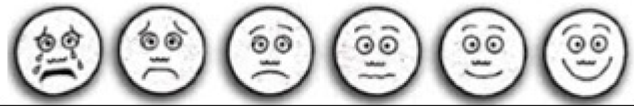
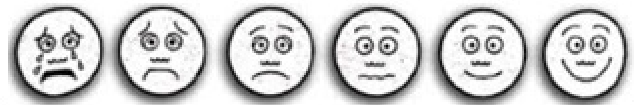
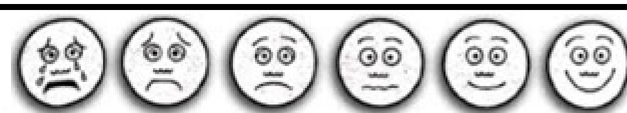
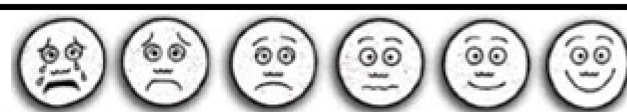
Name ____ Period ________

Algebra 3-4 Unit 6 Part 2
Logarithms

6.13	I can convert between logarithmic and exponential notation.	
6.14	I can apply the properties of logarithms.	
6.15-17	I can solve using logarithms and exponents.	
6.18-19	I can graph logarithms.	

My goal for this unit: ____

____

What I need to do to reach my goal: ___________________________________

____

____

Name ____ Period __________________

Algebra 3-4 Unit 6.13
Logs and Exponents

A logarithm is just another way to write an exponent!

Exponential Form	Logarithmic Form
$3^2 = 9$	$\log_{\square} \square = \square$
$4^3 = 64$	$\log_{\square} \square = \square$
$2^7 = 128$	

A logarithm (log) is another way of writing exponents.

Logarithmic Form	→	Exponential Form

Read as "**log base b of a equals x.**"

Note: If there is no number written as a subscript next to log, it is assumed to be a 10:
$\log a = b$ means $\log_{10} a = b$

Directions: *Write each exponential equation in logarithmic form.*

1. $2^6 = 64$	2. $4^{-2} = \frac{1}{16}$	3. $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$
4. $3^7 = 2187$	5. $12^2 = 144$	6. $5^3 = 125$

Directions: *Write each logarithmic equation in exponential form.*

7. $\log_7 49 = 2$	8. $\log_2 \frac{1}{16} = -4$	9. $\log_8 48 = x$
10. $\log_{10} 100{,}000 = 5$	11. $\log_4 1024 = 5$	12. $\log_9 729 = 3$

Directions: *Simplify without a calculator.*

13. $\log_4 16 = x$	14. $\log_8 1$	15. $\log_5 625$
16. $\log_4 x = 2$	17. $\log_9 x = 0.5$	18. $\log_2 y = 4$
19. $\log_4 2 = x$	20. $\log_8 2$	21. $\log_3 \frac{1}{9}$
22. $\log_4 64$	23. $\log_5 25 = x$	24. $\log_{10} 1000$
25. $\log_{15} 1$	26. $\log_{10} 100$	27. $\log_4 0.25$
28. $\log_2 16$	29. $\log_4 1$	30. $\log_9 81$
31. $\log_3 x = 4$	32. $\log_x 16 = 4$	33. $\log_x 25 = 2$

Name ____ Period ____________

Algebra 3-4 Unit 6.14
Properties of Logs

Product Property of Logarithms	$\log_b (mn) = \log_b m + \log_b n$
Quotient Property of Logarithms	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Property of Logarithms	$\log_b m^n = n \log_b m$

Condense into a single logarithm. Simplify if possible.

1. $\log_2 7 + \log_2 4$	**2.** $\log 25 + \log 4$	**3.** $\log_4 2x + \log_4 4x^2$

Expand using the product property.

4. $\log 6$	**5.** $\log_7 45$	**6.** $\log_2 (5x)$

Condense into a single logarithm. Simplify if possible.

7. $\log_3 24 - \log_3 8$	**8.** $\log_2 15 - \log_2 15$	**9.** $\log_4 x^9 - \log_4 x^2$

Expand using the quotient property.

10. $\log_8 4$	**11.** $\log_5 \frac{1}{3}$	**12.** $\log\left(\frac{m}{7}\right)$

Condense into a single logarithm. Simplify if possible.

13. $5 \cdot \log_4 2$	**14.** $7 \cdot \log_2 x$	**15.** $\frac{1}{3} \cdot \log 8$

Expand using the power property. Simplify if possible.

16. $\log_2 8^7$	**17.** $3 \cdot \log 4^{x-1}$	**18.** $\log_7 \sqrt{w}$

CONDENSING LOGS	Directions: Rewrite as a single logarithm. Simplify if possible.	
	19. $2 \cdot \log 6 - \log 9$	**20.** $4 \cdot \log_4 a + 2 \cdot \log_4 b$
	21. $7 \cdot \log_4 u - 3 \cdot \log_4 v^2$	**22.** $\log_2 15 + \log_2 4 - \log_2 6$
	23. $\log_3 4 + \log_3 y + \frac{1}{2} \cdot \log_3 49$	**24.** $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$
	25. $3 \cdot \log_2 4 - \log_2 32$	**26.** $2 \cdot \log 6 - \frac{1}{4} \cdot \log 16 + \log 3$
EXPANDING LOGS	Directions: Expand each logarithm.	
	27. $\log_6 \left(xyz^4\right)$	**28.** $\log_4 \left(\frac{a^9}{b}\right)$
	29. $\log_7 \left(q^4 r^2\right)^2$	**30.** $\log_2 \left(\frac{y}{z^5}\right)^2$
	31. $\log \sqrt{7x^3}$	**32.** $\log_3 \sqrt[4]{m^5 n^2}$

Name ______________________________ Period ____________

Algebra 3-4 Unit 6.15
Solving Using Logs and Exponents (Day 1)

One way to solve exponential equations, is to write both sides of the equation with the same base.

$2^{x+6} = 2^5$ same base ⇒ exponents are equal $x + 6 = 5$ $x = -1$ Check: $2^{-1+6} = 2^5$ ✓	$9^{2x-3} = 27$ different bases $(3^2)^{2x-3} = 3^3$ rewrite with the same base $3^{4x-6} = 3^3$ simplify $4x - 6 = 3$ same base ⇒ exponents are equal $4x = 9$ $x = 2.25$ Check: $9^{2(2.25)-3} = 27$ ✓

Directions: *Solve each equation for the unknown value showing all work using the method of writing each side of the equation using the same base. Check your answer.*

1. $2^{x+6} = 4$	2. $16^{3x} = 8^{x+6}$	3. $9^{2x} = 27^{x+4}$
4. $256^{0.5x} = 64^{2x+5}$	5. $\left(\frac{1}{2}\right)^{x} = 16^2$	6. $\left(\frac{1}{32}\right)^{2x} = 64$
7. $\left(\frac{1}{27}\right)^{x-6} = 27$	8. $216^{\frac{x}{3}} = 36^{2x+3}$	9. $\left(\frac{1}{9}\right)^{3x} = 27$
10. $16^{3x} = 64^{x+9}$	11. $81^{x} = 243^{x+2}$	12. $\left(\frac{1}{2}\right)^{3x} = 8^2$

Another way to solve exponential equations, is to take the log of both sides.

$5^{2x-3} = 18$ cannot use same base $\log 5^{2x-3} = \log 18$ take log of both sides $2x - 3(\log 5) = \log 18$ power property $2x - 3 = \frac{\log 18}{\log 5}$ isolate x $x = (\frac{\log 18}{\log 5} + 3) \div 2$ $x \approx 2.40$ Check: $5^{2(2.40)-3} = 18$ ✓	$e^{4x-9} = 56$ cannot use same base $\ln e^{4x-9} = \ln 56$ take ln of both sides $4x - 9 (\ln e) = \ln 56$ power property $4x - 9 = \ln 56$ $\ln e = 1$ $x = (\ln 56 + 9) \div 4$ isolate x $x \approx 3.26$ Check: $e^{4(3.26)-9} = 56$ ✓

Directions: *Solve each equation for the unknown value showing all work using the method of taking the log of both sides. Check your answer.*

13. $5^{2x} = 20$	14. $12^{2x-8} = 15$	15. $12^{x-1} = 20^2$
16. $3e^{2x-3} - 4 = 78$	17. $6e^{10x-8} - 4 = 34$	18. $8(10)^{7x-6} - 8 = 59$
19. $-6e^{-4x-1} + 3 = -37$	20. $8^{2x-5} = 48$	21. $4^{x+2} = 20$
22. $4^{2x} = 6$	23. $5^{5x-6} = 50$	24. $4e^{x+3} = 22$

Name ______________________________ Period ____________

Algebra 3-4 Unit 6.16
Solving Using Logs and Exponents (Day 2)

Solve logarithmic equations by applying the properties (if needed), then writing as an exponent. Solve resulting equation. Check.

$\log_2 (5x + 7) = 5$ $2^5 = 5x + 7$ write as an exponent $32 = 5x + 7$ solve for x $25 = 5x$ solve for x $x = 5$ ✓	$\log_4 x + \log_4 (x - 12) = 3$ $\log_4 (x(x - 12)) = 3$ properties of logs $4^3 = x^2 - 12x$ write as an exponent $x^2 - 12x - 64 = 0$ set equal to 0 $(x + 4)(x - 16) = 0$ factor $\cancel{x = -4}$ $x = 16$ ✓ $x \neq -4$

Directions: *Solve by applying the properties, writing as an exponent, then solving.*

1. $\log_3 (9x + 2) = 4$	2. $\log_4 x + \log_4 (x - 6) = 2$	3. $\log (5x - 11) = 2$
4. $\ln (4x - 1) = 3$	5. $\log_2 (x + 1) - \log_2 (x - 4) = 3$	6. $\ln (3x + 11) = 4$
7. $\log_6 x + \log_6 (x + 5) = 2$	8. $\log_4 (4x - 9) = 3$	9. $\log_5 (4x + 11) = 2$

Solve logarithmic equations by applying the properties then dropping the logs on each side, then solve. Check.

$\log_3 (7x + 3) = \log_3 (5x + 9)$ $7x + 3 = 5x + 9$ drop the logs $2x = 6$ solve for x $x = 3$ ✓	$\log_7 (x - 2) + \log_7 (x + 3) = \log_7 14$ $\log_7 ((x - 2)(x + 3)) = \log_7 14$ properties of logs $(x - 2)(x + 3) = 14$ drop the logs $x^2 + 3x - 2x - 6 = 14$ FOIL $x^2 + x - 20 = 0$ set equal to 0 $(x + 5)(x - 4) = 0$ factor $\cancel{x = -5}$ $x = 4$ ✓ $x \neq -5$

Directions: *Solve by applying the properties, dropping the logs on each side, then solving.*

10. $\log 5x = \log (2x + 9)$	11. $\log_4 (2x + 1) = \log_4 (x + 2) - \log_4 3$
12. $\log_8 x + \log_8 (x + 6) = \log_8 (5x + 12)$	13. $\ln (2x - 1) + \ln (x + 3) = \ln (x^2 + x - 7)$
14. $\log (x - 2) - \log (2x - 3) = \log 2$	15. $\log (10 - 4x) = \log (10 - 3x)$
16. $\log_6 (x + 4) + \log_6 (x - 2) = \log_6 4x$	17. $\log_9 (3x + 5) = \log_9 (7x - 12)$
18. $\log_9 (-11x + 2) = \log_9 (x^2 + 30)$	19. $\log_{12} (x^2 + 35) = \log_{12} (-12x - 1)$

Name ____ Period _______________

Algebra 3-4 Unit 6.17
Solving Using Logs and Exponents (Day 3)

Solve each equation. Use one of the 4 methods you have practiced the last few days:

1. Write exponents using the same base
2. Take the log of both sides
3. Use properties of logs then write as an exponent
4. Use properties of logs then drop the log on both sides

1. $-3(10)^{4-x} - 4 = -91$	2. $4^{-x} = 32$
3. $\log(4x - 2) = \log(-5x + 5)$	4. $\log_6 x + \log_6 (x - 9) = 2$
5. $3^{5x} = 27^{2x+1}$	6. $\left(\frac{1}{16}\right)^{x+5} = 8^2$
7. $5^{x-3} = 600$	8. $\ln(6x - 5) = 3$

9. $-7(10)^{8-10x}+9=4$	10. $\ln(x-3)-\ln(x-5)=\ln 5$
11. $\log_5 6+\log_5 2x^2=\log_5 48$	12. $3^{4x}=90$
13. $10e^{8x+1}-3=70$	14. $\log_4(3x-2)-\log_4(4x+1)=2$
15. In the year 2010, the population of a city was 22 million and was growing at a rate of about 2.3% per year. The function $p(t)=22(1.023)^t$ gives the population, in millions, t years after 2010. Use the model to determine in what year the population will reach 30 million. Round to the nearest year.	16. A sample of bacteria began with a population of 100 and grows over time at a rate of 35% per hour. Write a function to model this growth. How long before the population doubles?
17. In 2005, an orchard had 24,000 blueberries and the number has been growing at a rate of about 5% per year. The function $b(t)=24(1.05)^t$ gives the number of blueberries, in thousands, t years after 2005. Use the model to determine in what year the number will reach 55,000. Round to the nearest year.	18. A sample of cancer cells began with 400 cells and grows at a rate of 60% per hour. Write a function to model this growth. How long before the number of cells triples?

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Algebra 3-4 Unit 6.18
Graphs of Logarithms

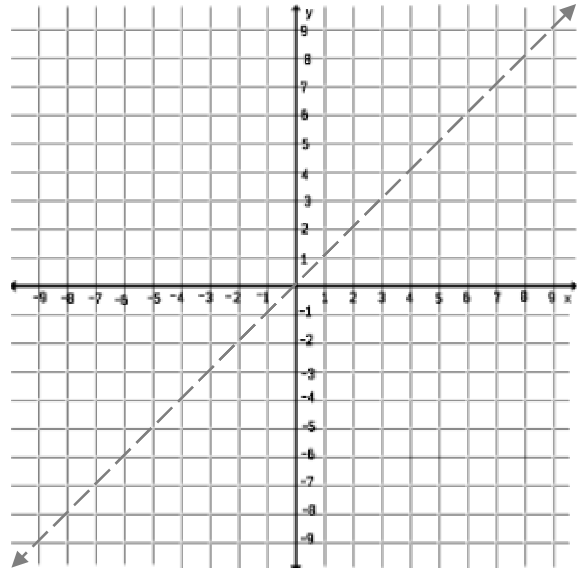
Complete the table of values for each of the following (use a graphing calculator or desmos) the use that to graph (on same graph, but different colors).

$y = 2^x$

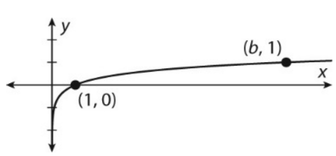
x	y
-3	
-2	
-1	
0	
1	
2	
3	

$y = \log_2 x$

x	y
$\frac{1}{8}$	
$\frac{1}{4}$	
$\frac{1}{2}$	
1	
2	
4	
8	

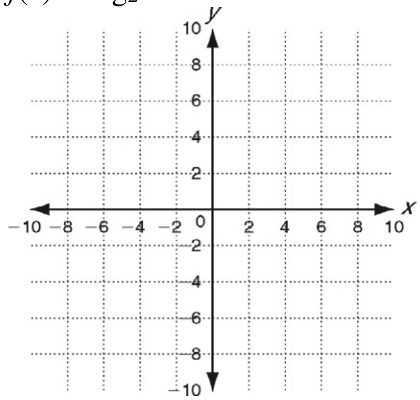


What relationship did you notice in the table of values?	What relationship did you notice on the graph?
What is the domain and the range for the first graph equation?	What is the domain and the range for the second equation?

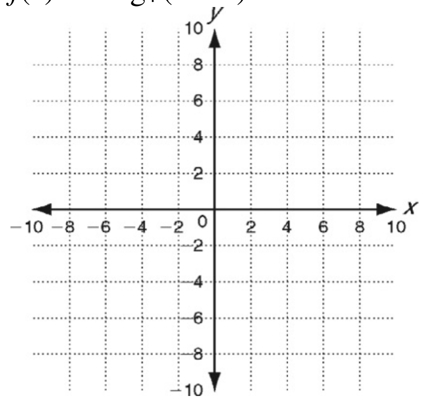
<table>
<tr><th></th><th>$f(x) = \log_b x$</th><th colspan="2">$g(x) = a\log_b(x-h)+k$</th></tr>
<tr><td rowspan="3">Graph</td><td rowspan="3"></td><td>a</td><td>$|a| > 1 \rightarrow$vertical stretch by $|a|$
$|a| < 1 \rightarrow$vertical compression by $|a|$
$a < 0 \rightarrow$reflection over x-axis</td></tr>
<tr><td>h</td><td>$h > 0 \rightarrow$ shift right h units
$h < 0 \rightarrow$ shift left h units</td></tr>
<tr><td>k</td><td>$k > 0 \rightarrow$ shift up k units
$k < 0 \rightarrow$ shift down k units</td></tr>
<tr><td>Vertical Asymptote</td><td>$x = 0$ ⟶</td><td colspan="2">$x = h$</td></tr>
<tr><td>Reference Point</td><td>(1, 0) ⟶</td><td colspan="2">(1 + h, k)</td></tr>
<tr><td>Reference Point</td><td>(b, 1) ⟶</td><td colspan="2">(b + h, a + k)</td></tr>
</table>

Directions: *Graph each function. Tell how the graph is transformed from the graph of its parent function.*

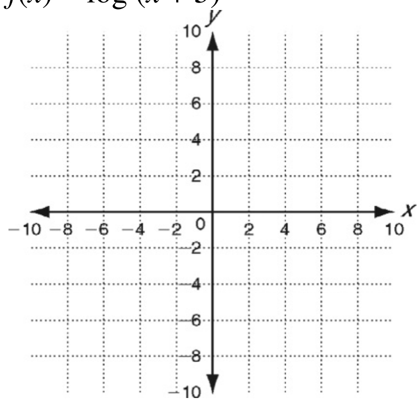
1. $f(x) = \log_2 x + 4$



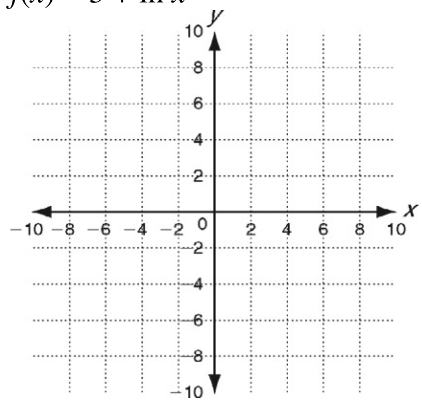
2. $f(x) = 3\log_4 (x + 6)$



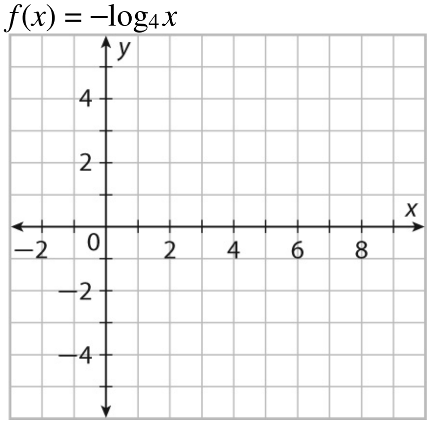
3. $f(x) = \log (x + 5)$



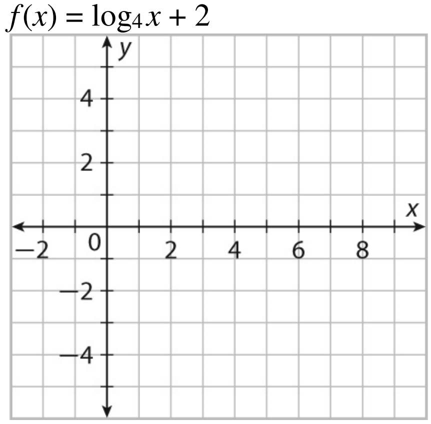
4. $f(x) = 3 + \ln x$



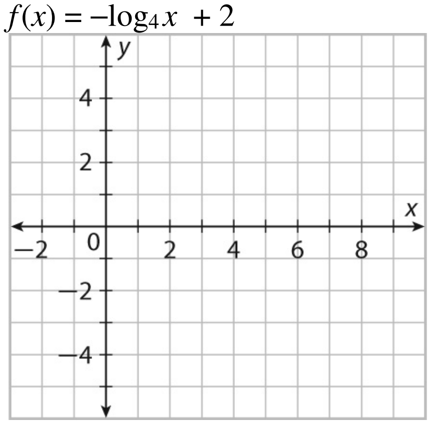
5. $f(x) = -\log_4 x$



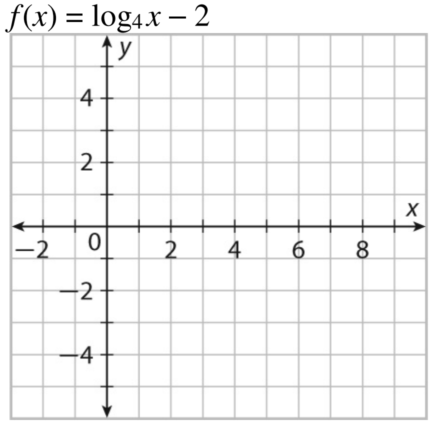
6. $f(x) = \log_4 x + 2$



7. $f(x) = -\log_4 x + 2$



8. $f(x) = \log_4 x - 2$



Name ____ Period ______________

Algebra 3-4 Unit 6.19
Graphs of Logarithms (Day 2)

Directions: *Write each transformed function.*

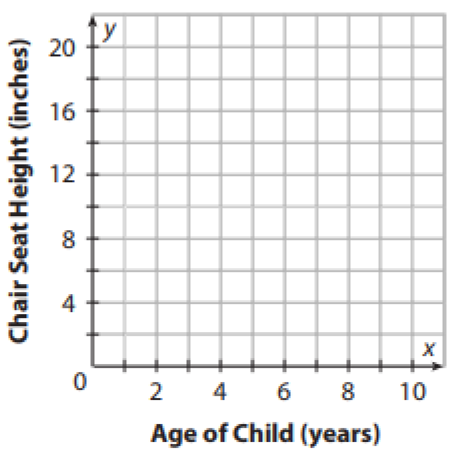
1. The function $f(x) = \log(x + 1)$ is reflected across the x-axis and translated down 4 units.	2. The function $f(x) = \log_8(x - 3)$ is compressed vertically by a factor of $\frac{2}{5}$ and translated up 11 units.
3. The function $f(x) = -\log_9(x + 4)$ is translated 4 units right and 1 unit down and vertically stretched by a factor of 7.	4. The function $f(x) = 3\ln(2x + 8)$ is vertically stretched by a factor of 3, translated 7 units up, and reflected across the x-axis.
5. The function $f(x) = -\log(5 - x) - 2$ is translated 6 units left, vertically compressed by a factor of $\frac{1}{3}$, and reflected across the x-axis.	6. The function $f(x) = 8\log_7 x - 5$ is compressed vertically by a factor of 0.5, translated right 1 unit, and reflected across the x-axis.
7. What transformations does the function $f(x) = -\ln(x + 1) - 2$ undergo to become the function $g(x) = \ln(x - 1)$?	8. The function $f(x) = \ln x$ is reflected across the x-axis.
9. The function $f(x) = \log_8 x$ is vertically compressed by a factor of 0.5.	10. The function $f(x) = \log_3 x$ is vertically stretched by a factor of 4.
11. The function $f(x) = \log x$ is shifted 3 units left and reflected across the x-axis.	12. The graph of the function $f(x) = \log_3 x$ is transformed by reflecting across the x-axis, translating 2 units left, and 4 unit down.

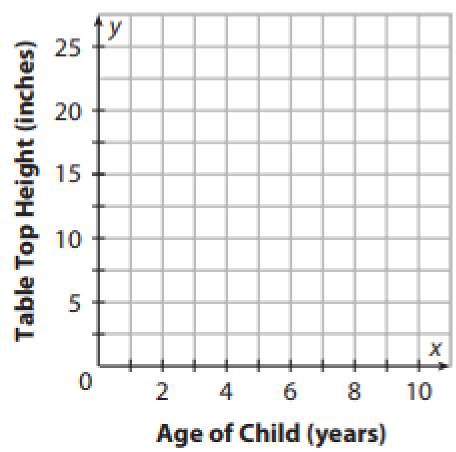
Directions: *Describe the transformation from the parent function to the given function.*

13. $g(x) = 5\log_2(x+2) - 1$	14. $g(x) = -\log(x+5) + 2$
15. $g(x) = 3\log_6(x-4) - 2$	16. $g(x) = -2\log_8(x+9) + 3$

Given the following data about the heights of chair seats and table tops for children, create scatterplots of the ordered pairs (age of child, chair seat height) (age of child, table top height).

Age of Child (years)	Chair Seat Height (inches)	Table Top Height (inches)
1	5	12
1.5	6.5	14
2	8	16
3	10	18
5	12	20
7.5	14	22
11	16	25





17. Explain if a logarithmic model would be appropriate for each data set.

18. Perform logarithmic regression for each data set.

19. Use your regression equation to predict the chair seat height for a child 14 years old and 50 years old. Explain if each is reasonable or not.

20. Use your regression equation to predict the table top height for a child 14 years old and 50 years old. Explain if each is reasonable or not.

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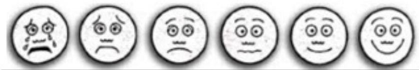
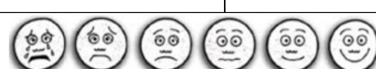
Algebra 3-4 Unit 6.20
Are You Ready for Unit 6 Part 2 Assessment?



I can apply logarithmic properties and rules.

1. Write as an exponent: ln $x = 8$	2. Write as an exponent: log $x = 3$
3. Write as a logarithm: $x^4 = 25$	4. Write as a logarithm: $e^3 = x$
5. Write as an exponent: $\log_3 x = 4$	6. Write as a logarithm: $10^x = 7$
7. Write as a single logarithm: $\log_3 8 + \log_3 7$	8. Write as a single logarithm: $\log_9 x - \log_9 y$
9. Write as a single logarithm: $\log_2 x + \log_2 y - \log_2 z$	10. Expand using the properties of logarithms. $\log \frac{a^2 b}{c^4}$
11. Expand using the properties of logarithms. $\log xy^3$	12. Expand using the properties of logarithms. $\log_3 \frac{xy^3}{a^3 b^2 c}$

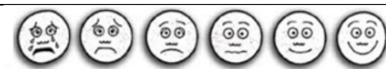


I can graph logarithmic equations.

13. Describe the transformations from $f(x) = \log_2(x)$ to $g(x) = -\log_2(x-3)$	14. Describe the transformations from $f(x) = \log_2(x)$ to $g(x) = 3\log_2(x+5) - 2$
15. Describe the transformations from $f(x) = \log_2(x)$ to $g(x) = -0.5\log_2(x) - 9$	16. Describe the transformations from $f(x) = \log_2(x)$ to $g(x) = \log_2(-x) + 6$
17. The graph of $f(x) = \log_2 x$ is transformed by translating up 2 units and left 4 units. What is the function of the transformed graph?	18. The graph of $f(x) = \log_2 x$ is transformed by reflecting over the x-axis, translating down 3 units and right 1 unit. What is the function of the transformed graph?

19. The graph of $f(x) = \log_2 x$ is transformed by a vertical stretch by a factor of 3 and translating down 5 units. What is the function of the transformed graph?

20. The graph of $f(x) = \log_2 x$ is transformed by a reflection over the x-axis and a vertical stretch by a factor of 5. What is the function of the transformed graph?



I can solve equations with logarithms and exponents.

21. Solve: $3^{2x-1} - 4 = 239$

22. Solve: $2^{3x+4} + 5 = 133$

23. Solve: $3e^{x} = 11$

24. Solve: $9 + 2e^{x+7} = 22$

25. Solve: $-8 + 4^{x-9} = 92$

26. Identify x in each:

$\ln(x) = 1.7$

$\ln(12) = x$

$e^{3.5} = x$

$e^{x} = 92$

27. The population of a town was 2,500 people in the year 2000. If it is growing exponentially at a rate of 8% per year, write an equation to model the growth.

Use your model to determine in what year the population will double what it was in the year 2000.

28. The population of a town was 2,500 people in the year 2000. If it is decreasing exponentially at a rate of 8% per year, write an equation to model the decay.

Use your model to determine in what year the population will reach 1,000 people.

29. The value of a painting can be modeled by the equation $V(t) = 250(0.93)^{t}$ where $t = 0$ in the year 2010 and the value is in thousands of dollars. What will the value of the painting be in the year 2020?

30. The value of a painting can be modeled by the equation $V(t) = 250(1.28)^{t}$ where $t = 0$ in the year 2010 and the value is in thousands of dollars. In approximately what year will the painting be valued at \$400,000?

Name Key Period ____________

Algebra 3-4 Unit 6.13
Logs and Exponents

A logarithm is just another way to write an exponent!

Exponential Form	Logarithmic Form
$3^2 = 9$	$\log_{\boxed{3}} \boxed{9} = \boxed{2}$
$4^3 = 64$	$\log_{\boxed{4}} \boxed{64} = \boxed{3}$
$2^7 = 128$	$\log_2 128 = 7$

A logarithm (log) is another way of writing exponents.

Logarithmic Form	→	Exponential Form
$\log_b a = x$		$b^x = a$

Read as "**log base b of a equals x.**"

Note: If there is no number written as a subscript next to log, it is assumed to be a 10:
$\log a = b$ means $\log_{10} a = b$

Directions: *Write each exponential equation in logarithmic form.*

1. $2^6 = 64$ $\log_2 64 = 6$	2. $4^{-2} = \frac{1}{16}$ $\log_4 \frac{1}{16} = -2$	3. $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ $\log_{1/3} \frac{1}{27} = 3$
4. $3^7 = 2187$ $\log_3 2187 = 7$	5. $12^2 = 144$ $\log_{12} 144 = 2$	6. $5^3 = 125$ $\log_5 125 = 3$

Directions: *Write each logarithmic equation in exponential form.*

7. $\log_7 49 = 2$ $7^2 = 49$	8. $\log_2 \frac{1}{16} = -4$ $2^{-4} = \frac{1}{16}$	9. $\log_8 48 = x$ $8^x = 48$
10. $\log_{10} 100{,}000 = 5$ $10^5 = 100{,}000$	11. $\log_4 1024 = 5$ $4^5 = 1024$	12. $\log_9 729 = 3$ $9^3 = 729$

Directions: *Simplify without a calculator.*

13. $\log_4 16 = x$ $4^x = 16$ $x = 2$	14. $\log_8 1 = x$ $8^x = 1$ $x = 0$	15. $\log_5 625 = x$ $5^x = 625$ $x = 4$
16. $\log_4 x = 2$ $4^2 = x$ $16 = x$	17. $\log_9 x = 0.5$ $9^{.5} = x$ $\sqrt{9} = x$ $3 = x$	18. $\log_2 y = 4$ $2^4 = y$ $16 = y$
19. $\log_4 2 = x$ $4^x = 2$ $x = \frac{1}{2}$	20. $\log_8 2 = x$ $8^x = 2$ $x = \frac{1}{3}$	21. $\log_3 \frac{1}{9} = x$ $3^x = \frac{1}{9}$ $x = -2$
22. $\log_4 64 = x$ $4^x = 64$ $x = 3$	23. $\log_5 25 = x$ $5^x = 25$ $x = 2$	24. $\log_{10} 1000 = x$ $10^x = 1000$ $x = 3$
25. $\log_{15} 1 = x$ $15^x = 1$ $x = 0$	26. $\log_{10} 100 = x$ $10^x = 100$ $x = 2$	27. $\log_4 0.25 = x$ $4^x = .25$ $x = -1$
28. $\log_2 16 = x$ $2^x = 16$ $x = 4$	29. $\log_4 1 = x$ $4^x = 1$ $x = 0$	30. $\log_9 81 = x$ $9^x = 81$ $x = 2$
31. $\log_3 x = 4$ $3^4 = x$ $x = 81$	32. $\log_x 16 = 4$ $x^4 = 16$ $x = 2$	33. $\log_x 25 = 2$ $x^2 = 25$ $x = 5$

Name Key Period ____________

Algebra 3-4 Unit 6.14
Properties of Logs

Product Property of Logarithms	$\log_b(mn) = \log_b m + \log_b n$
Quotient Property of Logarithms	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Property of Logarithms	$\log_b m^n = n \log_b m$

Condense into a single logarithm. Simplify if possible.

1. $\log_2 7 + \log_2 4$ $\log_2 28$	**2.** $\log 25 + \log 4$ $\log 100 = x$ $10^x = 100$ $x = 2$	**3.** $\log_4 2x + \log_4 4x^2$ $\log_4 8x^3$

Expand using the product property.

4. $\log 6$ $\log 2 + \log 3$	**5.** $\log_7 45$ $\log_7 5 + \log_7 3 + \log_7 3$	**6.** $\log_2(5x)$ $\log_2 5 + \log_2 x$

Condense into a single logarithm. Simplify if possible.

7. $\log_3 24 - \log_3 8$ $\log_3 3 = x$ $3^x = 3$ $x = 1$	**8.** $\log_2 15 - \log_2 15$ $\log_2 1 = x$ $2^x = 1$ $x = 0$	**9.** $\log_4 x^9 - \log_4 x^2$ $\log_4 x^7$

Expand using the quotient property.

10. $\log_8 4$ $\log_8 12 - \log_8 3$ (accept all correct)	**11.** $\log_5 \frac{1}{3}$ $\log_5 1 - \log_5 3$	**12.** $\log\left(\frac{m}{7}\right)$ $\log m - \log 7$

Condense into a single logarithm. Simplify if possible.

13. $5 \cdot \log_4 2$ $\log_4 2^5$ $\log_4 32$	**14.** $7 \cdot \log_2 x$ $\log_2 x^7$	**15.** $\frac{1}{3} \cdot \log 8$ $\log 8^{1/3}$ $\log 2$

Expand using the power property. Simplify if possible.

16. $\log_2 8^7$ $7 \log_2 8$ $7(3) = 21$	**17.** $3 \cdot \log 4^{x-1}$ $(3x-3) \log 4$	**18.** $\log_7 \sqrt{w}$ $\frac{1}{2} \log_7 w$

CONDENSING LOGS

Directions: Rewrite as a single logarithm. Simplify if possible.

19. $2 \cdot \log 6 - \log 9$

$\log 36 - \log 9$

$\log 4$

20. $4 \cdot \log_4 a + 2 \cdot \log_4 b$

$\log_4 a^4 + \log_4 b^2$

$\log_4 a^4 b^2$

21. $7 \cdot \log_4 u - 3 \cdot \log_4 v^2$

$\log_4 \frac{u^7}{v^6}$

22. $\log_2 15 + \log_2 4 - \log_2 6$

$\log_2 \frac{15 \cdot 4}{6}$

$\log_2 10$

23. $\log_3 4 + \log_3 y + \frac{1}{2} \cdot \log_3 49$

$\log_3 4y \cdot 7$

$\log_3 28y$

24. $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

$\log_5 \frac{2 \cdot 3}{3}$

$\log_5 2$

25. $3 \cdot \log_2 4 - \log_2 32$

$\log_2 \frac{64}{32}$

$\log_2 2 = x$

$2^x = 2$ $\quad x = 1$

26. $2 \cdot \log 6 - \frac{1}{4} \cdot \log 16 + \log 3$

$\log \frac{36 \cdot 3}{2}$

$\log 54$

EXPANDING LOGS

Directions: Expand each logarithm.

27. $\log_6 (xyz^4)$

$\log_6 x + \log_6 y + 4 \log_6 z$

28. $\log_4 \left(\frac{a^9}{b}\right)$

$9 \log_4 a - \log_4 b$

29. $\log_7 (q^4 r^2)^2$

$8 \log_7 q + 4 \log_7 r$

30. $\log_2 \left(\frac{y}{z^5}\right)^2$

$2 \log_2 y - 10 \log_2 z$

31. $\log \sqrt{7x^3}$

$\frac{1}{2}(\log 7 + 3 \log x)$

32. $\log_3 \sqrt[4]{m^5 n^2}$

$\frac{1}{4}[5 \log_3 m + 2 \log_3 n]$

Name Key Period ____________

Algebra 3-4 Unit 6.12
Solving Using Logs and Exponents (Day 1)

One way to solve exponential equations, is to write both sides of the equation with the same base.

$2^{x+6}=2^5$ same base $\Rightarrow$ exponents are equal $x+6=5$ $x=-1$ Check: $2^{-1+6}=2^5$ ✓	$9^{2x-3}=27$ different bases $(3^2)^{2x-3}=3^3$ rewrite with the same base $3^{4x-6}=3^3$ simplify $4x-6=3$ same base $\Rightarrow$ exponents are equal $4x=9$ $x=2.25$ Check: $9^{2(2.25)-3}=27$ ✓

***Directions:** Solve each equation for the unknown value showing all work using the method of writing each side of the equation using the same base. Check your answer.*

1. $2^{x+6}=4$

$2^{x+6}=2^2$
$x+6=2$
$\boxed{x=-4}$ ✓

2. $16^{3x}=8^{x+6}$

$(2^4)^{3x}=(2^3)^{x+6}$
$12x=3x+18$
$9x=18$
$\boxed{x=2}$ ✓

3. $9^{2x}=27^{x+4}$

$(3^2)^{2x}=(3^3)^{x+4}$
$4x=3x+12$
$\boxed{x=12}$ ✓

4. $256^{0.5x}=64^{2x+5}$

$(4^4)^{.5x}=(4^3)^{2x+5}$
$2x=6x+15$
$-15=4x$
$\boxed{x=-\frac{15}{4}=-3.75}$ ✓

5. $\left(\frac{1}{2}\right)^x=16^2$

$2^{-1x}=(2^4)^2$
$-x=8$
$\boxed{x=-8}$ ✓

6. $\left(\frac{1}{32}\right)^{2x}=64$

$(2^{-5})^{2x}=2^6$
$-10x=6$
$\boxed{x=-.6}$ ✓

7. $\left(\frac{1}{27}\right)^{x-6}=27$

$(3^{-3})^{x-6}=3^3$
$-3x+18=3$
$-3x=-15$
$\boxed{x=5}$ ✓

8. $216^{\frac{x}{3}}=36^{2x+3}$

$(6^3)^{\frac{x}{3}}=(6^2)^{2x+3}$
$x=4x+6$
$-6=3x$
$\boxed{x=-2}$ ✓

9. $\left(\frac{1}{9}\right)^{3x}=27$

$(3^{-2})^{3x}=3^3$
$-6x=3$
$\boxed{x=-\frac{1}{2}}$ ✓

10. $16^{3x}=64^{x+9}$

$(2^4)^{3x}=(2^6)^{x+9}$
$12x=6x+54$
$6x=54$
$\boxed{x=9}$ ✓

or

$(4^2)^{3x}=(4^3)^{x+9}$
$6x=3x+27$
$3x=27$
$\boxed{x=9}$ ✓

11. $81^x=243^{x+2}$

$(3^4)^x=(3^5)^{x+2}$
$4x=5x+10$
$\boxed{-10=x}$ ✓

12. $\left(\frac{1}{2}\right)^{3x}=8^2$

$(2^{-1})^{3x}=(2^3)^2$
$-3x=6$
$\boxed{x=-2}$ ✓

Another way to solve exponential equations, is to take the log of both sides.

$5^{2x-3} = 18$	cannot use same base	$e^{4x-9} = 56$	cannot use same base
$\log 5^{2x-3} = \log 18$	take log of both sides	$\ln e^{4x-9} = \ln 56$	take ln of both sides
$2x - 3(\log 5) = \log 18$	power property	$4x - 9\ (\ln e) = \ln 56$	power property
$2x - 3 = \dfrac{\log 18}{\log 5}$	isolate x	$4x - 9 = \ln 56$	$\ln e = 1$
$x = (\dfrac{\log 18}{\log 5} + 3) \div 2$		$x = (\ln 56 + 9) \div 4$	isolate x
$x \approx 2.40$		$x \approx 3.26$	
Check: $5^{2(2.40)-3} = 18$ ✓		Check: $e^{4(3.26)-9} = 56$ ✓	

Directions: *Solve each equation for the unknown value showing all work using the method of taking the log of both sides. Check your answer.*

13. $5^{2x} = 20$

$2x \log 5 = \log 20$

$2x = \dfrac{\log 20}{\log 5}$

$\boxed{x \approx 0.93}$ ✓

14. $12^{2x-8} = 15$

$2x-8\ (\log 12) = \log 15$

$2x-8 = \dfrac{\log 15}{\log 12}$

$\boxed{x \approx 4.54}$ ✓

15. $12^{x-1} = 20^2$

$x-1\ (\log 12) = \log 400$

$x-1 = \dfrac{\log 400}{\log 12}$

$\boxed{x \approx 3.41}$ ✓

16. $3e^{2x-3} - 4 = 78$
$+4$

$3e^{2x-3} = 82$

$e^{2x-3} = 27\frac{1}{3}$

$2x-3\ (\ln e) = \ln 27\frac{1}{3}$

$\boxed{x \approx 3.15}$ ✓

17. $6e^{10x-8} - 4 = 34$

$6e^{10x-8} = 38$

$e^{10x-8} = 6\frac{1}{3}$

$10x-8\ (\ln e) = \ln 6\frac{1}{3}$

$\boxed{x \approx .98}$ ✓

18. $8(10)^{7x-6} - 8 = 59$
$+8 \quad +8$

$8(10)^{7x-6} = 67$

$10^{7x-6} = 8.375$

$7x-6\ (\log 10) = \log 8.375$

$\boxed{x \approx .99}$ ✓

19. $-6e^{-4x-1} + 3 = -37$
$-3 \quad -3$

$-6e^{-4x-1} = -40$

$e^{-4x-1} = 6\frac{2}{3}$

$-4x-1\ \ln e = \ln 6\frac{2}{3}$

$\boxed{x \approx -.72}$ ✓

20. $8^{2x-5} = 48$

$2x-5\ (\log 8) = \log 48$

$\boxed{x \approx 3.43}$ ✓

21. $4^{x+2} = 20$

$x+2\ (\log 4) = \log 20$

$\boxed{x \approx .16}$ ✓

22. $4^{2x} = 6$

$2x \log 4 = \log 6$

$\boxed{x \approx .65}$ ✓

23. $5^{5x-6} = 50$

$5x-6\ (\log 5) = \log 50$

$\boxed{x \approx 1.69}$ ✓

24. $4e^{x+3} = 22$

$e^{x+3} = 5.5$

$x+3(\ln e) = \ln 5.5$

$\boxed{x \approx -1.30}$ ✓

Name *Key* Period ______________

Algebra 3-4 Unit 6.13
Solving Using Logs and Exponents (Day 2)

Solve logarithmic equations by applying the properties (if needed), then writing as an exponent. Solve resulting equation. Check.

$\log_2(5x+7)=5$ $2^5=5x+7$ write as an exponent $32=5x+7$ solve for x $25=5x$ solve for x $x=5$ ✓	$\log_4 x+\log_4(x-12)=3$ $\log_4(x(x-12))=3$ properties of logs $4^3=x^2-12x$ write as an exponent $x^2-12x-64=0$ set equal to 0 $(x+4)(x-16)=0$ factor $\cancel{x=-4}$ $x=16$ ✓ $x\neq -4$

Directions: *Solve by applying the properties, writing as an exponent, then solving.*

1. $\log_3(9x+2)=4$ $3^4=9x+2$ $81=9x+2$ $9x=79$ $\boxed{x=\frac{79}{9}}$ ✓	2. $\log_4 x+\log_4(x-6)=2$ $\log_4 x^2-6x=2$ $x^2-6x=4^2$ $x^2-6x-16=0$ $(x+2)(x-8)=0$ $\cancel{x=-2}$ $\boxed{x=8}$ ✓	3. $\log(5x-11)=2$ $10^2=5x-11$ $100=5x-11$ $111=5x$ $\boxed{x=22.5}$ ✓
4. $\ln(4x-1)=3$ $e^3=4x-1$ $\boxed{x\approx 5.27}$ ✓	5. $\log_2(x+1)-\log_2(x-4)=3$ $\log_2\frac{x+1}{x-4}=3$ $2^3=\frac{x+1}{x-4}$ $8(x-4)=x+1$ $8x-32=x+1$ $7x=33$ $\boxed{x=\frac{33}{7}}$ ✓	6. $\ln(3x+11)=4$ $e^4=3x+11$ $\boxed{x\approx 14.53}$ ✓
7. $\log_6 x+\log_6(x+5)=2$ $\log_6 x^2+5x=2$ $6^2=x^2+5x$ $x^2+5x-36=0$ $(x-4)(x+9)=0$ $\boxed{x=4}$ ✓ $\boxed{\cancel{x=-9}}$	8. $\log_4(4x-9)=3$ $4^3=4x-9$ $64=4x-9$ $4x=73$ $\boxed{x=18.25}$ ✓	9. $\log_5(4x+11)=2$ $5^2=4x+11$ $25=4x+11$ $4x=14$ $\boxed{x=3.5}$ ✓

Solve logarithmic equations by applying the properties then dropping the logs on each side, then solve. Check.

$\log_3 (7x + 3) = \log_3 (5x + 9)$

$7x + 3 = 5x + 9$ drop the logs

$2x = 6$ solve for x

$x = 3$ ✓

$\log_7 (x - 2) + \log_7 (x + 3) = \log_7 14$

$\log_7 ((x - 2)(x + 3)) = \log_7 14$ properties of logs

$(x - 2)(x + 3) = 14$ drop the logs

$x^2 + 3x - 2x - 6 = 14$ FOIL

$x^2 + x - 20 = 0$ set equal to 0

$(x + 5)(x - 4) = 0$ factor

$\cancel{x = -5}$ $x = 4$ ✓ $x \neq -5$

Directions: *Solve by applying the properties, dropping the logs on each side, then solving.*

10. $\log 5x = \log (2x + 9)$

$5x = 2x + 9$

$3x = 9$

$\boxed{x = 3}$ ✓

11. $\log_4 (2x + 1) = \log_4 (x + 2) - \log_4 3$

$\log_4 (2x+1) = \log_4 \frac{x+2}{3}$

$2x + 1 = \frac{x+2}{3}$

$6x + 3 = x + 2$

$5x = -1$

$\boxed{x = -\frac{1}{5}}$ ✓

12. $\log_8 x + \log_8 (x + 6) = \log_8 (5x + 12)$

$\log_8 (x^2 + 6x) = \log_8 (5x + 12)$

$x^2 + 6x = 5x + 12$

$x^2 + x - 12 = 0$

$(x - 3)(x + 4) = 0$

$\boxed{x = 3}$ ✓ $\cancel{x = -4}$

13. $\ln (2x - 1) + \ln (x + 3) = \ln (x^2 + x - 7)$

$\ln (2x^2 + 6x - x - 3) = \ln (x^2 + x - 7)$

$2x^2 + 5x - 3 = x^2 + x - 7$

$x^2 + 4x + 4 = 0$

$(x+2)(x+2) = 0$

$\cancel{x = -2}$ $\boxed{\text{no solution}}$

14. $\log (x - 2) - \log (2x - 3) = \log 2$

$\log \frac{x-2}{2x-3} = \log 2$

$\frac{x-2}{2x-3} = 2$

$4x - 6 = x - 2$

$3x = 4$

$\boxed{x = \frac{4}{3}}$ ✓

15. $\log (10 - 4x) = \log (10 - 3x)$

$10 - 4x = 10 - 3x$

$\boxed{0 = x}$ ✓

16. $\log_6 (x + 4) + \log_6 (x - 2) = \log_6 4x$

$\log_6 (x^2 + 2x - 8) = \log_6 4x$

$x^2 + 2x - 8 = 4x$

$x^2 - 2x - 8 = 0$

$(x + 2)(x - 4) = 0$

$\cancel{x = -2}$ $\boxed{x = 4}$ ✓

17. $\log_3 (3x + 5) = \log_3 (7x - 12)$

$3x + 5 = 7x - 12$

$17 = 4x$

$\boxed{x = \frac{17}{4}}$ ✓

18. $\log_9 (-11x + 2) = \log_9 (x^2 + 30)$

$-11x + 2 = x^2 + 30$

$x^2 + 11x + 28 = 0$

$(x + 4)(x + 7) = 0$

$\boxed{x = -4}$ $\boxed{x = -7}$ ✓

19. $\log_{12} (x^2 + 35) = \log_{12} (-12x - 1)$

$x^2 + 35 = -12x - 1$

$x^2 + 12x + 36 = 0$

$(x + 6)(x + 6) = 0$

$\boxed{x = -6}$ ✓

Name Key Period ______

Algebra 3-4 Unit 6.17
Solving Using Logs and Exponents (Day 3)

Solve each equation. Use one of the 4 methods you have practiced the last few days:

1. Write exponents using the same base
2. Take the log of both sides
3. Use properties of logs then write as an exponent
4. Use properties of logs then drop the log on both sides

1. $-3(10)^{4-x} - 4 = -91$

$10^{4-x} = 29$

$(4-x)\log 10 = \log 29$

$4 - \log 29 = x$

$\boxed{x \approx 2.5376}$

2. $4^{-x} = 32$

$(2^2)^{-x} = 2^5$

$-2x = 5$

$\boxed{x = -2.5}$

3. $\log(4x-2) = \log(-5x+5)$

$4x - 2 = -5x + 5$

$9x = 7$

$\boxed{x = \frac{7}{9}}$

4. $\log_6 x + \log_6 (x-9) = 2$

$\log_6 x^2 - 9x = 2$

$6^2 = x^2 - 9x$

$x^2 - 9x - 36 = 0$

$(x-12)(x+3) = 0$

$\boxed{x = 12}$ $\cancel{x = -3}$

5. $3^{5x} = 27^{2x+1}$

$3^{5x} = (3^3)^{2x+1}$

$5x = 6x + 3$

$\boxed{-3 = x}$

6. $\left(\frac{1}{16}\right)^{x+5} = 8^2$

$(2^{-4})^{x+5} = (2^3)^2$

$-4x - 20 = 6$

$-4x = 26$

$\boxed{x = -6.5}$

7. $5^{x-3} = 600$

$x - 3 \log 5 = \log 600$

$x = \frac{\log 600}{\log 5} + 3$

$\boxed{x \approx 6.9746}$

8. $\ln(6x-5) = 3$

$e^3 = 6x - 5$

$\boxed{x \approx 4.1809}$

9. $-7(10)^{8-10x}+9=4$ $10^{8-10x}=\frac{5}{7}$ $8-10x\log 10=\log\frac{5}{7}$ $\boxed{x\approx 0.8146}$ ✓	10. $\ln(x-3)-\ln(x-5)=\ln 5$ $\ln\frac{x-3}{x-5}=\ln 5$ $\frac{x-3}{x-5}=5$ $5x-25=x-3$ $4x=22$ $\boxed{x=5.5}$ ✓
11. $\log_5 6+\log_5 2x^2=\log_5 48$ $\log_5 12x^2=\log_5 48$ $12x^2=48$ $x^2=4$ $\boxed{x=2,-2}$ ✓ ✓	12. $3^{4x}=90$ $4x\log 3=\log 90$ $\boxed{x\approx 1.0240}$ ✓
13. $10e^{8x+1}-3=70$ $e^{8x+1}=7.3$ $8x+1=\ln 7.3$ $\boxed{x\approx .1235}$ ✓	14. $\log_4(3x-2)-\log_4(4x+1)=2$ $\log_4\frac{3x-2}{4x+1}=2$ $4^2=\frac{3x-2}{4x+1}$ $16(4x+1)=3x-2$ $64x+16=3x-2$ $61x=-18$ $\boxed{x=-\frac{18}{61}}$ ✓
15. In the year 2010, the population of a city was 22 million and was growing at a rate of about 2.3% per year. The function $p(t)=22(1.023)^t$ gives the population, in millions, t years after 2010. Use the model to determine in what year the population will reach 30 million. Round to the nearest year. $30=22(1.023)^t$ $t\approx 13.64$ 14 yrs in the year 2024	16. A sample of bacteria began with a population of 100 and grows over time at a rate of 35% per hour. Write a function to model this growth. $b(t)=100(1.35)^t$ How long before the population doubles? $t\approx 2.3$ about 2.3 hours
17. In 2005, an orchard had 24,000 blueberries and the number has been growing at a rate of about 5% per year. The function $b(t)=24(1.05)^t$ gives the number of blueberries, in thousands, t years after 2005. Use the model to determine in what year the number will reach 55,000. Round to the nearest year. $55=24(1.05)^t$ $t\approx 16.9969$ 17 yrs in the year 2022	18. A sample of cancer cells began with 400 cells and grows at a rate of 60% per hour. Write a function to model this growth. $c(t)=400(1.6)^t$ How long before the number of cells triples? $t\approx 2.34$ about 2.3 hours

Name Key Period ____________

Algebra 3-4 Unit 6.18
Graphs of Logarithms

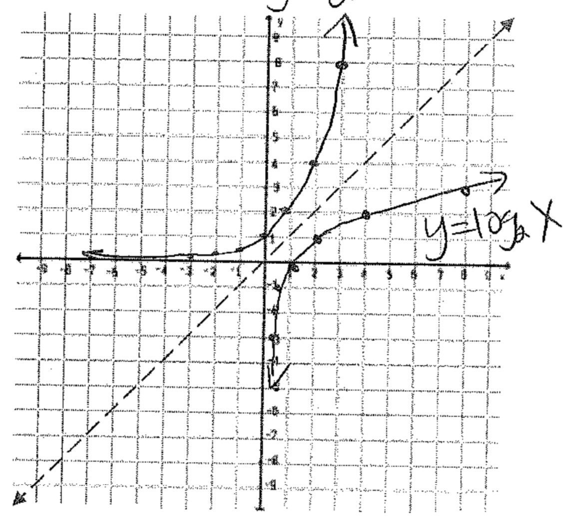
Complete the table of values for each of the following (use a graphing calculator or desmos) the use that to graph (on same graph, but different colors).

$y = 2^x$

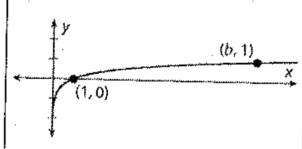
x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$y = \log_2 x$

x	y
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

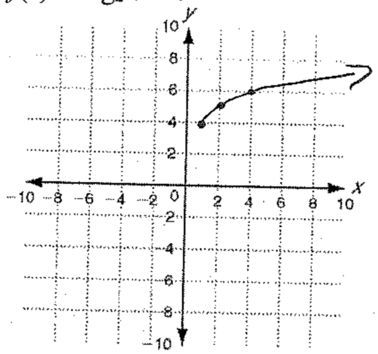


What relationship did you notice in the table of values? x and y are reversed	What relationship did you notice on the graph? reflection over line y=x
What is the domain and the range for the first graph equation? D: $(-\infty,\infty)$ R: $(0,\infty)$	What is the domain and the range for the second equation? D: $(0,\infty)$ R: $(-\infty,\infty)$

	$f(x) = \log_b x$	$g(x) = a\log_b(x-h)+k$	
Graph		a	$\lvert a\rvert > 1 \rightarrow$ vertical stretch by $\lvert a\rvert$ $\lvert a\rvert < 1 \rightarrow$ vertical compression by $\lvert a\rvert$ $a < 0 \rightarrow$ reflection over x-axis
		h	$h > 0 \rightarrow$ shift right h units $h < 0 \rightarrow$ shift left h units
		k	$k > 0 \rightarrow$ shift up k units $k < 0 \rightarrow$ shift down k units
Vertical Asymptote	$x = 0$ $\longrightarrow$		$x = h$
Reference Point	$(1, 0)$ $\longrightarrow$		$(1 + h, k)$
Reference Point	$(b, 1)$ $\longrightarrow$		$(b + h, a + k)$

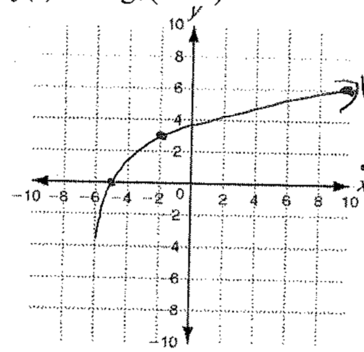
Directions: *Graph each function. Tell how the graph is transformed from the graph of its parent function.*

1. $f(x) = \log_2 x + 4$



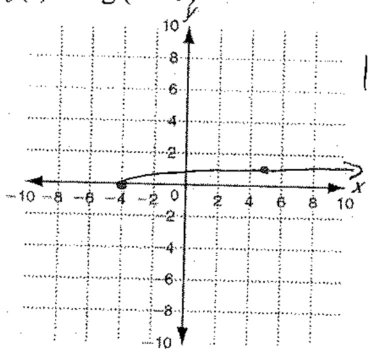
up 4

2. $f(x) = 3\log_4 (x + 6)$



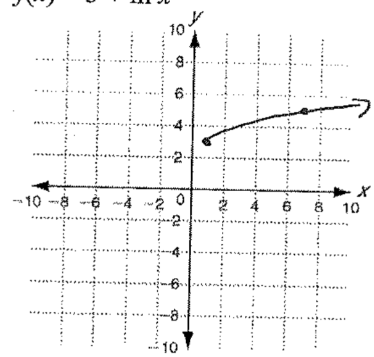
vert stretch factor 3
left 6

3. $f(x) = \log (x + 5)$



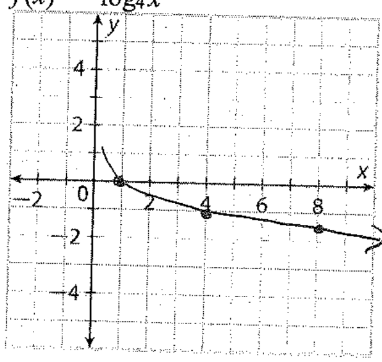
left 5

4. $f(x) = 3 + \ln x$



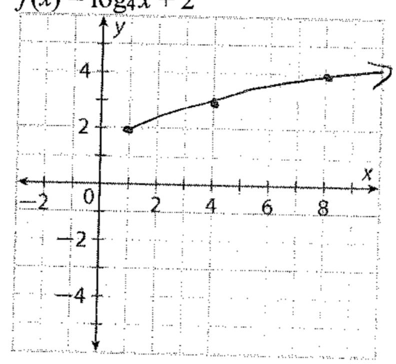
up 3

5. $f(x) = -\log_4 x$



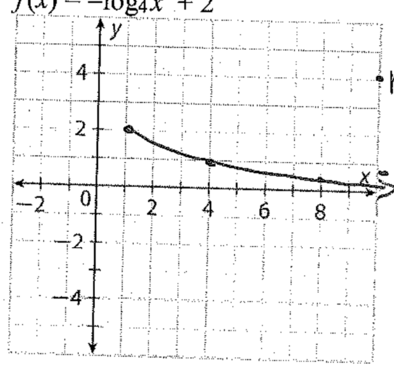
reflect x-axis

6. $f(x) = \log_4 x + 2$



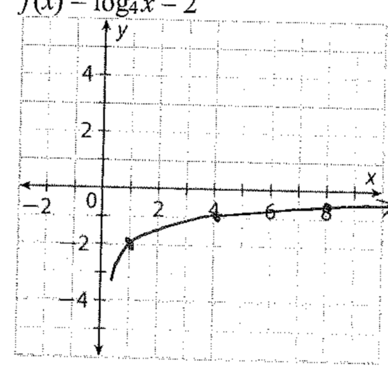
up 2

7. $f(x) = -\log_4 x + 2$



reflect x-axis
up 2

8. $f(x) = \log_4 x - 2$



down 2

Name Key Period ______

Algebra 3-4 Unit 6.16
Graphs of Logarithms (Day 2)

Directions: *Write each transformed function.*

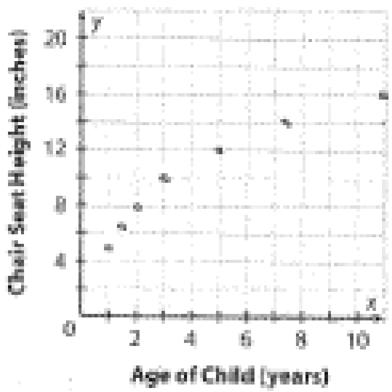
1. The function $f(x) = \log(x+1)$ is reflected across the x-axis and translated down 4 units. $g(x) = -\log(x+1) - 4$	2. The function $f(x) = \log_8(x-3)$ is compressed vertically by a factor of $\frac{2}{5}$ and translated up 11 units. $g(x) = \frac{2}{5}\log_8(x-3) + 11$
3. The function $f(x) = -\log_9(x+4)$ is translated 4 units right and 1 unit down and vertically stretched by a factor of 7. $g(x) = -7\log_9 x - 1$	4. The function $f(x) = 3\ln(2x+8)$ is vertically stretched by a factor of 3, translated 7 units up, and reflected across the x-axis. $g(x) = -9\ln(2x+8) + 7$
5. The function $f(x) = -\log(5-x) - 2$ is translated 6 units left, vertically compressed by a factor of $\frac{1}{3}$, and reflected across the x-axis. $g(x) = \frac{1}{3}\log(11-x) - 2$	6. The function $f(x) = 8\log_7 x - 5$ is compressed vertically by a factor of 0.5, translated right 1 unit, and reflected across the x-axis. $g(x) = -4\log_7(x-1) - 5$
7. What transformations does the function $f(x) = -\ln(x+1) - 2$ undergo to become the function $g(x) = \ln(x-1)$? 2 units up 2 units right reflect x-axis	8. The function $f(x) = \ln x$ is reflected across the x-axis. $g(x) = -\ln(x)$
9. The function $f(x) = \log_8 x$ is vertically compressed by a factor of 0.5. $g(x) = .5\log_8 x$	10. The function $f(x) = \log_3 x$ is vertically stretched by a factor of 4. $g(x) = 4\log_3 x$
11. The function $f(x) = \log x$ is shifted 3 units left and reflected across the x-axis. $g(x) = -\log(x+3)$	12. The graph of the function $f(x) = \log_5 x$ is transformed by reflecting across the x-axis, translating 2 units left, and 4 unit down. $g(x) = -\log_5(x+2) - 4$

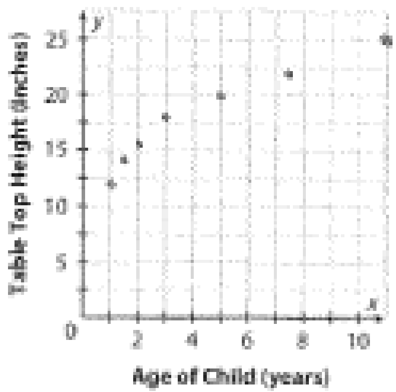
Directions: *Describe the transformation from the parent function to the given function.*

13. $g(x) = 5\log_2(x+2) - 1$ vert stretch by 5 left 2 down 1	14. $g(x) = -\log(x+5) + 2$ reflect x-axis left 5 up 2
15. $g(x) = 3\log_6(x-4) - 2$ vert stretch by 3 right 4 down 2	16. $g(x) = -2\log_4(x+9) + 3$ reflect x-axis vert stretch by 2 left 9 up 3

Given the following data about the heights of chair seats and table tops for children, create scatterplots of the ordered pairs (age of child, chair seat height) (age of child, table top height).

Age of Child (years)	Chair Seat Height (inches)	Table Top Height (inches)
1	5	12
1.5	6.5	14
2	8	16
3	10	18
5	12	20
7.5	14	22
11	16	25





17. Explain if a logarithmic model would be appropriate for each data set. yes $r \approx .999$ $\qquad$ $r \approx .996$
18. Perform logarithmic regression for each data set. $S(a) = 4.829 + 4.586 \ln x$ $\qquad$ $T(a) = 12.047 + 5.191 \ln x$
19. Use your regression equation to predict the chair seat height for a child 14 years old and 50 years old. Explain if each is reasonable or not. prob not reasonable age 14 seat ≈ 16.9 in reasonable $\qquad$ age 50 seat ≈ 22.77 in
20. Use your regression equation to predict the table top height for a child 14 years old and 50 years old. Explain if each is reasonable or not. not reasonable age 14 table ≈ 25.7 in reasonable $\qquad$ age 50 table ≈ 32.4 in

Name Key Period ____________

Algebra 3-4 Unit 6.20
Are You Ready for Unit 6 Part 2 Assessment?

I can apply logarithmic properties and rules. ____________

1. Write as an exponent: $\ln x = 8$ $e^8 = x$	2. Write as an exponent: $\log x = 3$ $10^3 = x$
3. Write as a logarithm: $x^4 = 25$ $\log_x 25 = 4$	4. Write as a logarithm: $e^3 = x$ $\ln x = 3$
5. Write as an exponent: $\log_3 x = 4$ $3^4 = x$	6. Write as a logarithm: $10^x = 7$ $\log 7 = x$
7. Write as a single logarithm: $\log_3 8 + \log_3 7$ $\log_3 56$	8. Write as a single logarithm: $\log_9 x - \log_9 y$ $\log_9 \frac{x}{y}$
9. Write as a single logarithm: $\log_2 x + \log_2 y - \log_2 z$ $\log_2 \frac{xy}{z}$	10. Expand using the properties of logarithms. $\log \frac{a^2 b}{c^4}$ $2\log a + \log b - 4\log c$
11. Expand using the properties of logarithms. $\log xy^3$ $\log x + 3\log y$	12. Expand using the properties of logarithms. $\log_3 \frac{xy^3}{a^3 b^2 c}$ $\log_3 x + 3\log_3 y - 3\log_3 a - 2\log_3 b - \log_3 c$

I can graph logarithmic equations. ____________

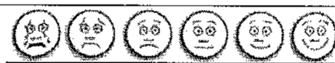
13. Describe the transformations from $f(x) = \log_2(x)$ to $g(x) = -\log_2(x-3)$ • reflect x-axis • right 3	14. Describe the transformations from $f(x) = \log_2(x)$ to $g(x) = 3\log_2(x+5) - 2$ • vert stretch factor 3 • left 5 • down 2
15. Describe the transformations from $f(x) = \log_2(x)$ to $g(x) = -0.5\log_2(x) - 9$ • reflect x-axis • vert comp factor .5 • down 9	16. Describe the transformations from $f(x) = \log_2(x)$ to $g(x) = \log_2(-x) + 6$ • reflect y-axis • up 6
17. The graph of $f(x) = \log_2 x$ is transformed by translating up 2 units and left 4 units. What is the function of the transformed graph? $f(x) = \log_2(x+4) + 2$	18. The graph of $f(x) = \log_2 x$ is transformed by reflecting over the x-axis, translating down 3 units and right 1 unit. What is the function of the transformed graph? $f(x) = -\log_2(x-1) - 3$

19. The graph of $f(x) = \log_2 x$ is transformed by a vertical stretch by a factor of 3 and translating down 5 units. What is the function of the transformed graph?

$f(x) = 3\log_2(x) - 5$

20. The graph of $f(x) = \log_2 x$ is transformed by a reflection over the x-axis and a vertical stretch by a factor of 5. What is the function of the transformed graph?

$f(x) = -5\log_2 x$



I can solve equations with logarithms and exponents.

21. Solve: $3^{2x-1} - 4 = 239$

$3^{2x-1} = 243$

$3^{2x-1} = 3^5$

$2x - 1 = 5$

$2x = 6$

$\boxed{x = 3}$

22. Solve: $2^{3x+4} + 5 = 133$

$2^{3x+4} = 128$

$2^{3x+4} = 2^7$

$3x + 4 = 7$

$3x = 3$

$\boxed{x = 1}$

23. Solve: $3e^x = 11$

$e^x = \frac{11}{3}$

$\ln \frac{11}{3} = x$

$\boxed{x \approx 1.2993}$

24. Solve: $9 + 2e^{x+7} = 22$

$e^{x+7} = 6.5$

$\ln 6.5 = x + 7$

$\boxed{x \approx -5.1282}$

25. Solve: $-8 + 4^{x-9} = 92$

$4^{x-9} = 100$

$(x-9)(\log 4) = \log 100$

$x - 9 = \frac{2}{\log 4}$

$\boxed{x \approx 12.3219}$

26. Identify x in each:

$\ln(x) = 1.7$ 5.4739

$\ln(12) = x$ 2.4849

$e^{3.5} = x$ 33.1155

$e^x = 92$ 4.5218

27. The population of a town was 2,500 people in the year 2000. If it is growing exponentially at a rate of 8% per year, write an equation to model the growth.

$2500(1.08)^t$

Use your model to determine in what year the population will double what it was in the year 2000.

$5000 = 2500(1.08)^t$

$t = 9$ yrs

$\boxed{2009}$

28. The population of a town was 2,500 people in the year 2000. If it is decreasing exponentially at a rate of 8% per year, write an equation to model the decay.

$2500(.92)^t$

Use your model to determine in what year the population will reach 1,000 people.

$1000 = 2500(.92)^t$

$t = 11$ yrs

$\boxed{2011}$

29. The value of a painting can be modeled by the equation $V(t) = 250(0.93)^t$ where $t = 0$ in the year 2010 and the value is in thousands of dollars. What will the value of the painting be in the year 2020?

$250(.93)^{10}$

$\boxed{\$121,000}$

30. The value of a painting can be modeled by the equation $V(t) = 250(1.28)^t$ where $t = 0$ in the year 2010 and the value is in thousands of dollars. In approximately what year will the painting be valued at \$400,000?

$400 = 250(1.28)^t$

$t \approx 1.9$ yrs

$\boxed{2012}$