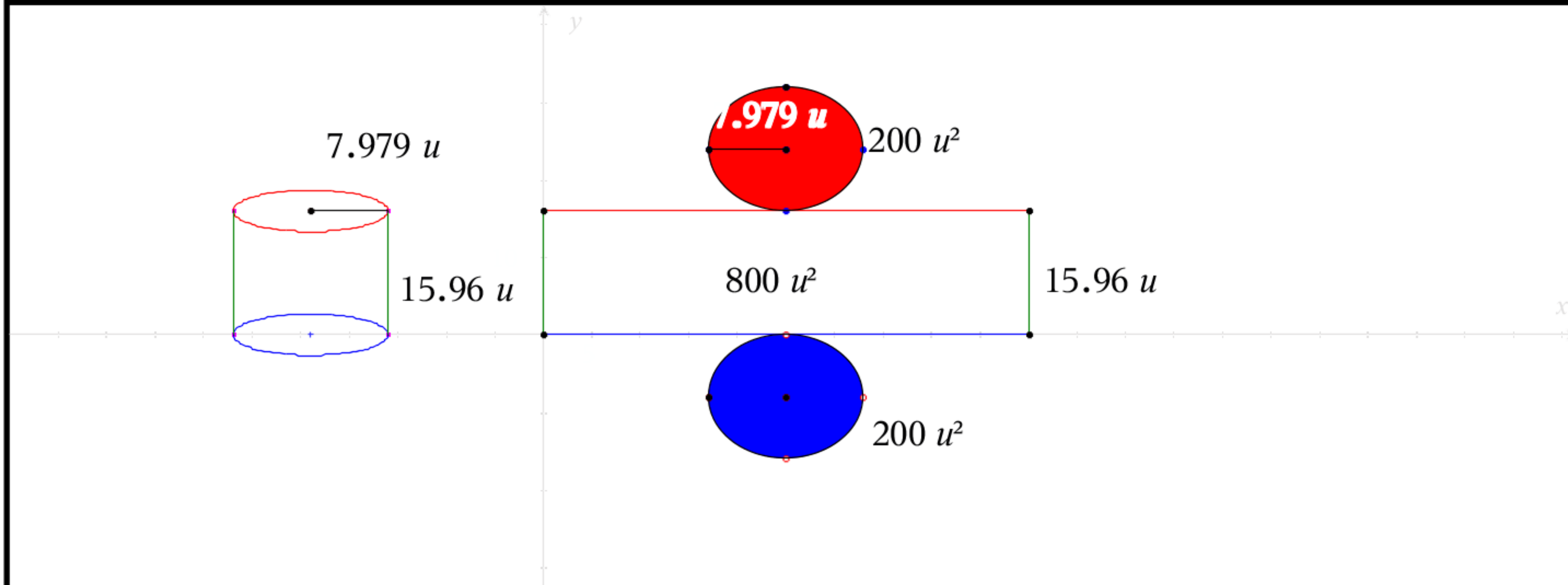



*What we found!*

*The dimensions of the can that maximize volume of a can with surface area=* 1200

$$r = x = \sqrt{\frac{1200}{6\pi}} = \frac{10\cdot\sqrt{2}}{\sqrt{\pi}} \approx 7.979$$

$$h = \frac{1200}{2\cdot\pi}\cdot\frac{1}{\sqrt{\frac{1200}{6\pi}}} - \sqrt{\frac{1200}{6\pi}} = \frac{20\cdot\sqrt{2}}{\sqrt{\pi}} \approx 15.96$$




*What we found*!

*The maximum volume of the can that maximize volume of a can with surface area of* 1200

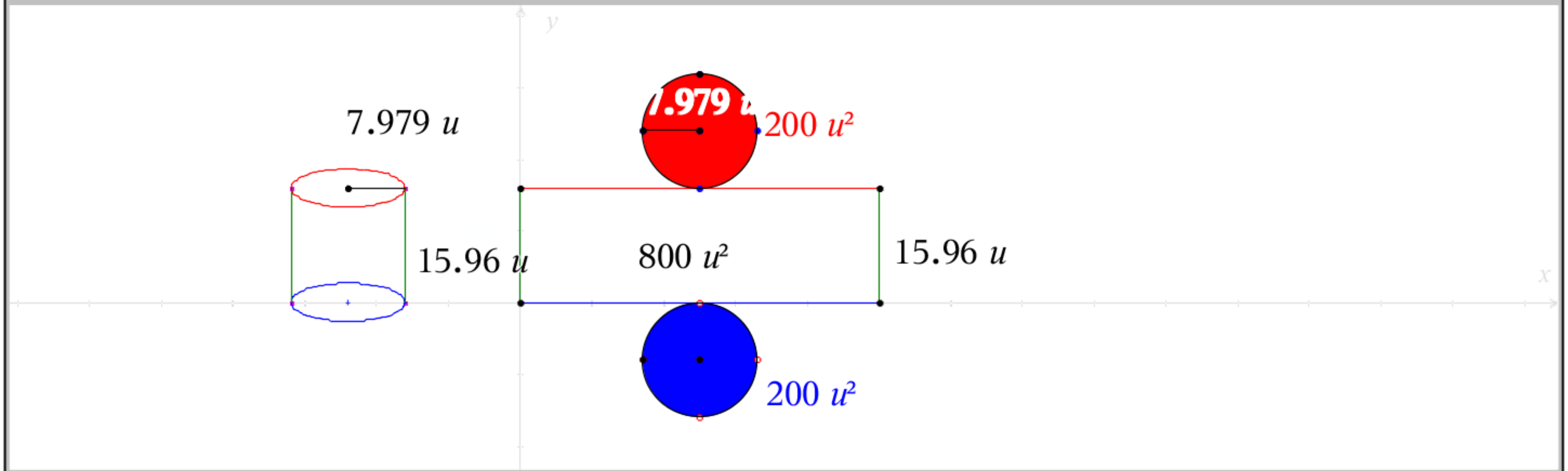
Exact *Volume* *Approximate Volume*

$$V=\pi\left(\frac{10\cdot\sqrt{2}}{\sqrt{\pi}}\right)^2\left(\frac{20\cdot\sqrt{2}}{\sqrt{\pi}}\right)$$

3192

3191.538

$$V = 200\cdot\frac{20\cdot\sqrt{2}}{\sqrt{\pi}} \approx 3191.538$$




*What we found!* *The parts of the can that maximize volume of a can with surface area of* 1200

*Base Area* $=\pi\cdot\left(\dfrac{10\cdot\sqrt{2}}{\sqrt{\pi}}\right)^2=$ 200 ≈ 200.

*radius*$=\dfrac{10\cdot\sqrt{2}}{\sqrt{\pi}}$ ≈ 7.979

*Lateral Area* $=2\pi\left(\dfrac{10\cdot\sqrt{2}}{\sqrt{\pi}}\right)\cdot\left(\dfrac{20\cdot\sqrt{2}}{\sqrt{\pi}}\right)=$ 800 ≈ 800.

*height*$=\dfrac{20\cdot\sqrt{2}}{\sqrt{\pi}}$ ≈ 15.96

*total surface area* = *LA* +2*B* = 800 +2( 200 )= 1200