

Logarithmic Equations

Solve each equation.

1) $\log 5x = \log (2x + 9)$

{3} $5x = 2x + 9$
 $x = 3$

2) $\log (10 - 4x) = \log (10 - 3x)$

{0} $10 - 4x = 10 - 3x$
 $0 = x$

3) $\log (4p - 2) = \log (-5p + 5)$

{ $\frac{7}{9}$ } $4p - 2 = -5p + 5$
 $9p = 7$
 $p = \frac{7}{9}$

4) $\log (4k - 5) = \log (2k - 1)$

{2} $4k - 5 = 2k - 1$
 $2k = 4$
 $k = 2$

5) $\log (-2a + 9) = \log (7 - 4a)$

{-1} $-2a + 9 = 7 - 4a$
 $2a = -2$
 $a = -1$

6) $2\log_7 -2r = 0$

{ $-\frac{1}{2}$ } $\log_7 -2r = 0$
 $7^0 = -2r$
 $1 = -2r$
 $-\frac{1}{2} = r$

7) $-10 + \log_3 (n + 3) = -10$

{-2} $\log_3 n + 3 = 0$
 $3^0 = n + 3$
 $1 = n + 3$
 $-2 = n$

8) $-2\log_5 7x = 2$

{ $\frac{1}{35}$ } $\log_5 7x = -1$
 $5^{-1} = 7x$
 $\frac{1}{5} = 7x$
 $\frac{1}{35} = x$

9) $\log -m + 2 = 4$

{-100} $\log -m = 2$
 $10^2 = -m$
 $-100 = m$

10) $-6\log_3 (x - 3) = -24$

{84} $\log_3 x - 3 = 4$
 $3^4 = x - 3$
 $81 = x - 3$
 $84 = x$

11) $\log_{12} (v^2 + 35) = \log_{12} (-12v - 1)$

{-6} $v^2 + 35 = -12v - 1$
 $v^2 + 12v + 36 = 0$
 $(v + 6)^2 = 0$
 $v = -6$

12) $\log_9 (-11x + 2) = \log_9 (x^2 + 30)$

{-7, -4} $-11x + 2 = x^2 + 30$
 $0 = x^2 + 11x + 28$
 $0 = (x + 7)(x + 4)$
 $x = -7 \quad x = -4$

13) $\log(16 + 2b) = \log(b^2 - 4b)$

$\{8, -2\}$

$16 + 2b = b^2 - 4b$

$0 = b^2 - 6b - 16$

$0 = (b-8)(b+2)$

$b = 8 \quad b = -2$

14) $\ln(n^2 + 12) = \ln(-9n - 2)$

$\{-2, -7\}$

$n^2 + 12 = -9n - 2$

$n^2 + 9n + 14 = 0$

$(n+7)(n+2) = 0$

$n = -7 \quad n = -2$

15) $\log x + \log 8 = 2$

$\left\{\frac{25}{2}\right\}$

$\log 8x = 2$

$10^2 = 8x$

$100 = 8x$

17) $\log 2 + \log x = 1$

$\{5\}$

$\log 2x = 1$

$10^1 = 2x$

$5 = x$

16) $\log x - \log 2 = 1$

$\{20\}$

$\log \frac{x}{2} = 1$

$10^1 = \frac{x}{2}$

$20 = x$

18) $\log x + \log 7 = \log 37$

$\left\{\frac{37}{7}\right\}$

$\log 7x = \log 37$

$7x = 37$

$x = \frac{37}{7}$

19) $\log_8 2 + \log_8 4x^2 = 1$

$\{1, -1\}$

$\log_8 8x^2 = 1$

$8^1 = 8x^2$

$1 = x^2$

$\pm 1 = x$

20) $\log_9(x+6) - \log_9 x = \log_9 2$

$\{6\}$

$\log_9 \frac{x+6}{x} = \log_9 2$

$\frac{x+6}{x} = 2$

$x+6 = 2x$

$6 = x$

21) $\log_6(x+1) - \log_6 x = \log_6 29$

$\left\{\frac{1}{28}\right\}$

$\log_6 \frac{x+1}{x} = \log_6 29$

$\frac{x+1}{x} = 29$

$x+1 = 29x$

$\frac{1}{28} = x$

22) $\log_5 6 + \log_5 2x^2 = \log_5 48$

$\{2, -2\}$

$\log_5 12x^2 = \log_5 48$

$12x^2 = 48$

$x^2 = 4$

$x = \pm 2$

23) $\ln 2 - \ln(3x+2) = 1$

$\left\{\frac{2-2e}{3e}\right\}$

$\ln \frac{2}{3x+2} = 1$

$e^1 = \frac{2}{3x+2}$

$3ex + 2e = 2$

$3ex = \frac{2-2e}{e}$

24) $\ln(-3x-1) - \ln 7 = 2$

$\left\{\frac{-7e^2-1}{3}\right\}$

$\ln \frac{-3x-1}{7} = 2$

$e^2 = \frac{-3x-1}{7}$

$7e^2 = -3x-1$

$\frac{7e^2+1}{-3} = \frac{-3x-1}{-3}$

25) $\ln(x-3) - \ln(x-5) = \ln 5$

$\left\{\frac{11}{2}\right\}$

$\ln \frac{x-3}{x-5} = \ln 5$

$\frac{x-3}{x-5} = 5$

$x-3 = 5x-25$
 $22 = 4x$
 $\frac{11}{2} = x$

26) $\ln(4x+1) - \ln 3 = 5$

$\left\{\frac{3e^5-1}{4}\right\}$

$\ln \frac{4x+1}{3} = 5$

$e^5 = \frac{4x+1}{3}$

$3e^5 = 4x+1$

$\frac{3e^5-1}{3} = x$