CP Algebra 2

Unit 5B Exponentials and Logarithms

(Book Chapter 8)

Learning Targets:

Exponential Models	1. I can apply exponential functions to real world situations.
Graphing	2. I can graph parent exponential functions and describe and graph transformations of exponential functions. 3. I can write equations for graphs of exponential functions.
Logarithms	4. I can rewrite equations between exponential and logarithm form. 5. I can write and evaluate logarithmic expressions. 6. I can graph logarithmic equations.
Operations with Logarithms	7. I can use properties of exponents to multiply, divide, and exponentiate with logarithms. 8. I can simplify and expand expressions using logarithms properties.
Solving	9. I can solve exponential and logarithm equations. 10. I can apply solving exponential and logarithm equations to real world situations.
Understanding	11. I can apply my knowledge of exponential and logarithmic functions to solve new and non-routine problems.

NAME ____________________ PERIOD _________ Teacher ___________

Exploring Exponential Models

Name ________________________________

Date: ___________

After this lesson and practice, I will be able to …

- ☐ apply exponential functions to real world situations. (LT 1)
- ☐ graph parent exponential functions and describe and graph transformations of exponential functions (LT 2a)

In the M&M activity, you discovered the formula for _________________________ functions. In today's lesson, we will continue our introduction of this important family of functions and explore how exponential functions can be used to model many real-life scenarios.

Definition 1: **Exponential Function** – The general form of an exponential function is________________

where _____ is the ______-intercept (the "starting value") and ______ is the _____________ or

______________factor.

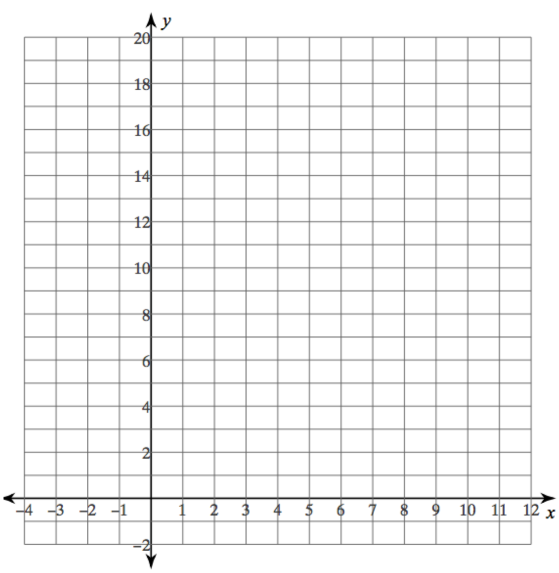
Both exponential growth and decay are modeled by this equation.

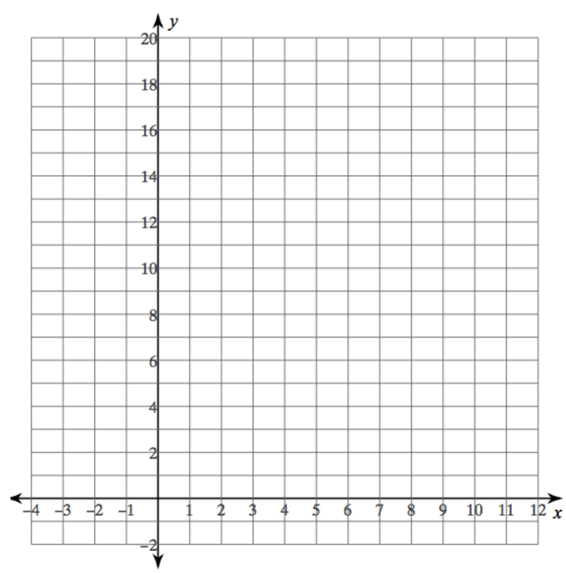
- If b > _______, then the equation models exponential ___________.

- If b < ________ (but greater than _______), then the equation models exponential ___________.

Example 1: Graph each function.

A) $y = 2^x$ B) $y = 3(2)^x$ C) $y = 20\left(\frac{1}{2}\right)^x$ D) $y = 10\left(\frac{1}{5}\right)^x$





Observe: An _______________ occurs at _________. An ______________ is a line a graph approaches as *x* or *y* approach large absolute values.

Example 2: Most automobiles depreciate as they get older. Suppose an automobile that originally costs \$14,000 depreciates by one-fifth of its value every year. What is the value of the automobile after 4 years? After 5.5 years?
Use the formula:

- Notice, the value of the car after 5.5 years is not _____________ between the values for years 5 and 6.

This is because the function is ______________________________________, not _________.

Oftentimes, rates of growth or decay are given in the form of _______________________. When this is the case, you can represent the growth or decay factor by _________________________ if *r* is a percent increase or ________________________ if *r* is a percent decrease.

Example 3: Given the percent growth or decay (where + indicates growth, and – indicates decay), find r (expressed as a decimal) and b, the growth/decay factor:

+30%	–75%	+2%	+110%	–3%
r = ______	r = ______	r = ______	r = ______	r = ______
b = ______	b = ______	b = ______	b = ______	b = ______

Example 4: Given the following equations, find the percent growth/decay:

$y = 100(0.12)^x$ $\qquad$ $y = 30(1.67)^x$ $\qquad$ $y = 24\left(\frac{3}{4}\right)^x$ $\qquad$ $y = 4(5)^x$

- First, find r, by using $|b-1|$.

r = ______ $\qquad$ r = ______ $\qquad$ r = ______ $\qquad$ r = ______

- Now write the rate in percent form, and use + to indicate growth, and – to indicate decay.

_______ $\qquad$ _______ $\qquad$ _______ $\qquad$ ______

Your Turn 1: The value of a video game depreciates exponentially over time. Suppose a video game costs \$60 when it is first released and loses 7% of its value every month after it is released.
a) Write an equation modeling the value of the video game after *n* months.
b) How much do you expect the video game to be worth after one year?

Your Turn 2: The population of Algebratown increases exponentially over time. Suppose the population of Algebratown is currently 12,000 and is increasing by 3.6% each year.
a) Write an equation modeling the population of Algebratown after *n* years.
b) What do you expect the population of Algebratown to be after 20 years?

Activity: Representing Linear and Exponential Growth

Simple and Compound Interest Applications

In the above activity, you compared ___________ and ________________ functions through the applications of __________ and ______________ interest. Since compound interest, represented by _________________ functions, can be calculated several different ways, you will learn today how to solve investment problems involving several types of interest.

Simple Interest – Calculates a percentage of the ___________ investment and adds it on each year.

Example 5: You invest $2000 into an account that pays 4% simple interest per year. How much money will your account have after 3 years?

Compound Interest – Calculates a percentage of the amount in the account and adds it on each time interval (i.e. day, month, quarter). In essence, you earn interest on your ___________.
Use the formula:

Compound Interest Terminology	
Semi-annually	
Quarterly	
Monthly	
Weekly	
Daily	

Example 6: $500 is deposited into an account that pays 9.5% annual interest. What is the balance in the account after 3 years if the interest is compounded…

a) monthy?

b) weekly?

Example 7: How much must you deposit into an account that pays 6.5% interest, compounded semi-annually, to have a balance of $5000 in 15 years?

Continuously Compounded Interest – Calculates a percentage of the amount in the account and continuously adds it on.
Use formula:

Important: ____ is a ___________. It is a number that frequently occurs in many real-life phenomena.

Example 6 continued! $500 is deposited into an account that pays 9.5% annual interest. What is the balance in the account after 3 years if the interest is compounded continuously?

Example 8: How much must be deposited in order to attain $10,000 after 20 years in an account that earns 10.5% annual interest, compounded continuously?

Example 9: How long will it take to double $500 in an account that pays 3% annual interest? For now, solve this question by graphing.

Final Check: Exponential Models and Graphing LT 1 and LT 2a

LT 1. I can apply exponential functions to real world situations

1. Without graphing, determine whether each function represents exponential growth or decay. Then give the percent increase or percent decrease, using a + or – sign to indicate increase or decrease.

a. $f(x) = 5(\frac{3}{4})^x$ b. $w(t) = 25(1.08)^t$ c. $y = 7.1^x$ d. $h(x) = 0.05(3.5)^x$

Circle one: Growth or decay Growth or decay Growth or decay Growth or decay

% inc/dec: _______ % inc/dec: _______ % inc/dec: _______ % inc/dec: _______

2. Your parents purchased a new car in 2004 for $26,000. If the value of the car depreciates by 15% each year...

a. Write an exponential decay model for *V*, the value of the car, after *t* years.

$V(t) =$ ____________________

b. Use the model to find the value of the car in 2016. ________________

3. A certain town had a population of approximately 52,000 people in 2000. If the population growth is about 1.5% per year...

a. Write an exponential growth model for *P*, the population, after *t* years, where $t = 0$ represents the year 2000.

$P(t) =$ ____________________

b. What is the expected population in 2018? _____________

4. For each percentage rate of increase or decrease, find the corresponding growth or decay factor (Hint: First find r by taking the number out of percent form.)

a. +22% b. –3% c. –0.5% d. +250% e. +0.8%

_________ _________ _________ _________ _________

Growth or decay Growth or decay Growth or decay Growth or decay Growth or decay

For #5-8, show the substitution into the formula. Find the money values to the nearest cent.

continuous compounding formula: $A = Pe^{rt}$ compound interest formula: $A = P\left(1+\frac{r}{n}\right)^{nt}$

5. If $3,000 was initially deposited, find the amount of money in an account after 10 years of earning 3.4% interest compounded quarterly.

6. Compute the minimum principal necessary to have $50,000 in 18 years in an account that compounds monthly and earns 4.5% interest.

7. If $1,000 is invested into an account earning 3.4% interest, compounded continuously, what is the balance in the account after 3 years?

8. How long will it take an investment to triple in an account that pays 8.5% interest compounded continuously? Use your graphing calculator.

Practice Assignment

☐ Apply exponential functions to real world situations and graph parent exponential functions (LT 1-2a).
- Practice 8-1 Worksheet
- Worksheet LT1

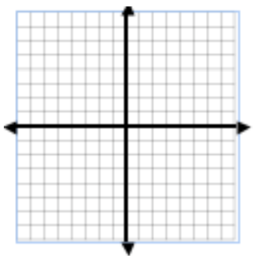
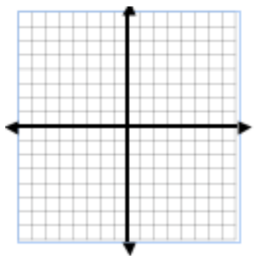
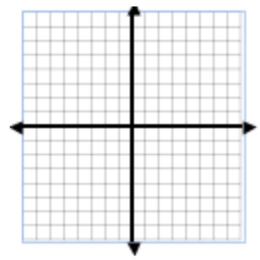
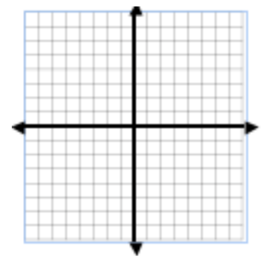
Practice 8-1 Exploring Exponential Models

Without graphing, determine whether each equation represents exponential growth or exponential decay.

1. $y = 72(1.6)^x$ **2.** $y = 24(0.8)^x$ **3.** $y = 3\left(\frac{6}{5}\right)^x$ **4.** $y = 7\left(\frac{2}{3}\right)^x$

Sketch the graph of each function. Identify the horizontal asymptote.

5. $y = (0.3)^x$ **6.** $y = 3^x$ **7.** $y = 2\left(\frac{1}{5}\right)^x$ **8.** $y = \frac{1}{2}(3)^x$

y=________________ **y=**________________ **y=**________________ **y=**________________

9. A new car that sells for \$18,000 depreciates 25% each year. Write a function that models the value of the car. Find the value of the car after 4 yr.

10. A new truck that sells for \$29,000 depreciates 12% each year. Write a function that models the value of the truck. Find the value of the truck after 7 yr.

11. The bear population increases at a rate of 2% per year. There are 1573 bears this year. Write a function that models the bear population. How many bears will there be in 10 yr?

12. An investment of \$75,000 increases at a rate of 12.5% per year. Find the value of the investment after 30 yr.

13. The population of an endangered bird is decreasing at a rate of 0.75% per year. There are currently about 200,000 of these birds. Write a function that models the bird population. How many birds will there be in 100 yr?

For each annual rate of change, find the corresponding growth or decay factor.

17. +45% **18.** –10% **19.** –40% **20.** +200%

For each function, find the annual percent increase or decrease that the function models.

21. $y = 1700(0.75)^x$ **22.** $y = 30.698\left(\frac{5}{8}\right)^x$ **23.** $y = 984.5(1.73)^x$

24. The value of a piece of equipment has a decay factor of 0.80 per year. After 5 yr, the equipment is worth \$98,304. What was the original value of the equipment?

CPA2 Worksheet LT1

LT 1: 1. I can apply exponential functions to real world situations

1-6 Find the amount in each account for the given conditions:

1. Principal: \$2000
Annual interest: 5.1%
Compound monthly for 3 years

2. Principal: \$2000
Annual interest: 5.1%
Compound continuously for 3 years

3. Principal: \$400
Annual interest: 7.6%
Compound quarterly for 1.5 years

4. Principal: \$400
Annual interest: 7.6%
Compound continuously for 1.5 years

5. Principal: \$950
Annual interest: 6.5%
Compound semi-annually for 10 years

6. Principal: \$950
Annual interest: 6.5%
Compound continuously for 10 years

7. A student wants to save for college in 5 years. How much should be put into an account that earns 5.1% annual interest compounded continuously?

8. How long would it take to double your principal at annual interest rate of 7% compounded continuously?

9. Suppose you invest $1000 at an annual interest rate compounded monthly.
a) How much would you have in the account after 4 years?

b) How much more would you have in the interest were compounded continuously?

10. An account that was neglected for 6 years has all $550 withdrawn. If it eared 3.5% annual interest compounded quarterly, how much was the initial deposit?

CPAlg2 Worksheet LT1 Answers

1) $2329.89
2) $2330.65
3) $447.82
4) $448.30
5) $1801.05
6) $1819.76
7) $7749.16
8) 9.902 years
9) a) $1245.45
   b) $1246.07 - $1245.45 = $0.63
10) $448.89

Practice 8.1 Answers:

1. growth 2. decay 3. growth 4. decay

9. $y = 18{,}000(0.75)^x$ $5,695.31

10. $y = 29{,}000(0.88)^x$ $11,851.59

11. $y = 1{,}573(1.02)^x$ 1,917 bears

12. $y = 75{,}000(1.125)^x$ $2,568,247.87

13. $y = 200{,}000(0.9925)^x$ 94,207 birds

14. $y = 2(0.65)^x$ 15. $y = 10(0.8)^x$

16. $y = 0.7(1.2)^x$ 17. 1.45

18. 0.9 19. 0.6 20. 3

21. –25% 22. –37.5%

23. +73% 24. $300,000

Graphs of Exponential Functions

Name ______________________________

Date: ___________

After this lesson and practice, I will be able to …

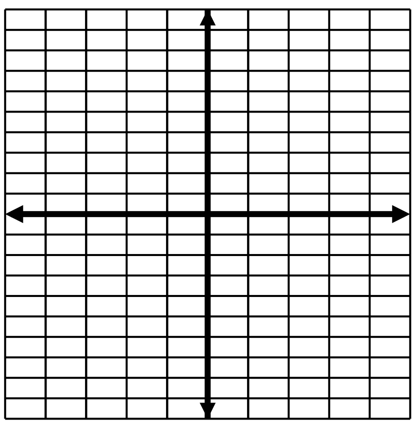
- ☐ graph parent exponential functions and describe and graph transformations of exponential functions. (LT 2)
- ☐ write equations for graphs of exponential functions. (LT 3)

One of the major themes throughout this course has been applying _____________________ to graphs of parent functions. In today's lesson, you will learn how to graph transformations of _____________________ functions.

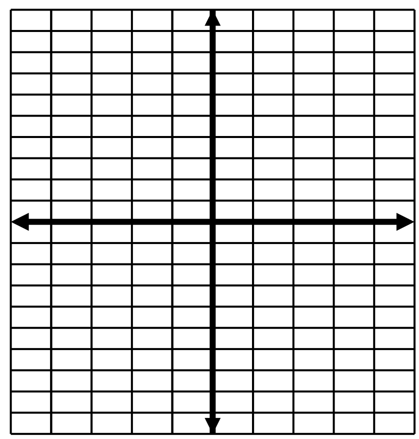
The graph of the parent exponential function, _______, will depend on the _____ of the power.

Example 1: Graph each exponential function. (no calc) State the equation of the asymptote.

a) $y = 2^x$ ASY:______________



b) $y = \left(\frac{1}{2}\right)^x$ ASY:______________



Graphing Transformations of Exponential Functions

1) Plot the parent exponential functions by making a table of values. Use x = ____________ for _________

or $x =$ ___________________ for ______________.

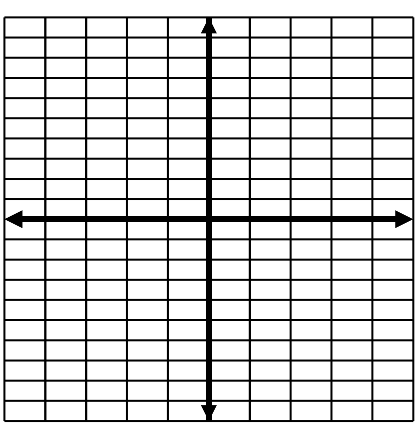
2) If your graph has a vertical shift up or down, draw the ______________ at the vertical shift.

3) Apply any vertical ____________ or _____________. *You may want to change your y-scale.*

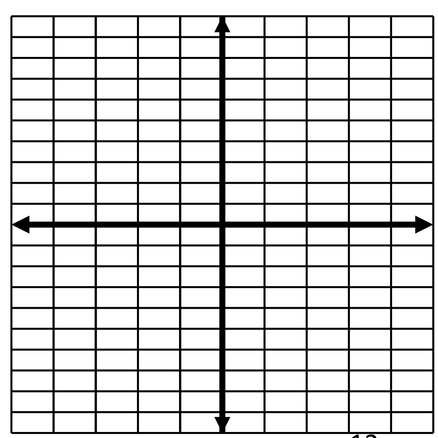
4) Apply all vertical and horizontal _________________.

Example 2: Graph each function. (no calc). State the equation of the asymptote.

a) $y = 2\cdot 3^{x-1} - 4$

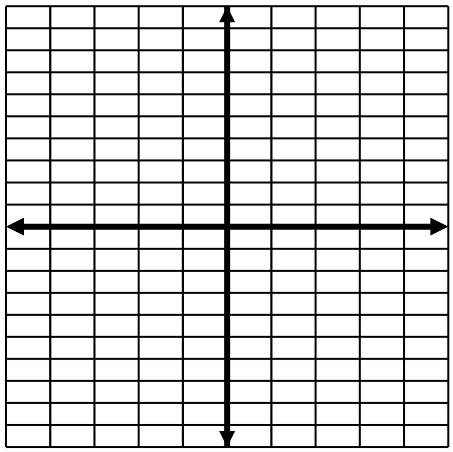


b) $y = -10(0.5)^{x-1} + 2$

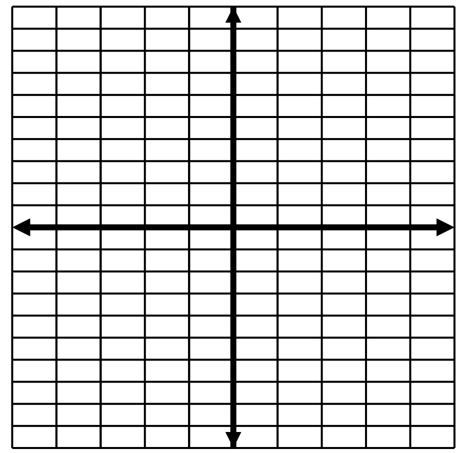


Your Turn 1: Graph each function. Sketch the parent function using a dashed line and then graph the transformation function using a solid line. Include the transformations of at least three "key points." Change the y-scale if necessary. (no calc). State the equation of the asymptote.

a) $y = -2^{x+3} + 6$



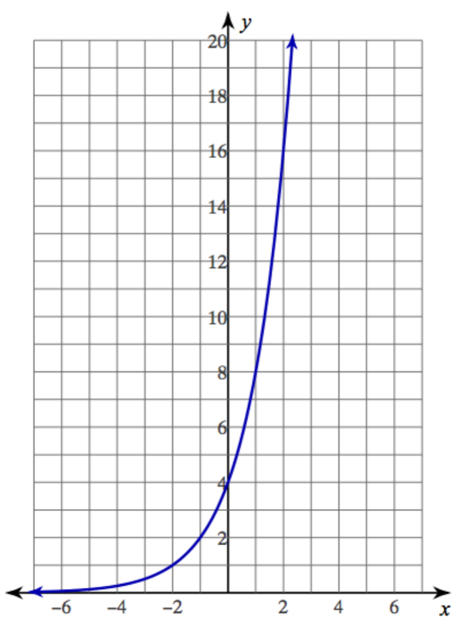
b) $y = -6\left(\frac{1}{3}\right)^{x-4} - 3$



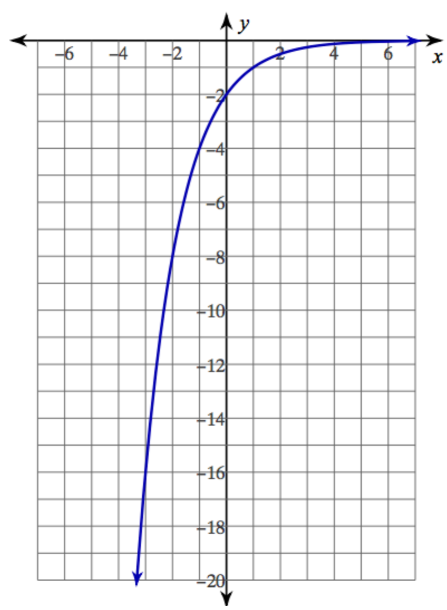
Example 3: The parent function for each graph below is of the form $y = ab^x$. Write the parent function. (no calc)

Steps: Find the y-intercept. a=_____. Find another point and find the ratio of the growth/decay. b=_

a)



b)



Extension: Writing Equations from data points.

Write an exponential equation in the form $y = ab^x$ that passes through the points (2, 4) and (3, 16).

- Write two equations in general form, one using Point 1 and the other using Point 2.

- Set the equations equal to each other and solve for *b*.

- Substitute your *b* value back into one of the original equations to solve for *a*.

- Write your final equation using your values for *a* and *b*.

<u>*Your Turn 2:*</u> Write an exponential equation in the form $y = ab^x$ that passes through the points (4, 8) and (6, 32).

Final Check: Graphing LT 2 and 3

LT2. I can graph parent exponential functions and describe and graph transformations of exponential functions.

For # 1-4, graph the parent function on the left-hand grid. Graph the two transformed functions on the right-hand grid, being sure to label them with their capital letter. List the transformations below the equations. Include the asymptotes as dashed lines. Plot at least two points per graph - preferably more if they fit on the graph paper. Fill in each scale.

1.) parent: $y = 2^x$

x	y

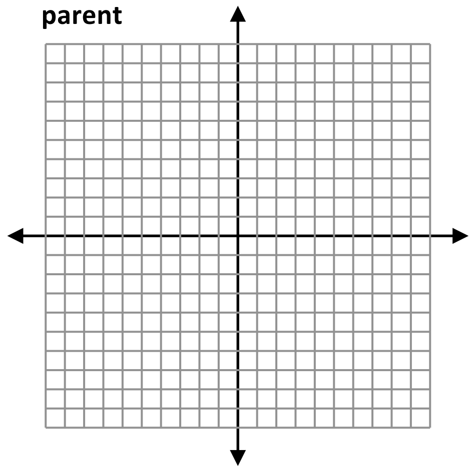
A) $y = 2^x - 5$

____

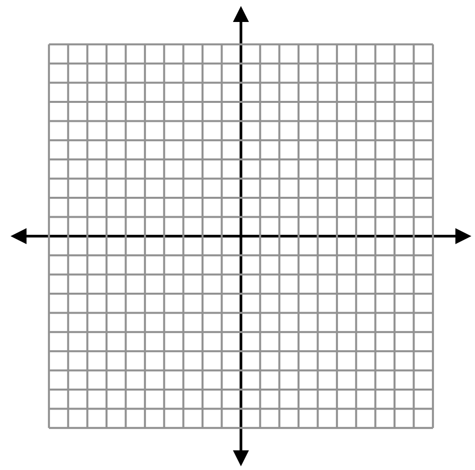
B) $y = 2^{(x+6)}$

____

parent



asy: _____________



A) asy:_____________

B) asy:_____________

2.) parent: $y = \left(\frac{1}{3}\right)^x$

x	y

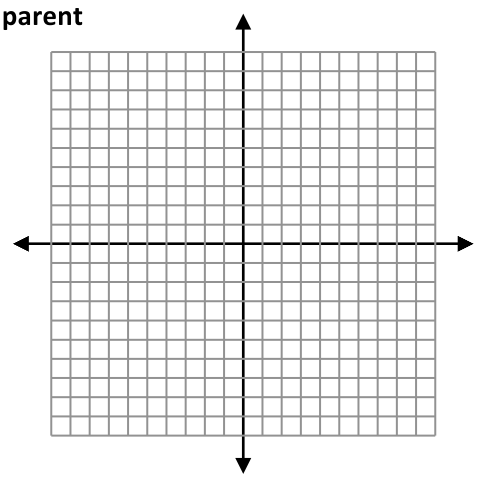
C) $y = -\left(\frac{1}{3}\right)^x$

____

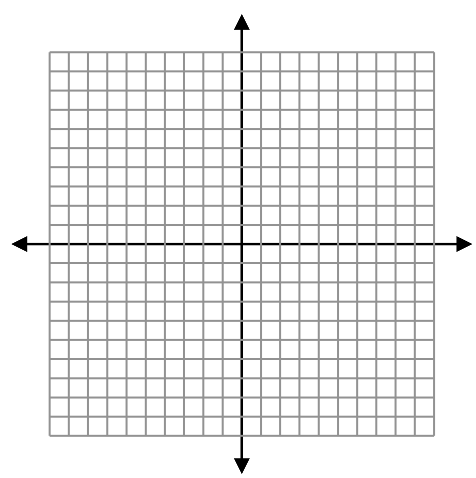
D) $y = -3\left(\frac{1}{3}\right)^x$

____

parent



asy: _____________



C) asy:_____________

D) asy:_____________

3.) parent: $y=\left(\frac{1}{2}\right)^{x}$

x	y

E) $y=100\left(\frac{1}{2}\right)^{x}$

____

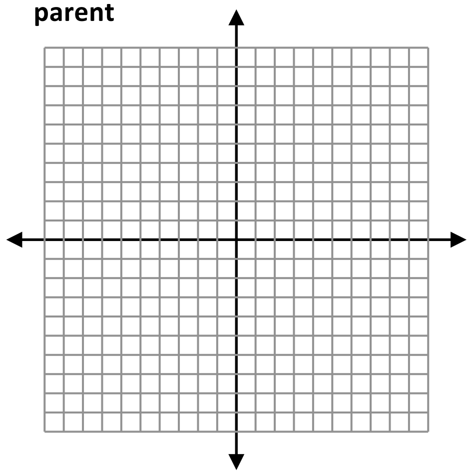
F) $y=100\left(\frac{1}{2}\right)^{(x-3)}-200$

____

____

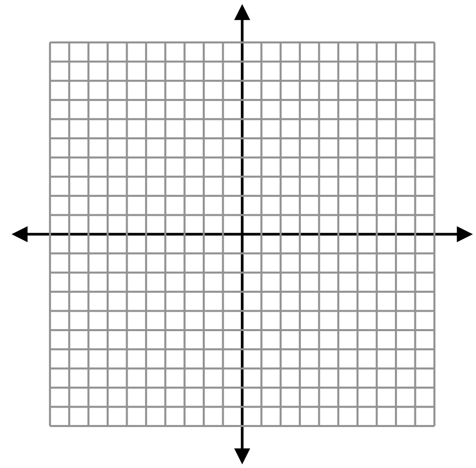
____

parent



x-scl=_______ y-scl=_______

asy: ____________



x-scl= 1 y-scl=_______

E) asy:____________

F) asy: ____________

4.) parent: $y=(4)^{x}$

x	y

G) $y=\frac{1}{2}(4)^{(x+5)}$

____

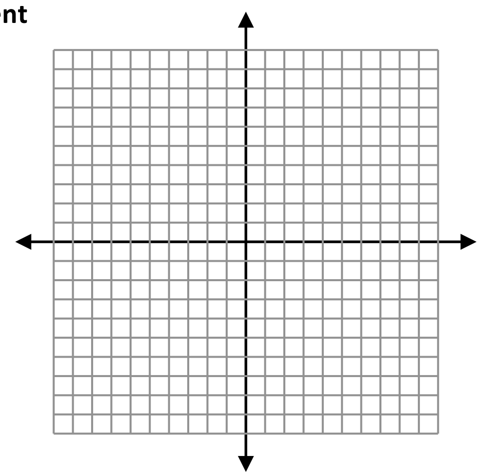
____

H) $y= -(4)^{(x)}+3$

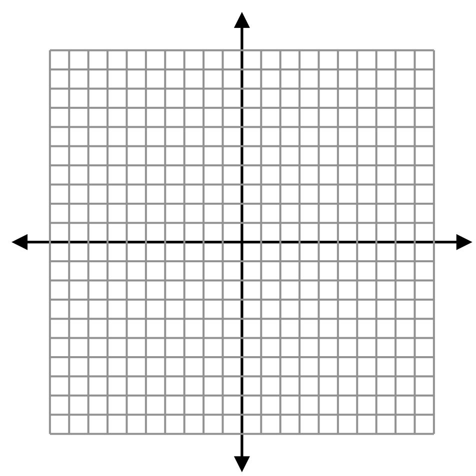
____

____

parent



asy: ____________



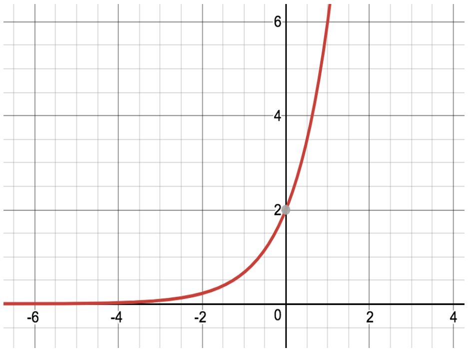
G) asy:____________

H) asy:____________

LT 3. I can write equations for graphs of exponential functions.

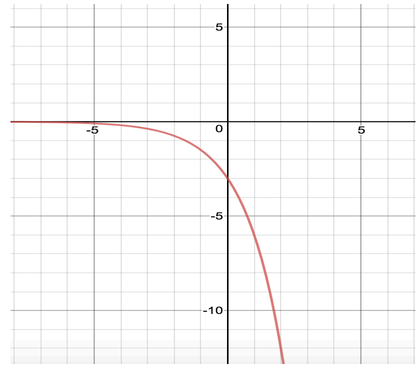
5. Write an exponential function for a graph:

a)



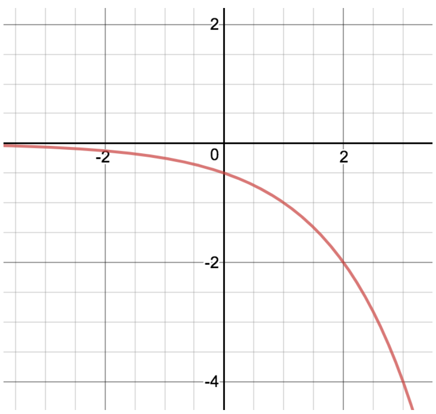
y= ___________________________

b)



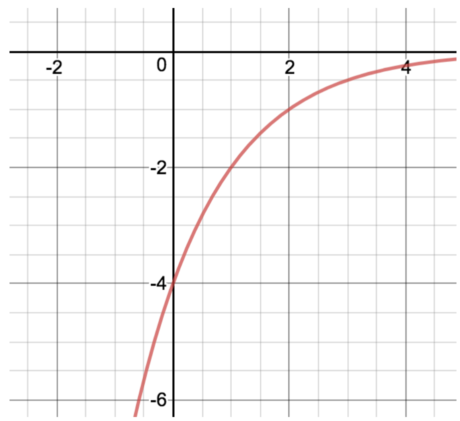
y=______________________

c)



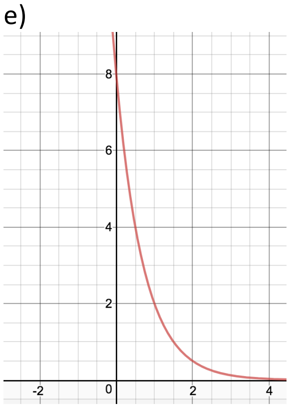
y= ___________________________

d)



y=______________________

e)



y= ___________________________

f) includes points (-1,.25) and (2, 6.75)

y=______________________

Practice Assignment

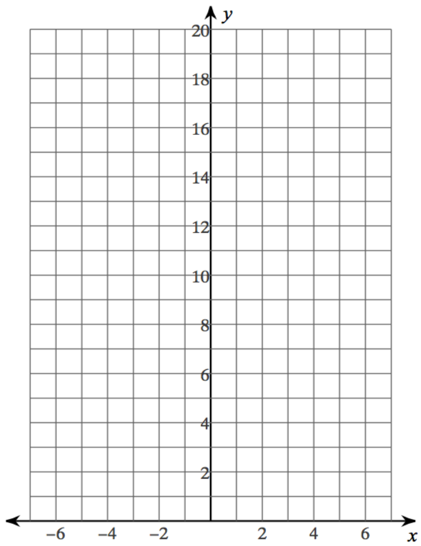
- ☐ I can graph parent exponential functions and describe and graph transformations of exponential functions. (LT 2)
- ☐ I can write equations for graphs of exponential functions. (LT 3)
    - o Worksheet LT2 and 3

CPA2 Worksheet LT2 and 3

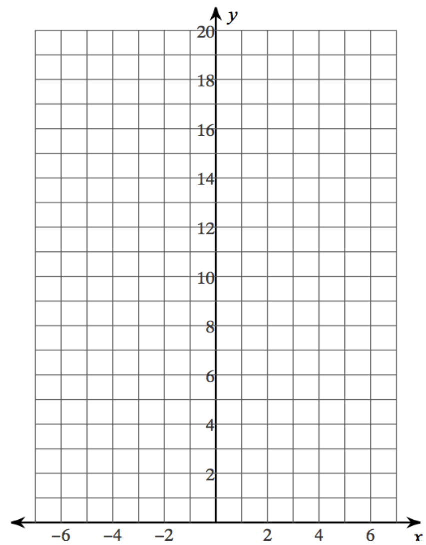
LT 2. I can graph parent exponential functions and describe and graph transformations of exponential functions.

Graph each function. Sketch the parent function using a dashed line and then graph the transformation function using a solid line. Include the transformations of at least three "key points." Change the y-scale if necessary.

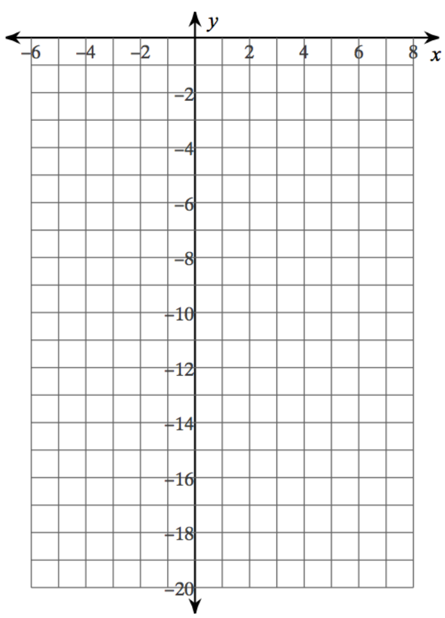
1) $y = 5 \cdot 2^x$



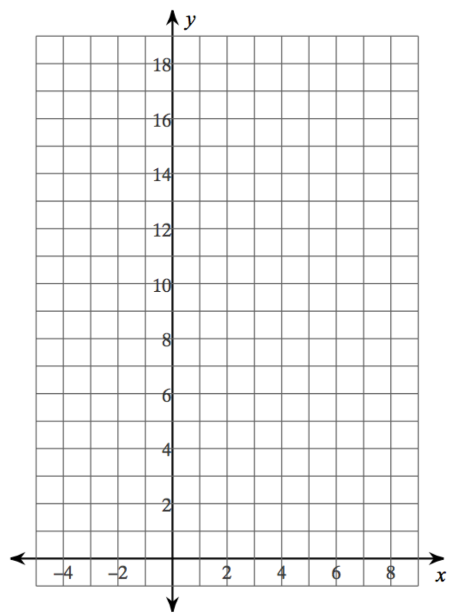
2) $y = 2 \cdot \left(\frac{1}{3}\right)^x$



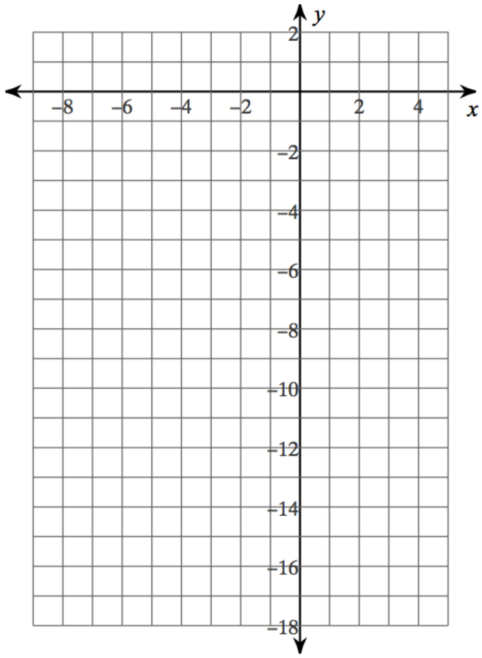
3) $y = -2 \cdot \left(\frac{1}{2}\right)^{x-1} - 2$



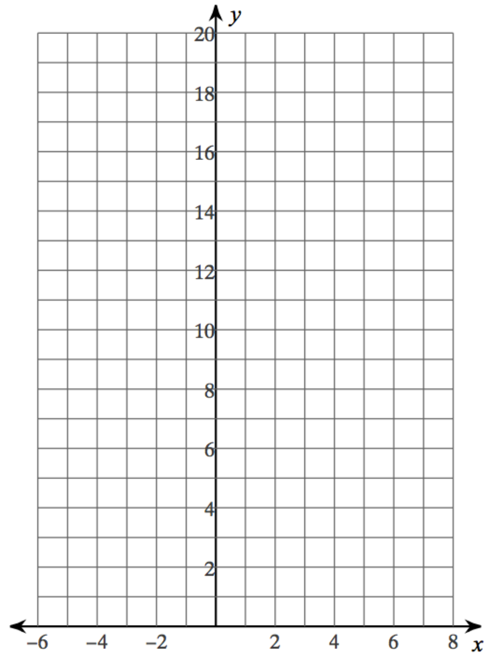
4) $y = 3 \cdot 2^{x-2} - 1$



5) $y = -5 \cdot 2^{x+2} + 2$



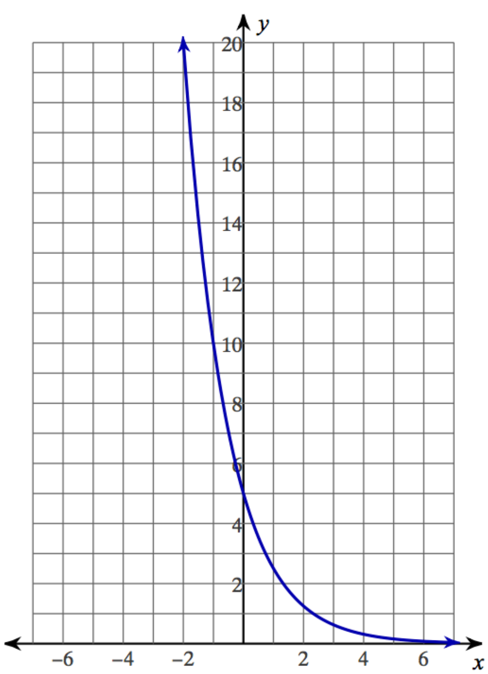
6) $y = 5 \cdot \left(\frac{1}{2}\right)^{x-1} + 1$



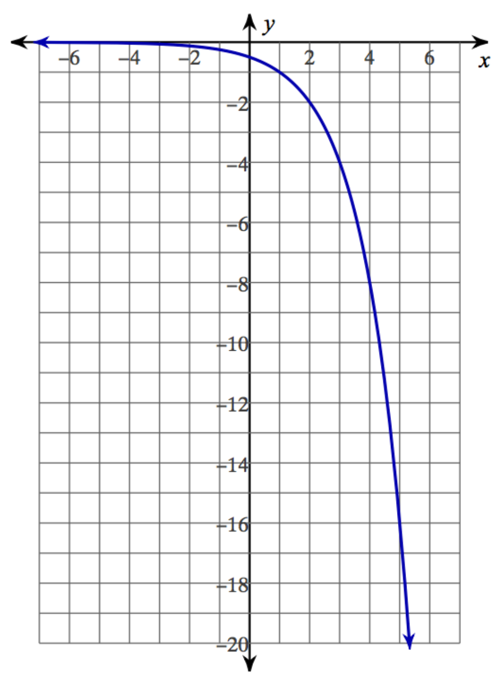
LT 3. I can write equations for graphs of exponential functions.

Write an equation for each graph.

7)



8)



9. Write an exponential function $y = ab^x$ for a graph that includes the given points.

a) (0, 2), (1, 1.3) **b.** (–1, 12.5), (4, 4.096) **c.** (1, 0.84), (2, 1.008)

Answers Worksheet LT2 and 3

1) Vertical Stretch 5 Base 2 (growth) Asymptote: y = 0 Points: (-1,2.5) (0,5) (1,10) (2,20)	2) Vertical Stretch 2 Base ⅓ (decay) Asymptote y = 0 Points: (-2,18) (-1,6) (0,2) (1,⅔)	3) Vertical Stretch 2 Reflect over x axis Base ½ (decay) Asymptote y = -2 Points: (-1,10) (0,-6) (1,-4) (2,-3)	4) Vertical Stretch 4 Down 1 Right 2 Base 2 Asymptote y = -1 Points: (0,-¼) (1,½) (2,2) (3,5) (4,11)
5) Vertical Stretch 5 Reflect over x-axis Left 2 Up 2 Base 2 Asymptote y = 2 Points: (0,-18) (-1,-8) (-2,-3) (-3,½)	6) Vertical Stretch 5 Right 1 Up 1 Base ½ Asymptote y = 1 Points (-1,21) (0,11) (1,6) (2,3.5)	7) Decay (0,5) and (-1,10) $y=a(b)^x$ $5=ab^0$ $a = 5$ $10=ab^{-1}$ $10=5b^{-1}$ $b=½$ $y=5(½)^x$	8) Growth Reflection over x axis (1,-1) and (2,-2) $-1=ab^1$ $a= -1/b$ $-2=ab^2$ $10=5b^{-1}$ $a = -2/b^2$ $-1/b=-2/b^2$ $-b^2 = -2b$ $b = 2$ $a = -½$ $y = -½ (2)^x$
9a) $y=2(.65)^x$	9b) $y=10(0.8)^x$	9c) $y=.7(1.2)^x$	

Logarithmic Functions as Inverses

After this lesson and practice, I will be able to…

- Rewrite expressions between exponential and logarithmic form. (LT 4)
- Write and evaluate logarithmic expressions. (LT 5)

Warm Up:

Solve each equation.

1. $8 = x^3$ 2. $x^{1/4} = 2$ 3. $4^6 = 2^x$

4. Philth E. Rich invested $12,000 in an account that paid monthly compound interest 10 years ago. Today the account is worth $17,890. What interest rate did he earn over the 10 years of his investment?

Now suppose you invest $10,000 in an account that pays an annual interest rate of 7%, how long would it take to double your money?

At this point, you could use your graphing calculator to answer questions like these. In this lesson you will learn about the function that can be used to solve __________________equations.

Exponential functions are one-to-one. Therefore exponential functions have an inverse function. The inverse of an exponential function is the ________________ ______________.

<u>I can rewrite equations between exponential and logarithmic form. (LT 4)</u>

Definition:

If $y = a^x$, then $\log_a y = x$, where $a \neq 1$ and $a > 0$

NOTE: The positive number a raised to any power x cannot equal a number y less then or equal to zero. Therefore, the logarithm of a negative number or zero is undefined.

The expression $\log_a x$ is called a ***logarithm*** and is read as "*the base a logarithm of x*". The function $f(x) = \log_a x$ is the logarithmic function with base a.

The solution to the equation ____________________, or ______ is the power to which a must be raised to produce ____________.

- The most important thing to remember is that logarithms are exponents.
- $\log_a y = x$ is just another way of saying a raised to the x equals y.

Writing in Logarithmic Form

Example: Write each in logarithmic form

1. $25 = 5^2$ 2. $8 = 2^3$ 3. $81 = 3^4$

Your Turn:

4. $125 = 5^3$ 5. $32 = 2^5$ 6. $216 = 6^3$

I can write and evaluate logarithmic expressions. (LT 5)

To evaluate ______________, you can write them in ________________ __________.
Strategies for Evaluating Logarithms:

1. Set the expression equal to________________________
2. Rewrite the equations in ________________________form.
3. Write each side of the equation with the same ________________________
4. Set the ________________________equal to each other and solve for x.

Example: Evaluate the following.

7. $\log_8 16$ 8. $\log_9 27$ 9. $\log_{10} 100$

Your turn:

10. $\log_5 25$ 11. $\log_9 3$ 12. $\log_2 \frac{1}{8}$

Although the base of a logarithm can be any number, there are two bases for logarithms that are used frequently.

The two bases for logarithms that are used most frequently are ___________, referred to as the

__________________ _________ and___________, referred to as the _____________ ______________ or ____.

Finding a common (base 10) or natural (base e) logarithm

Evaluate each logarithm

a. $\log 1000$ b. $\ln e$ c. $\log 10$

d. $\log \frac{1}{10}$ e. $\ln 1$ f. $\ln \frac{1}{e}$

Final Check: Logs
LT 4. I can rewrite equations between exponential and logarithm form.
LT 5. I can write and evaluate logarithmic expressions.

Learning Target 4: I can rewrite equations between exponential and logarithm form.

1. Translate each logarithm equation into an equivalent exponential equation.

a) $\log_9 c = b$ _______________ b) $\log h = j$ _______________ c) $\ln 8 = 3y$ ___________

2. Translate each exponential equation into an equivalent logarithm equation.

a) $10^w = m$ ______________ b) $p^f = v$ _______________ c) $e^{5x} = 12$ __________ or ___________

Learning Target 5: I can write and evaluate logarithmic expressions.

3. Evaluate each logarithm. Show all work.

a) $\log_2 64$ _______________ b) $\log_{1/3} 27$ _______________ c) $log_5 \frac{1}{25}$ _______________

d) $log_4 \frac{1}{2}$ _______________ e) $log_5 5$ _______________ f) $\log 1$ _______________

Practice Assignment:

- I can rewrite equations between exponential and logarithm form. (LT 4)
- I can write and evaluate logarithmic equations. (LT 5)

Worksheet: Logarithms as Inverses

CPA2 Worksheet LT4 and 5 Logs as Inverses

LT 4. I can rewrite equations between exponential and logarithm form.
LT 5. I can write and evaluate logarithmic expressions.

Rewrite each equation in Logarithmic Form:

1. $15^2 = 225$ 2. $19^2 = 361$ 3. $3^a = b$

4. $32^{1/5} = 2$ 5. $y^x = 6$ 6. $\left(\frac{29}{19}\right)^y = x$

Rewrite each equation in exponential form.

7. $\log_{289} 17 = \frac{1}{2}$ 8. $\log_{64} 8 = \frac{1}{2}$ 9. $log_{\frac{6}{11}} b = a$

10. $\log_v 84 = u$ 11. $\log_{16} u = v$ 12. $\log_7 b = a$

Evaluate each expression.

13. $\log_5 25$ 14. $\log_2 4$ 15. $\log_6 36$ 16. $\log_2 8$ 17. $\log_4 64$

18. $\log_2 \frac{1}{4}$ 19. $log_6 \frac{1}{36}$ 20. $\log_3 3$ 21. $\log_7 343$ 22. $\log_5 125$

Use a calculator to approximate each to the nearest thousandth.

23. log 23 24. ln 8 25. ln 23 26. log 1.7 27. ln 4.8

28. log 54 29. log 30 30. log 19

ANSWERS:

1) $\log_{15} 225 = 2$ 2) $\log_{19} 361 = 2$ 3) $\log_3 b = a$ 4) $\log_{32} 2 = \frac{1}{5}$

5) $\log_y 6 = x$ 6) $\log_{\frac{29}{19}} x = y$ 7) $289^{\frac{1}{2}} = 17$ 8) $64^{\frac{1}{2}} = 8$

9) $\left(\frac{6}{11}\right)^a = b$ 10) $v^u = 84$ 11) $16^v = u$ 12) $7^a = b$

13) 2 14) 2 15) 2 16) 3
17) 3 18) −2 19) −2 20) 1
21) 3 22) 3 23) 1.362 24) 2.079
25) 3.135 26) 0.23 27) 1.569 28) 1.732
29) 1.477 30) 1.279

Graphing Logarithmic Equations

After this lesson and practice, I will be able to…

- Graph logarithmic equations. (LT 6)

In order to use logarithms to help solve ______________________ equations, it is important to observe and important relationship between logarithms and exponential functions.

Investigation:

1. Find the inverse of each of the following, algebraically.

a.) $y = \log_5 x$ b.) $y = 6^x$ c.) $y = \log_{1/3} x$

2. Use the above answers to predict the inverses of $y = \log_8 x$ and $y = 0.7^x$.

As you observed from the investigation, a logarithm function is the ______________ of an _______________ function. Thus, you can apply what you know about graphing ______________________ _____________ to graph logarithm functions.

<u>Graphing Logarithmic Functions (LT 6)</u>

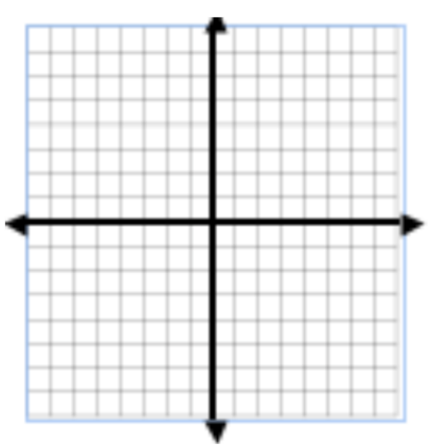
Graphing the Parent Logarithmic Function

1) Plot the parent ______________ function using familiar techniques.

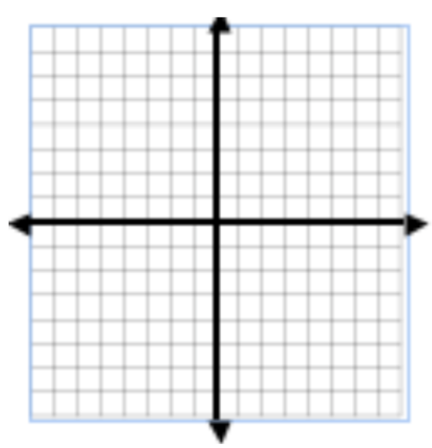
2) Draw the line of _________________ (_______). Reflect the parent function points across this line.

<u>Example 1</u>: Graph each logarithm function. State the equation of the asymptote.

a) $y = \log_2 x$



b) $y = \log_3 x$



Notice, all parent _________________ functions have an _________________ at _______.

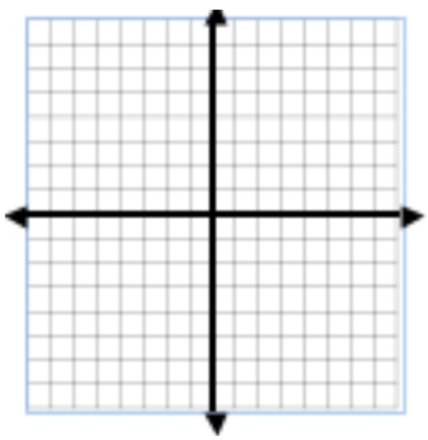
Graphing Transformations Parent Logarithmic Functions

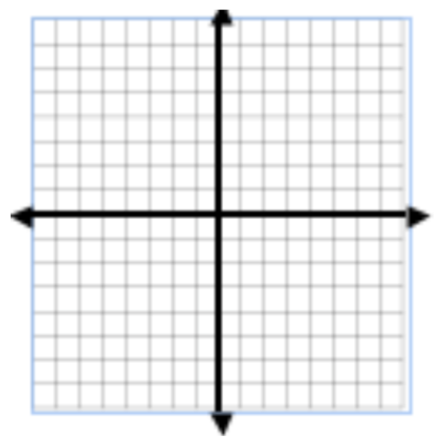
1) Plot the parent _______________ function using the above techniques.
2) If your graph has a horizontal shift up or down, draw the ______________ at the horizontal shift.
3) Apply any vertical ______________ or _______________.
4) Apply all vertical and horizontal ___________________.

Example 2: Graph each logarithm function. State the equation of the asymptote.

a) $y = \log_6(x-2)+3$

b) $y = 2\log_3(x+1)-2$





Consider the general shapes of the graphs of exponential and logarithmic functions. Find the domain and range of a parent exponential and logarithmic function, using $y = 2^x$ and y = $\log_2 x$

Exponential :

Domain

Range:

Logarithmic:

Domain:

Range:

Example 3: Without graphing, find the domain and range of the following.

A) $y = \log_3 x$

Domain: ______________

Range: ______________

B) $y = \log_3(x-2)$

Domain: ______________

Range: ______________

C) $y = \log_3(x+4)-1$

Domain: ___________

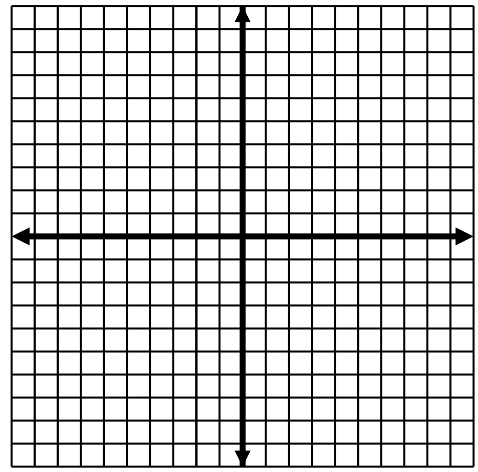
Range: _____________

Final Check: Graphing Logs

LT 6. I can graph logarithmic equations.

1. Graph each function. Sketch the parent logarithm function using a dashed line and then graph the transformation function using a solid line. Include the transformations of at least two "key points." Sketch and label the asymptote of the transformed function. List the ordered pairs of your transformed key points. Then find the domain and range

a) $y = 2\log_6(x+3) - 5$

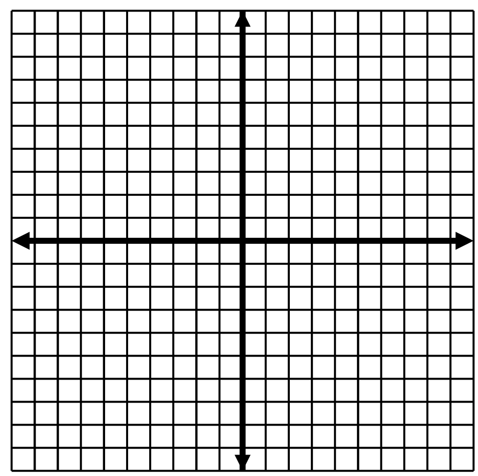


Key Points: ______________________________

Asy: ____________

Domain: ______________ Range: ________________

b) $y = -\log_5(x) - 3$

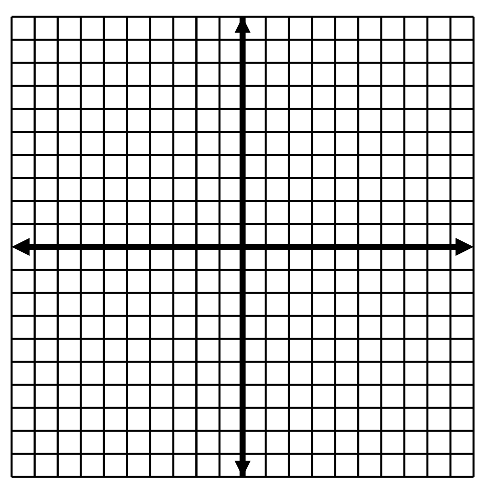


Key Points: ______________________________

Asy: ____________

Domain: ______________ Range: ________________

c) $y = 3\log_2(x-1)$



Key Points: ______________________________

Asy: ____________

Domain: ______________ Range: ________________

Practice Assignment:

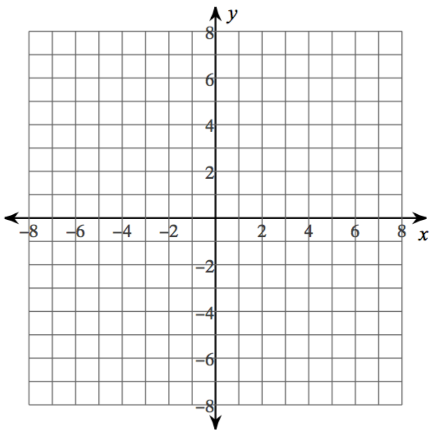
- Worksheet LT 6

CPA2 Worksheet LT6 Graphing

LT 6. I can graph logarithmic equations.

Sketch each graph and identify the domain and range. State the equation of the asymptote.

1) $y = \log\left(x - 1\right) - 4$

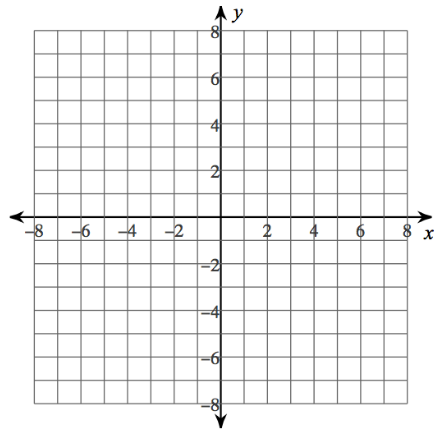


Asy: ______________________

Domain: __________________

Range: ___________________

2) $y = \log_4\left(x - 1\right) + 4$

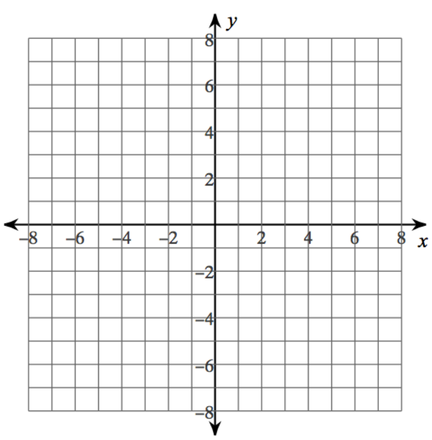


Asy: ______________________

Domain: __________________

Range: ___________________

3) $y = \log_2\left(x + 4\right) - 1$

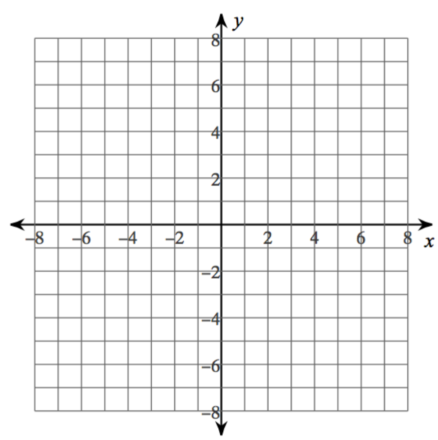


Asy: ______________________

Domain: __________________

Range: ___________________

4) $y = \log_6\left(x - 1\right) + 5$

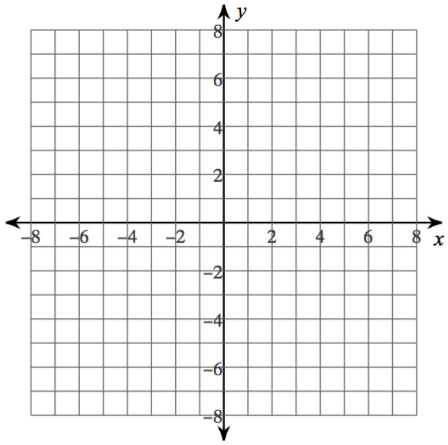


Asy: ______________________

Domain: __________________

Range: ___________________

5) $y = \log_2(x+5) - 3$

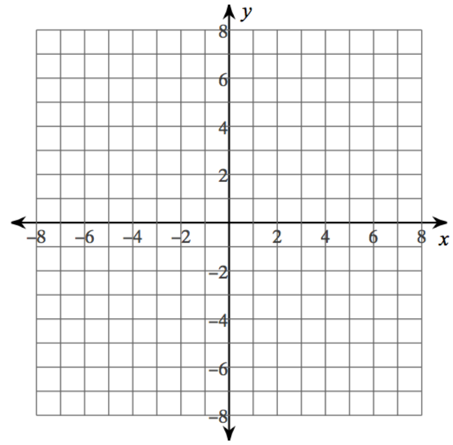


Asy: ______________________

Domain: __________________

Range: ___________________

6) $y = \log_6(x+4) - 5$

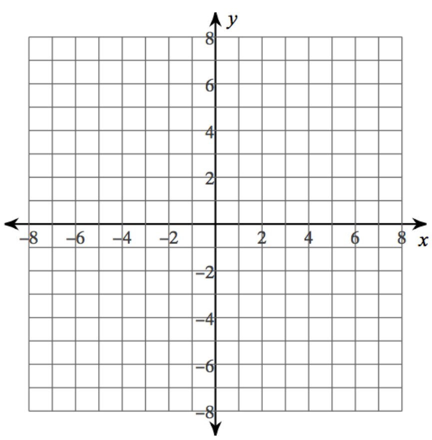


Asy: ______________________

Domain: __________________

Range: ___________________

7) $y = \log_3(x+2) + 3$

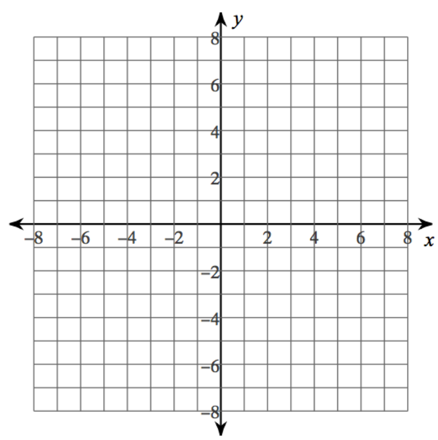


Asy: ______________________

Domain: __________________

Range: ___________________

8) $y = \log_4(x+4) + 1$



Asy: ______________________

Domain: __________________

Range: ___________________

Operations with Logarithms

Name ___________________________________

Date: ____________

After this lesson and practice, I will be able to …

- ☐ use properties of exponents to multiply, divide, an exponentiate with logarithms. (LT 7)
- ☐ simplify and expand expressions using logarithms. (LT 8)

Before you can use logarithms to solve ____________________ equations, it is important that you first learn several helpful properties of logarithms.

Investigation: Properties of Logarithms

Log Form	Decimal Form
$\log 2$	
$\log 3$	
$\log 4$	
$\log 5$	
$\log 6$	
$\log 8$	
$\log 9$	
$\log 10$	
$\log 12$	
$\log 15$	
$\log 18$	
$\log 20$	

1. Use your calculator to fill in the table. Round to three decimal places.

2. Using the values from the table, attempt to find two logarithms whose sum is another logarithm on the table. For example, what is $\log 2 + \log 4$ equal to? Does it match another logarithm from the table? Write your statements in the form $\log\_\_\_ + \log\_\_\_ = \log\_\_\_$.

3. Complete the conjecture: $\log M + \log N = \log\_\_\_\_\_\_$.

4. Using the values from the table, attempt to find two logarithms whose difference is another logarithm on the table. For example, what is $\log 8 - \log 2$ equal to? Does it match another logarithm from the table? Write your statements in the form $\log\_\_\_\_ - \log\_\_\_\_ = \log\_\_\_\_$.

5. Complete the conjecture: $\log M - \log N = \log\_\_\_\_\_\_$.

6. Using the values from the table, attempt to find an integer and a logarithm from your table whose product is another logarithm on the table. For example, what is $3 \cdot \log 2$ equal to? Does it match another logarithm from the table? Write your statements in the form $\_\_\_\_\_\log\_\_\_\_\_ = \log\_\_\_\_\_$.

7. Complete the conjecture: $x \log M = \log\_\_\_\_\_$.

Let's summarize the properties you discovered in this activity!

***Property 1*: Log of a Product–**

***Property 2:* Log of a Quotient –**

***Property 3:* Log of a Power –**

You can use these properties to rewrite logarithm expressions, which will be helpful in solving logarithm equations.

Example 1: Rewrite each expression as a single logarithm.

A) $\log_2 8 - \log_2 4$ B) $\log_4 7 + \log_4 10$ C) $2\log_3 x + 4\log_3 y$

Your Turn 1: Rewrite each expression as a single logarithm. Write in simplest form.

A) $2\log_5 3 + \log_5 4$ B) $\log_4 64 - \log_4 16$ C) $6\log_n m - 3\log_n p$

Example 2: Express each expression as sum or difference of logarithms.

A) $\log_9 \dfrac{x}{y}$

B) $\log 5x^4$

C) $\log\sqrt{\dfrac{2x}{y}}$

Your Turn 2: Rewrite each expression as a single logarithm.

A) $\log_5 xyz^2$

B) $\log\dfrac{a^2b^5}{c^3d}$

C) $\log_2 \dfrac{3\sqrt{x}}{\sqrt[3]{y}}$

Final Check: Operations with Logs LT 7 and LT 8
LT 7. I can use properties of exponents to multiply, divide, and exponentiate with logarithms.
LT 8. I can simplify and expand expressions using logarithms properties.

1. Rewrite the expression as a single logarithm and simplify completely. Rewrite all rational exponents as radicals.

a) $\frac{1}{2}\log x - \log 2 - 7\log m$

b) $3\log_a 2 - 4\log_a p + 5\log_a d - 2\log_a w$

c) $\frac{1}{3}\log a - 2\log 3 + \log b - \log 10$

d) $-\log_a 2 - 4\log_a m + 3\log_a 5 - 2\log_a p$

2. Expand completely to express the expression as sum or difference of logarithms. Your answer should not include any exponents.

a) $\log_5\left(\frac{\sqrt[6]{x}}{5y^3}\right)$

b) $\log 4x^5$

c) $\log\left(\frac{3zx^5}{10\sqrt{y}}\right)^2$

d) $\log\sqrt{\frac{6b}{ac}}$

e) $\ln\left(\frac{a^3}{5\sqrt[6]{b}}\right)$

f) $\ln e\sqrt[3]{2}$

Practice Assignment

- ☐ I can use properties of exponents to multiply, divide, an exponentiate with logarithms. (LT 7)
- ☐ I can simplify and expand expressions using logarithms. (LT 8)
  - o Worksheet LT 7 and 8

CPA2 Worksheet LT 7 and 8 Operations with Logs

LT 7. I can use properties of exponents to multiply, divide, and exponentiate with logarithms.
LT 8. I can simplify and expand expressions using logarithms properties.

Condense each expression to a single logarithm.

1) $6\log_6 x$

2) $3\log_5 11$

3) $\log_7 x - 4\log_7 y$

4) $\log_9 11 + 2\log_9 10$

5) $4\log_6 x + 24\log_6 y$

6) $12\log_9 a + 3\log_9 b$

7) $\log_4 x + 3\log_4 y + 4\log_4 z$

8) $20\log_4 w + 20\log_4 u - 5\log_4 v$

Expand each logarithm.

9) $\log_4 \sqrt[3]{11}$

10) $\log_5 \sqrt[3]{x}$

11) $\log_3 \left(\frac{7}{2}\right)^2$

12) $\log_5 \frac{7}{10^3}$

13) $\log_3 \frac{u^3}{v^3}$

14) $\log_8 (11 \cdot 10^6)^6$

15) $\log (c \cdot a^5 \cdot b)^5$

16) $\log (x^4 y^5 \cdot z)$

ANSWERS LT 7 and LT 8:

1) $\log_6 x^6$

2) $\log_5 11^3$

3) $\log_7 \frac{x}{y^4}$

4) $\log_9 \left(11 \cdot 10^2\right)$ $\log_9 1100$

5) $\log_6 \left(y^{24}x^4\right)$

6) $\log_9 \left(b^3a^{12}\right)$

7) $\log_4 \left(xz^4y^3\right)$

8) $\log_4 \frac{w^{20}u^{20}}{v^5}$

9) $\frac{1}{3}\log_4 11$

10) $\frac{1}{3}\log_5 x$

11) $2\log_3 7 - 2\log_3 2$ $\log_3 49 - \log_3 4$

12) $\log_5 7 - 3\log_5 10$

13) $3\log_3 u - 3\log_3 v$

14) $6\log_8 11 + 36\log_8 10$

15) $5\log c + 25\log a + 5\log b$

16) $4\log x + 5\log y + \log z$

Solving

Name ______________________________

Date: ___________

After this lesson and practice, I will be able to …

- ☐ solve exponential and logarithm equations. (LT 9)
- ☐ apply solving exponential and logarithm equations to real world situations. (LT 10)

Having learned about logarithms and various properties of logarithms, you are finally prepared to solve exponential equations ____________________.

<u>Summary of Logarithm Properties</u>

If $10^x = y$, then _______________.

In general, $\log_b a = x$ is equivalent to _________________.

Logarithms are _________________.

<u>Common Logarithms</u>

$\log 10 =$ $\qquad$ $\log 1 =$

$\log_b b =$ $\qquad$ $\log_b 1 =$

Solving Exponential Equations

<u>Example 1</u>: Solve algebraically.

A) $5^x = 96$ $\qquad$ B) $3(7)^{5x} - 2 = 148$

> ***<u>Solving Exponential Equations</u>***
> *METHOD 1*
> *1) Isolate the exponential term.*
> *2) Take the _________ or ____________log of both sides.*
> *3) Solve for x.*

<u>Your Turn 1</u>: Solve algebraically.

A) $6^{x-5} = 73$ $\qquad$ B) $5(3)^{x+4} = 100$

There exists another method for solving exponential equations that involves evaluating a logarithm of ______ base.

> **Logarithm Change-of-Base Formula**
> $\log_b a = \underline{\qquad\qquad}$ where $a > 0, b > 0$.

<u>Example 2</u>: Use the change-of-base formula to evaluate each logarithm.

A) $\log_9 20$ $\qquad$ B) $\log_3 12$ $\qquad$ C) $\log_4 2016$

Example 3: Solve algebraically using the change-of-base formula.

A) $12^x = 702$

B) $0.5(4)^{x+7} + 13 = 208$

> ***Solving Exponential Equations***
> *METHOD 2*
> *1) Isolate the exponential term.*
> *2) Rewrite the equation into ______________________ form.*
> *3) Solve for x using the change-of-base formula.*

Your Turn 2: Solve algebraically using the change-of-base formula. $6^{7x} + 3 = 74$

Solving Logarithmic Equations with a Single Logarithm

Example 4: Solve algebraically.

A) $\log(5x-2) = 4$

B) $4\log_5 3x - 7 = 10$

> ***Solving Logarithmic Equations***
> *1) Isolate the logarithmic term.*
> *2) Rewrite the equation into ______________________ form.*
> *3) Solve for x.*

Your Turn 3: Solve algebraically.

A) $2\log(2x-1) = -1$

B) $5\log_8(4x+9) - 12 = -4$

Solving Logarithmic Equations with a Multiple Logarithms
<u>*Example 5*</u>: Solve algebraically.

A) $2\log x - \log 3 = 2$

B) $\log_3 21 = \log_3 7 + \log_3 x$

> ***<u>Solving Logarithmic Equations</u>***
> *1) ____________ logarithm expressions if necessary.*
> *2) Isolate the logarithm*
> *3a) If the logarithm is equal to a number, solve as before.*
> *3b) If the logarithm is equal to another logarithm, you may ________________________ the logarithms and solve for x.*

<u>*Your Turn 4*</u>: Solve algebraically.

A) $\log_5 3 - \log_5 x = \log_5 \frac{3}{4}$

B) $\frac{2}{3}\log_4 x = \log_4 16$

Applying Exponential and Logarithmic Equations
<u>*Example 6*</u>: Suppose you invest \$5000 at 4.25% annual interest, compounded weekly. How long will it take to double your money?

Final Check: Operations with Logs LT 9 and LT 10

LT 9. I can solve exponential and logarithm equations.

LT 10. I can apply solving exponential and logarithm equations to real world situations.

1. Use the change of base formula to evaluate each expression.
   Round to four decimal places. *Show your work.*

a) $\log_2 61$ b) $\log_5 90$ c) $\log_{11} 136$ d) $\log_{11} \frac{1}{2}$ e) ln 12

____________ ____________ ____________ ____________ ____________

2. Solve each equation algebraically. Round to four decimal places if necessary. *Show all work.*

a) $5^{10x} = 20$

b) $12\log(3x-1) = 36$

i) $15 \ln(8x) = 38$

____________ ____________ ____________

c) $-2(1.6)^{2x+1} = -10$

d) $4\log_5(x-1) - 3 = 9$

f) $\ln x + \ln 2 = 7$

____________ ____________ ____________

e) $\ln(2x-3) = 4$

f) $\log_{16}(8x-2) = \log_{16}(3x+13)$

g) $e^{x+3} = 30$

____________ ____________ ____________

h) $\log_3(x+1)-\log_3 2=3$

k) $\ln\left(\frac{x+2}{5}\right) = 9$

____

____

l) $4 - 2e^x = -23$

m) $12e^{3x-2} = 8$

____

____

Practice Assignment

- ☐ I can solve exponential and logarithm equations. (LT 9)
  - Worksheet LT 9
- ☐ I can apply solving exponential and logarithm equations to real world situations. (LT 10)
  - Worksheet LT 10

CPA2 Worksheet LT 9 Solving

LT 9. I can solve exponential and logarithm equations.

Solve each equation algebraically. Give the exact answer or round to 4 decimal places if necessary.

1) $16^x = 28$

2) $9^k = 39$

3) $9 \cdot 3^{x+10} = 20$

4) $19^{n+9} - 7 = 62$

5) $-3 \cdot 16^{9n} - 1 = -2$

6) $5 \cdot 10^{4.6x} + 8 = 80$

7) $\log_{18} (10 - 2b) = \log_{18} (5b - 4)$

8) $\log_5 (-b + 7) = \log_5 (2b - 8)$

9) $5 \log_6 7m = -10$

10) $4 + \log_4 -4n = 5$

11) $\log 4 + \log x = \log 13$

12) $\log x - \log 5 = 1$

13) $\log_4 (x - 8) + \log_4 6 = \log_4 37$

14) $\log_5 2 + \log_5 -x = 2$

CPA2 Worksheet LT 10 Solving (real world)

LT 10. I can apply solving exponential and logarithm equations to real world situations.

1. Suppose you invest $10,000 at 5.1% annual interest, compounded weekly. How long will it take to double your money?

2. A parent raises a child's allowance by 20% each year. If the allowance is $8 now, when will it reach $20?

3. Protactinium-234*m*, a toxic radioactive metal with no known use, has a half-life of 1.17 minutes. How long does sit take for a 10-mg sample to decay to 2 mg?

4. As Algebratown gets smaller, the population of its high school decreases by 12% each year. The student body has 125 students now. In how many years will it have about 75 years?

ANSWERS:

1.About 13.6 years 2. About 5 years (5.0257) 3. About 2.72 minutes 4. About 4 years

Understanding

Name ___________________________

Date: __________

After this lesson and practice, I will be able to …

- ☐ apply my knowledge of exponential and logarithmic functions to solve new and non-routine problems. (LT 11)

TIME OF DEATH

Newton's Law of Cooling

$$kt = \ln\frac{T - S}{T_0 - S}$$

T_0 = initial body temperature
T = second body temperature
S = environment temperature
t= time passed
k = constant

A dead body of found in a hotel room at midnight.
The temperature in the room was 60 degrees F.
The body temperature at that time was 80 degrees F.
At 2 am the same morning the body temperature was 75 degrees.
Normal body temperature is 98.6 degrees F.

Find the time of death using Newton's Law of Cooling.

Step 1: Substitute given information to find k.

t = 2 T = 75 T_0 = 80 S = 60 k = ?

Step 2: Substitute given information, normal body temp and k to find new t.

t = ? T = 80 T_0 = 98.6 S = 60 k = __________

Step 3: Use calculated t and time the body was found to estimate time of death. (Estimate to hours and minutes)

2. Spread of a Virus

On a college campus of 5000 students, one student returned from a vacation with a contagious three-day flu virus. The spread of the virus through the student body can be modeled by:

$$y = \frac{5000}{1 + 4.999e^{-.8t}}$$

Where y is the total number infected after t days. How many students will have been infected by the end of five days? Sketch a graph of this function. Make a table of values for this function. What happens "in the long run"?

3. Median Age of First Marriages

For 1970 through 1990, the median age, A, of Americans at their first marriage can be modeled by the two equations for below for women and men respectively:

$$y = 20.4 + \frac{3.1}{1 + 43e^{-0.36t}}$$

$$y = 22.4 + \frac{3.1}{1 + 63e^{-0.36t}}$$

where t = 0 represents 1970. Find the median age of women and men at their first marriage in :

(a) 1970 (b) 1980 (c) 1990

4. The number N of bacteria present in a culture at time t (in hours) obeys the equation $N = 1000e^{0.01t}$. After how many hours will the population equal 1500? 2000?

Example: At 9 am on October 19, 2009 a body was found in room 327 at the University Center. The room is kept at a constant temperature of 72° F. The medical examiner was called and he arrived in eight minutes. The first thing he did was to take the temperature of the body. It was 83° F. Thirty minutes later the temperature of the body was taken again and it was now 78° F. Help the police by telling them when the person was murdered.

$$kt = \ln\frac{T - S}{T_0 - S}$$

6. A spacecraft can attain a stable orbit 300 km above Earth if it reaches a velocity of 7.7 km/s. The formula for a rocket's maximum velocity *v* in kilometers per second is $v = -0.0098t + c\ln R$. The booster rocket fires for *t* seconds, and the velocity of the exhaust is *c* km/s. The ratio of the mass of the rocket filled with fuel to its mass without fuel is *R*.

a) Suppose a rocket used to propel a spacecraft has a mass ratio of 25, an exhaust velocity of 2.8 km/s, and a firing time of 100 s. Can the spacecraft obtain a stable orbit 300 km above Earth?

b) Suppose a booster rocket's mass ratio is 15, the exhaust velocity is 2.1 km/s, and the firing time is 30 s. Find the maximum velocity of the spacecraft. Can it achieve a stable orbit 300 km above Earth?

7. You make an investment of $5000 into an account that will pay 8% interest, compounded quarterly. How much time must pass before you have $6000 in the account?

8. The population of bighorn sheep in Mexico was approximately 6,200 in 1971. By 1999, about 2,300 remained. Assume this data fits an exponential model and use your calculator to determine when only 750 bighorn sheep remain in Mexico.

9. Recall that the Richter scale is a measure of earthquake magnitude. The energy released in an earthquake of magnitude M is $E \bullet 30^M$. An earthquake in Alaska of magnitude 9.2 released 164 times the energy of a quake in California. Find the magnitude of the CA quake.

10. **Sound INTENSITY:**

Recall that loudness, measured in decibels (dB) is $L = 10\log\left(\frac{I}{I_0}\right)$ **where *I* is the intensity of the sound, and I_0 is the threshold of sound, at 10^{-12} W/m^2.**

A conversation measures 60 dB. Find the intensity of this sound in W/m^2.

A jazz concert measures 90 dB. Find the intensity of this sound in W/m^2.

How many times as intense is the jazz concert compared to the conversation?

11.

A detective is called to the scene of a crime where a dead body has just been found. She arrives on the scene at 10:23 pm and begins her investigation. Immediately, the temperature of the body is taken and is found to be 80° F. The detective checks the programmable thermostat and finds that the room has been kept at a constant 68° F for the past 3 days.

After evidence from the crime scene is collected, the temperature of the body is taken once more and found to be 78.5° F. This last temperature reading was taken exactly one hour after the first one. The next day the detective is asked by another investigator, **"What time did our victim die?"** Assuming that the victim's body temperature was normal (98.6° F) prior to death, what is her answer to this question? Newton's Law of Cooling can be used to determine a victim's time of death.

$$T(t) = T_e + (T_0 - T_e)\, e^{-kt}, \quad \text{OR} \qquad kt = \ln\frac{T - S}{T_0 - S}$$