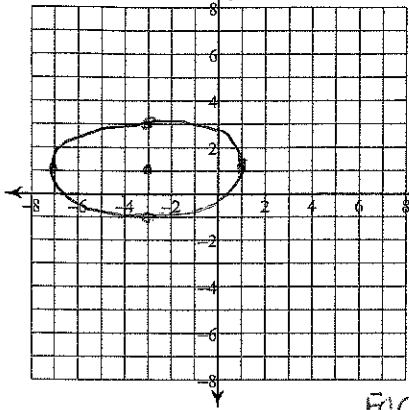


Ellipses

Identify the center, vertices, co-vertices, and foci of each. Then sketch the graph.

1) $\frac{(x+3)^2}{16} + \frac{(y-1)^2}{4} = 1$

Horizontal Major



Center: (-3, 1)

a = 4

b = 2

Vert: (1, 1)
(-7, 1)

Covert: (-3, 3)
(-3, -1)

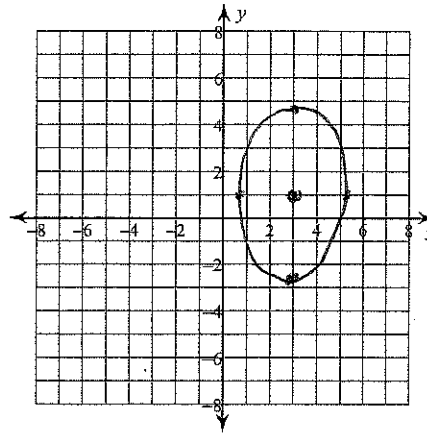
$c^2 = a^2 - b^2$

$c^2 = 16 - 4 = 12$

$c = \pm\sqrt{12} = \pm 2\sqrt{3}$

FOCI: $(-3 + 2\sqrt{3}, 1)$
 $(-3 - 2\sqrt{3}, 1)$

2) $\frac{(x-3)^2}{5} + \frac{(y-1)^2}{15} = 1$



center: (3, 1)
Vertical Major

$a = \sqrt{15}$

$b = \sqrt{5}$

Vertices: $(3, 1 + \sqrt{15})$
 $(3, 1 - \sqrt{15})$

CoVertices: $(3 + \frac{\sqrt{5}}{2}, 1)$
 $(3 - \frac{\sqrt{5}}{2}, 1)$

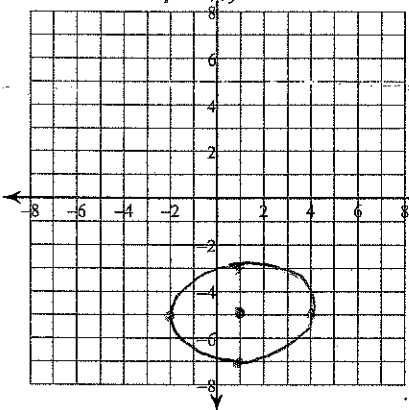
$c^2 = 15 - 5 = 10$

$c = \sqrt{10}$

FOCI: $(3, 1 + \sqrt{10})$
 $(3, 1 - \sqrt{10})$

3) $\frac{(x-1)^2}{9} + \frac{(y+5)^2}{4} = 1$

Horiz Major Axis



center: (1, -5)

a = 3

b = 2

Vert: (4, -5)
(-2, -5)

Covert: (1, -3)
(1, -7)

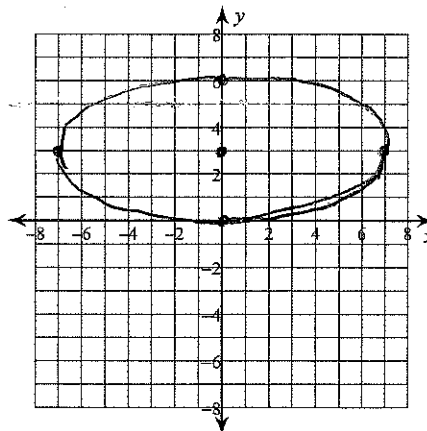
$c^2 = a^2 - b^2$

$c^2 = 9 - 4 = 5$

$c = \pm\sqrt{5}$

FOCI: $(1 + \sqrt{5}, -5)$
 $(1 - \sqrt{5}, -5)$

4) $\frac{x^2}{49} + \frac{(y-3)^2}{9} = 1$



center: (0, 3)
Horiz. Major Axis

a = 7

b = 3

Vertices: (7, 3)
(-7, 3)

Covert: (0, 6) (0, 0)

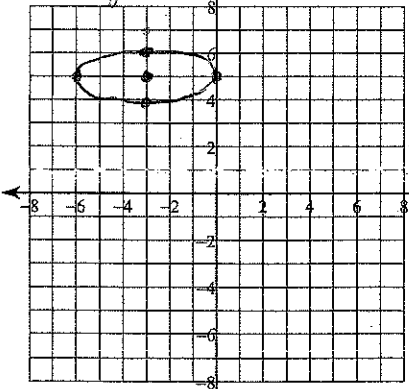
$c^2 = 49 - 9 = 40$

$c = \pm 2\sqrt{10}$

FOCI: $(2\sqrt{10}, 3)$ $(-2\sqrt{10}, 3)$

5) $x^2 + 9y^2 + 6x - 90y + 225 = 0$

Horiz.



center: (-3, 5)

a = 3

b = 1

Vert: (0, 5) (-6, 5)

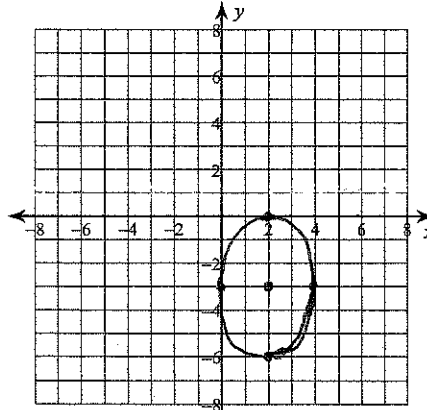
Covert: (-3, 6)
(-3, 4)

$c^2 = 9 - 1 = 8$

$c = \pm\sqrt{8} = \pm 2\sqrt{2}$

FOCI: $(-3 + 2\sqrt{2}, 5)$
 $(-3 - 2\sqrt{2}, 5)$

6) $9x^2 + 4y^2 - 36x + 24y + 36 = 0$



$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$

center: (2, -3)
Vertical Major

$a = 3$
 $b = 2$

Vertices: (2, 0) (2, -6)

Covert: (4, -3) (0, -3)

$c^2 = 9 - 4 = 5$ $c = \pm\sqrt{5}$

FOCI: $(2, -3 + \sqrt{5})$
 $(2, -3 - \sqrt{5})$

$x^2 + 6x + 9 + 9y^2 - 90y = -225 + 9$
 $(x+3)^2 + 9(y-5)^2 = -225 + 9 + 225$
 $\frac{(x+3)^2}{9} + \frac{(y-5)^2}{1} = 1$

$9x^2 - 36x + 4y^2 + 24y = -36$
 $9(x^2 - 4x + 4) + 4(y^2 + 6y + 9) = -36 + 36 + 36$
 $9(x-2)^2 + 4(y+3)^2 = 36$

Identify the center, vertices, co-vertices, foci, length of the major axis, length of the minor axis, length of the latus rectum, and eccentricity of each.

7) $3x^2 + 35y^2 - 60x + 140y - 85 = 0$

$$3x^2 - 60x + 35y^2 + 140y = 85$$

$$3(x^2 - 20x + 100) + 35(y^2 + 4y + 4) = 85 + 300 + 140$$

$$\frac{3(x-10)^2}{525} + \frac{35(y+2)^2}{525} = \frac{525}{525}$$

$$\frac{(x-10)^2}{175} + \frac{(y+2)^2}{15} = 1$$

$c^2 = 175 - 15 = 150$
 $c = \sqrt{150} = 5\sqrt{6}$
FOCI: $(10 \pm 5\sqrt{6}, -2)$

$e = \frac{\sqrt{150}}{\sqrt{175}} = \frac{\sqrt{6}}{\sqrt{7}} \approx .92$

Horiz. $a = \sqrt{175} = 5\sqrt{7}$

Vert: $(10 + 5\sqrt{7}, -2)$ Co-Vert: $(10, -2 + \sqrt{15})$
 $(10, -2 - \sqrt{15})$
 $(10 - 5\sqrt{7}, -2)$ Length major: $10\sqrt{7}$ minor: $2\sqrt{15}$

8) $36x^2 + 5y^2 - 90y - 495 = 0$

$$36x^2 + 5y^2 - 90y = 495$$

$$36x^2 + 5(y^2 - 18y + 81) = 495 + 405$$

$$\frac{36x^2}{900} + \frac{5(y-9)^2}{900} = \frac{900}{900}$$

$$\frac{x^2}{25} + \frac{(y-9)^2}{180} = 1$$

$e = \frac{\sqrt{155}}{\sqrt{180}} \approx .93$

Center: $(0, 9)$
 Vertical $a = \sqrt{180} = 6\sqrt{5}$ $b = 5$
 Vertices: $(0, 9 + 6\sqrt{5})$ covert: $(5, 9)$
 $(0, 9 - 6\sqrt{5})$ $(-5, 9)$
FOCI: $(0, 9 \pm \sqrt{155})$

Use the information provided to write the standard form equation of each ellipse.

9) Vertices: $(6, -6), (-10, -6)$ major axis = 16 $a = 8$
 Foci: $(-2 + 2\sqrt{7}, -6), (-2 - 2\sqrt{7}, -6)$ $c = 2\sqrt{7}$
 $c^2 = a^2 - b^2$ $b^2 = a^2 - c^2 = 64 - 28 = 36$
 Center $(-2, -6)$
 $\frac{(x+2)^2}{64} + \frac{(y+6)^2}{36} = 1$

10) Vertices: $(13, 9), (-3, 9)$ major = 16 $a = 8$
 Foci: $(5 + 2\sqrt{7}, 9), (5 - 2\sqrt{7}, 9)$ $c = 2\sqrt{7}$
 Center: $(5, 9)$
 $\frac{(x-5)^2}{64} + \frac{(y-9)^2}{36} = 1$
 $c^2 = a^2 - b^2$
 $b^2 = a^2 - c^2$
 $b^2 = 64 - 28$
 $b^2 = 36$

11) Vertices: $(5, 9), (-13, 9)$ major axis = 18 $a = 9$
 Co-vertices: $(-4, 14), (-4, 4)$ minor = 10 $b = 5$
 Center $(-4, 9)$
 $\frac{(x+4)^2}{81} + \frac{(y-9)^2}{25} = 1$

Foci: $(3, 10 + \sqrt{105}), (3, 10 - \sqrt{105})$ $c = \sqrt{105}$
 Co-vertices: $(11, 10), (-5, 10)$
 Center $(3, 10)$ minor axis length
 $\frac{(x-3)^2}{64} + \frac{(y-10)^2}{169} = 1$
 $2b = 16$ $b = 8$ (w/x)
 $c^2 = a^2 - b^2$
 $b^2 + c^2 = a^2$
 $64 + 105 = a^2$
 $169 = a^2$

13) Foci: $(\frac{4\sqrt{35} + 7}{2}, \frac{3}{2}), (\frac{-4\sqrt{35} + 7}{2}, \frac{3}{2})$
 Endpoints of minor axis: $(\frac{7}{2}, \frac{7}{2}), (\frac{7}{2}, -\frac{1}{2})$
 Center $(\frac{7}{2}, \frac{3}{2})$ $c = 2\sqrt{35}$ $b = 2$
 $c^2 + b^2 = a^2$
 $140 + 4 = a^2$
 $144 = a^2$
 $\frac{(x-\frac{7}{2})^2}{144} + \frac{(y-\frac{3}{2})^2}{4} = 1$

14) Center: $(-8, 5)$ $a = 10$ (w/ y's)
 Vertex: $(-8, 15)$
 Focus: $(-8, 5 + \sqrt{51})$ $c = \sqrt{51}$
 $\frac{(x+8)^2}{49} + \frac{(y-5)^2}{100} = 1$
 $c^2 = a^2 - b^2$
 $b^2 = 100 - 51 = 49$

15) Endpoints of major axis: $(-4, -6), (-16, -6)$ major = 12 $a = 6$
 Endpoints of minor axis: $(-10, -2), (-10, -10)$ minor = 8 $b = 4$
 Center $(-10, -6)$
 $\frac{(x+10)^2}{36} + \frac{(y+6)^2}{16} = 1$

16) Eccentricity = $\frac{\sqrt{15}}{4} = \frac{c}{a}$ $\frac{\sqrt{15}}{4} a = c$ $b = 3$
 Center: $(-5, 5)$
 Co-vertex: $(-8, 5)$
 $c^2 = a^2 - b^2$
 $(\frac{\sqrt{15}}{4} a)^2 = a^2 - 9$
 $\frac{15}{16} a^2 = a^2 - 9$
 $9 = \frac{1}{16} a^2$
 $144 = a^2$ (w/ y's)
 $\frac{(x+5)^2}{9} + \frac{(y-5)^2}{144} = 1$