Day 1: Solving Logarithmic Equations

SWBAT: Solve Equations Involving Logs

Warm - Up

Solve: $16^{x-1} = 8^x$

A) $x = \frac{1}{4}$

B) $x = -1$

C) $x = \frac{4}{3}$

D) $x = 4$

In chapter 7, we solved exponential equations by writing each side of the equation to the same base. Often that is possible only by using logarithms.

Since $f(x) = \log_b x$ is a function:

► **If $x_1 = x_2$, then $\log_b x_1 = \log_b x_2$.**

For example, solve $8^x = 32$ for x. There are two possible methods.

METHOD 1

Write each side of the equation to the base 2.

$$8^x = 32$$
$$(2^3)^x = 2^5$$
$$3x = 5$$
$$x = \frac{5}{3}$$

Check: $8^{\frac{5}{3}} = (\sqrt[3]{8})^5 = 2^5 = 32$ ✔

METHOD 2

Take the log of each side of the equation and solve for the variable.

$$8^x = 32$$
$$\log 8^x = \log 32$$
$$x \log 8 = \log 32$$
$$x = \frac{\log 32}{\log 8}$$

ENTER: LOG 32) ÷ LOG 8 ENTER MATH ENTER ENTER

DISPLAY:

```
LOG(32)/LOG(8
          1.666666667
ANS►FRAC
                  5/3
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Concept 1: Taking the log of both sides to solve an exponential equation

(Can't change to the same base)

Solve for the variable in each problem.
If there is some reason the base is not isolated before you begin, ISOLATE the base first before you take the log of both sides.

Solve each of the following equations to the nearest tenth:

1. $3^x = 31$	**2.** $2^x + 3 = 102$
3. $7^{n+7} + 3 = 90$	**4.** $4^{x+2} - 2 = 12$
5. $2^{x+1} = 3^{1-x}$	**6.** $e^{x-1} = 6$

Solve the equation using two different methods: $3^{4x-2} = 27^{x+2}$

Method 1: Common base	*Method 2: Logarithms*

Concept 2: Equations with more than one log on one side.

If an equation has more than one "log," use the rules for log distribution to CONDENSE the logs on a side, and then proceed as you would otherwise. Usually this will involve converting the equation to exponential form.

Solve each of the following equations.

1. $\log 9 + \log x = 1$	2. $\ln (x + 2) + \ln 2 = 3$
3. $\log_6 x - \log_6 (x-3) = 1$	4. $\log_5 9 - \log_5 (x+10) = 1$
5. $\log (3x + 7) = 0$	6. $\log_4 (x^2 + 3x) - \log_4 (x + 5) = 1$

****When solving logarithmic equations, we need to check for extraneous solutions.*

7. $\log (x+2) + \log (x + 5) = 1$
8. $2 \ln (x+2) - \ln(-x) = 0$

Concept 3: Equations with logs on both sides of the equations

If an equation has more than one "log," use the rules for log distribution to CONDENSE the logs on one or both sides of the equation. If the bases are the same, cross out the logs and solve the remaining equation.

Solve each of the following equations:

1. $\log_8(x+2) \ \ \log_8 x = \log_8 12$	2. $log_4 x + log_4(x+6) = log_4 16$
3. $\ln 12 - \ln x = \ln 3$	4. $\log x + \log (x+5) = \log 6$
5. $\log_2 (x+2) + \log_2 (x+1) = \log_2 (x) + \log_2 (x+4)$	

Exit ticket:
Solve for all value of x:

$$\log_{x+3} \frac{x^3 + x - 2}{x} = 2$$

Summary/Closure

To solve most exponential equations:

1. Isolate the exponential expression.
2. Take log or ln of both sides.
3. Solve for the variable.

Remember:

$\log a^r = r \log a$

$(\ln a^r = r \ln a)$

To solve most logarithmic equations:

1. Isolate the logarithmic expression. (you may need to use the properties to create one logarithmic term)
2. Rewrite in exponential form (with a common base)
3. Solve for the variable.

Things to remember about logs:

$\log_b 1 = 0$	$\ln 1 = 0$
$\log_b b = 1$	$\ln e = 1$
$\log_b b^x = x$	$\ln e^x = x$

Homework: page 6

Homework #1

Solve each equation. Round your answers to the nearest ten-thousandth.

1) $5^{b+5} + 9 = 60$

2) $20^{n-3} + 3 = 101$

3) $7 \cdot 8^{x+8} = 53$

4) $6 \cdot 18^{-8x} = 70$

5) $9e^{r+3} = 10$

Solve each equation.

6) $\log_2 3 + \log_2 4x = 3$

7) $\log_4 x - \log_4 (x-3) = 3$

8) $\log_8 4x^2 - \log_8 9 = 2$

9) $\log_3 4x^2 - \log_3 10 = \log_3 10$

10) $\log_6 x + \log_6 (x+17) = \log_6 38$

11) $\log_9 6 + \log_9 -3x = 1$

12) $\log_9 2 - \log_9 4x = \log_9 31$

13) $\log_2 4x^2 - \log_2 9 = 4$

14) $\log_6 (x^2 + 8) - \log_6 2 = 2$

15) $\log_3 10 - \log_3 4x = \log_3 34$

Day 2
SWBAT solve more complicated logarithmic equations

Solve each of the following equations. Round all non – integral answers to the nearest hundredth. Be sure to check for extraneous solutions.

1. $3^{2x-3} = 2^{x+4}$

2. $5^{2+x} - 5^{x} = 10$

3. $\log_2 (x+5) = 4$

4. Drew said that the equation $\log_2[(x+1)^4] = 8$ cannot be solved because he expanded $(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$ and realized that he cannot solve the equation $x^4 + 4x^3 + 6x^2 + 4x + 1 = 2^8$. Is he correct? Explain how you know.

5. $\log((2x+5)^2) = 4$

6. $\log (x + 10) - \log (x-1) = 2$

7. $\log_2(x+1) + \log_2(x-1) = 3$

8. $\log(x^2 + 7x + 12) - \log(x + 4) = 0$

9. $2\ln(x+2) - \ln(-x) = 0$

10. $\ln(x+2) = \ln(12) - \ln(x+3)$

11. $\log(x-1) + \log(x-4) = 1$

12. $\log\left(\sqrt{(x+3)^3}\right) = \frac{3}{2}$
13. $\ln(4x^2 - 1) = 0$
14. $\log_2(x^2 - 16) - \log_2(x - 4) = 1$

Homework: pages 13 -14

Answer Key to Day 2 - Classwork

1. 4.03
2. $5^{2+x} - 5^x = 10$

$$5^x(5^2 - 1) = 10$$
$$5^x = \frac{10}{24}$$
$$x = \log_5\left(\frac{10}{24}\right)$$
$$x = \frac{\log\left(\frac{10}{24}\right)}{\log(5)}$$
$$x = \frac{\log(10) - \log(24)}{\log(5)}$$
$$x \approx -0.5440$$

3. $\log_2(x + 5) = 4$

$$2^4 = x + 5$$
$$16 = x + 5$$
$$x = 11$$

4. *If we apply the logarithmic properties, this equation is solvable.*

$$\log_2[(x + 1)^4] = 8$$
$$4\log_2(x + 1) = 8$$
$$\log_2(x + 1) = 2$$
$$x + 1 = 2^2$$
$$x = 3$$

Check: If $x = 3$, *then* $\log_2[(3 + 1)^4] = 4\log_2(4) = 4 \cdot 2 = 8$, *so 3 is a solution to the original equation.*

5. $2 \cdot \log(2x + 5) = 4$

$$\log(2x + 5) = 2$$
$$10^2 = 2x + 5$$
$$100 = 2x + 5$$
$$95 = 2x$$
$$x = \frac{95}{2}$$

Check: Since $2\left(\frac{95}{2}\right) + 5 \neq 0$, *we know that* $\log\left(\left(2 \cdot \frac{95}{2} + 5\right)^2\right)$ *is defined.*

Thus, $\frac{95}{2}$ *is the solution to the equation.*

6. $$\log\left(\frac{x+10}{x-1}\right) = 2$$

$$\frac{x+10}{x-1} = 10^2$$

$$x + 10 = 100(x-1)$$

$$99x = 110$$

$$x = \frac{10}{9}$$

Is $\frac{10}{9}$ a valid solution? Explain how you know.

- *Yes;* $\log\left(\frac{10}{9}+10\right)$ *and* $\log\left(\frac{10}{9}-1\right)$ *are both defined, so* $\frac{10}{9}$ *is a valid solution.*

Why could we not rewrite the original equation in exponential form using the definition of the logarithm immediately?

- *The equation needs to be in the form* $\log_b(Y) = L$ *before using the definition of a logarithm to rewrite it in exponential form, so we had to use the logarithmic properties to combine terms first.*

7. $$\log_2((x+1)(x-1)) = 3$$

$$\log_2(x^2-1) = 3$$

$$2^3 = x^2 - 1$$

$$0 = x^2 - 9$$

$$0 = (x-3)(x+3)$$

- Is -3 a valid solution?
  - *Because* $-3+1=-2$, $\log_2(-3+1) = \log_2(-2)$ *is undefined, so* -3 *not a valid solution. The value* -3 *is an extraneous solution, and this equation has only one solution:* 3.
- What should we look for when determining whether or not a solution to a logarithmic equation is extraneous?
  - *We cannot take the logarithm of a negative number or* 0*, so any solution that would result in the input to a logarithm being negative or* 0 *cannot be included in the solution set for the equation.*

Thus, $x = 3$ or $x = -3$. These solutions need to be checked to see if they are valid.

- Is 3 a valid solution?
  - $\log_2(3+1) + \log_2(3-1) = \log_2(4) + \log_2(2) = 2 + 1 = 3$, *so 3 is a valid solution.*

8. $$\log\left(\frac{x^2+7x+12}{x+4}\right) = 0$$

$$\frac{x^2+7x+12}{x+4} = 10^0$$

$$\frac{x^2+7x+12}{x+4} = 1$$

$$x^2+7x+12 = x+4$$

$$0 = x^2+6x+8$$

$$0 = (x+4)(x+2)$$

$$x = -4 \text{ or } x = -2$$

Check: If $x=-4$*, then* $\log(x+4) = \log(0)$*, which is undefined. Thus,* -4 *is an extraneous solution. Therefore, the only solution is* -2*.*

9.

$$\ln((x+2)^2) - \ln(-x) = 0$$

$$\ln\left(\frac{(x+2)^2}{-x}\right) = 0$$

$$1 = \frac{(x+2)^2}{-x}$$

$$-x = x^2 + 4x + 4$$

$$0 = x^2 + 5x + 4$$

$$0 = (x+4)(x+1)$$

$$x = -4 \text{ or } x = -1$$

Check: Thus, we get $x = -4$ or $x = -1$ as solutions to the quadratic equation. However, if $x = -4$, then $\ln(x+2) = \ln(-2)$, so -4 is an extraneous solution. Therefore, the only solution is -1.

10.

$$\ln(x+2) + \ln(x+3) = \ln(12)$$

$$\ln\big((x+2)(x+3)\big) = \ln(12)$$

$$(x+2)(x+3) = 12$$

$$x^2 + 5x + 6 = 12$$

$$x^2 + 5x - 6 = 0$$

$$(x-1)(x+6) = 0$$

$$x = 1 \text{ or } x = -6$$

Check: If $= -6$, then the expressions $\ln(x+2)$ and $\ln(x+3)$ are undefined. Therefore, the only valid solution to the original equation is 1.

11.

$$\log\big((x-1)(x-4)\big) = 1$$

$$\log(x^2 - 5x + 4) = 1$$

$$x^2 - 5x + 4 = 10$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x = 6 \text{ or } x = -1$$

Check: Since the left side of the equation is not defined for $x = -1$, this is an extraneous solution.

Therefore, the only valid solution is 6.

12. x = 7

13. $\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$

14. **No solution**

Day 2 Homework: Solve each of the following.

1. If $\log_2(x^2 - 1) = \log_2 8$, then the solution set for x is

1) $\{3, -3\}$
2) $\{-3\}$
3) $\{3\}$
4) $\{\ \}$

2. Solve for x: $2(4)^{2x} + 8 = 40$

3. Solve for x: $\log_5 (x+3) = 1 - \log_5(x-1)$

4. $\ln(2x-3) + \ln(x+4) = \ln(2x^2 + 11)$

5. $2\log_3 x - \log_3(x+6) = 1$

6. $\log_7(x+4) - \log_7(x-4) = \log_7 5$

7. Leave answer in terms of e:
$\ln(2x+1) = 2 - \ln x$

8. $4 \cdot (5)^{x+1} - 3 = 99$

Summary:

Algebra2/Trig: Solving Equations with Logarithms (reference)

Equation contains logs

Use log distribution rules to write equation using "log" once on each side of equations

log on **one side** of the equation

Log on **both sides** of the equation

Cross out logs and solve.

Equation contains roots

Solve each of the following:

1. $3 + \sqrt[3]{x} = 20$
2. $5\sqrt[7]{x^2} - 12 = 5$
3. $10^x = 20$
4. $12^{y-2} = 30$
5. $5^{2x} = 125^{x+3}$
6. log (3x + 1) = 5
7. log 6 – log 3x = -2
8. log 2x + log x = 11
9. log (x + 1) + log 3 = 6
10. 2 log x – log 3 = 2

Exponential form of an equation

Exponent is a variable

Exponent is a value

ISOLATE THE BASE.
Can you re-write both sides in terms of the same base?
YES
NO

Isolate variable, and raise both sides to the reciprocal power. Solve for the variable

Re-write both sides of the equation using the same base and solve

"Take the log" of both sides and solve for the variable

Answers
1. x = 4913 2. 72.473 3. X=1.301 4. X= 3.369 5. X=-9 6. X=33333 7. X=200
8. x=223606.8 9. X=333332.333 10. X=17.32

Answer Key

Day 1 HW: page 6

Answers:

1) -2.557
2) 4.5305
3) -7.0265
4) -0.1062
5) -2.8946
6) $\left\{\frac{2}{3}\right\}$
7) $\left\{\frac{64}{21}\right\}$
8) $\{12, -12\}$
9) $\{5, -5\}$
10) $\{2\}$
11) $\left\{-\frac{1}{2}\right\}$
12) $\left\{\frac{1}{62}\right\}$
13) $\{6, -6\}$
14) $\{8, -8\}$
15) $\left\{\frac{5}{68}\right\}$

Day 2 HW: pages 13 and 14

1. {3,-3}

2. x = 1

3.

$$\log_5(x+3) = 1 - \log_5(x-1)$$
$$\log_5(x+3) + \log_5(x-1) = 1$$
$$\log_5(x+3)(x-1) = 1$$
$$(x+3)(x-1) = 5^1$$
$$x^2 + 2x - 3 = 5$$
$$x^2 + 2x - 8 = 0$$
$$(x+4)(x-2) = 0$$
$$x = -4 \quad or \quad x = 2$$

4. $\ln(2x-3)+\ln(x+4)=\ln(2x^2+11)$

$\ln(2x-3)(x+4)=\ln(2x^2+11)$

$e^{\ln(2x-3)(x+4)}=e^{\ln(2x^2+11)}$

$(2x-3)(x+4)=2x^2+11$

$\cancel{2x^2}+5x-12=\cancel{2x^2}+11$

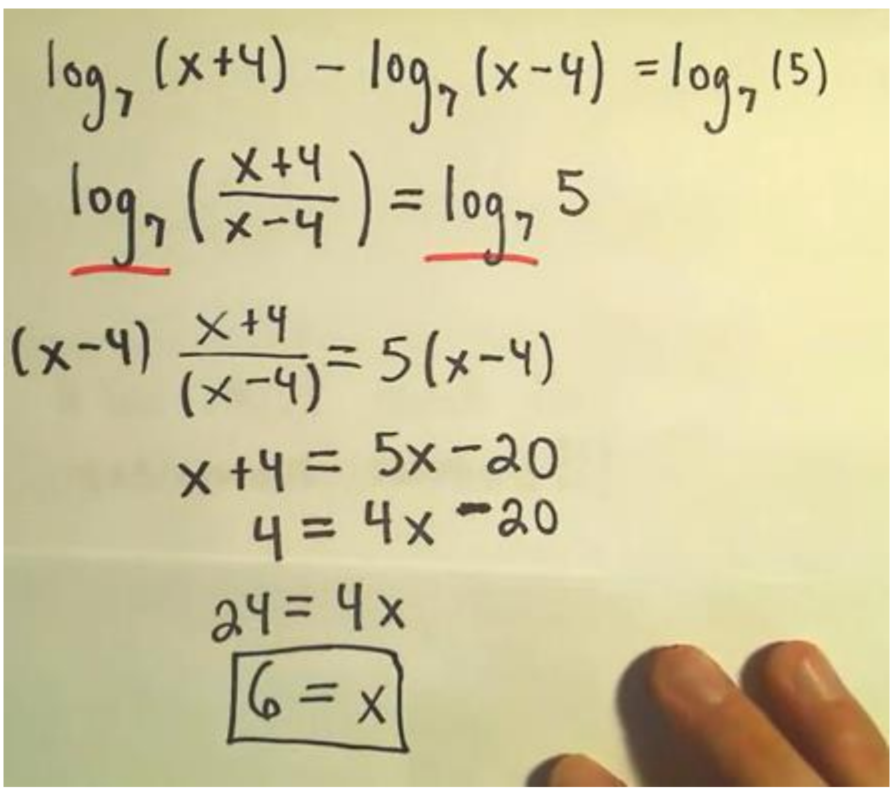
$5x-12=11$

$5x=23$

$x=\dfrac{23}{5}$

5. Answer : x = 6 (x= -3 is extraneous)

6.



7. $\ln(2x+1) = 2 - \ln x$

$$= \frac{-1 \pm \sqrt{1+8e^2}}{4}$$

8. 1.01

Solving Logarithmic Equations

Assignments

Day 1: Solving Logarithmic Equations - Pages 1 -5
HW: Page 6

Day 2: Solving Logarithmic Equations - More Practice - Pages: 7 – 12
HW: Pages 13 and 14