

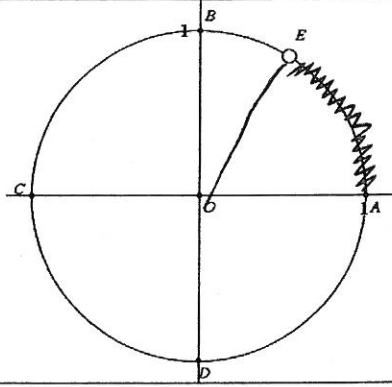
Angles in Trigonometry Guided Notes 1

What is a radian?

method of measuring angles based on radius.

What is the approximate degree measure of a radian?

$$1 \text{ radian} = \frac{180}{\pi} \approx 57.3^\circ$$



Draw the radius OE

Shade the arc EA

How do you convert degrees to radians?

$$\text{Degrees} \cdot \frac{\pi}{180}$$

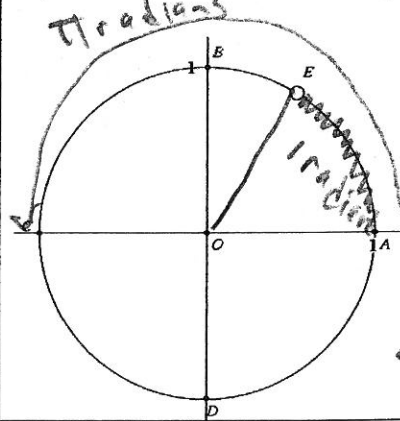
How do you convert radians to degrees?

$$\text{Radians} \cdot \frac{180}{\pi}$$

What is the difference between exact radian measure and approximate radian measure?

1 radian no approx exact  
 $\pi$  radian no approx exact  
 $\pi$  radian  $\approx 3.14$  radians  
 1 radian  $\approx 57.3^\circ$

Explain the difference between 1 radian and  $\pi$  radians



$\angle EOA = 1 \text{ radian}$   
 $\angle COA = 1\pi \text{ radians}$   
 $\angle COA = 180^\circ$   
 $\angle EOA \approx 57.3^\circ$

$$\pi \text{ radians} = 180^\circ$$

It is suggested that each of these exact radian measurements' conversion to degrees is important enough to commit to memory

Exact Radian Measure								
0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
Degree Measure								
$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
Exact Radian Measure								
$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi$
Degree Measure								
$180^\circ$	$210^\circ$	$225^\circ$	$240^\circ$	$270^\circ$	$300^\circ$	$315^\circ$	$330^\circ$	$360^\circ$

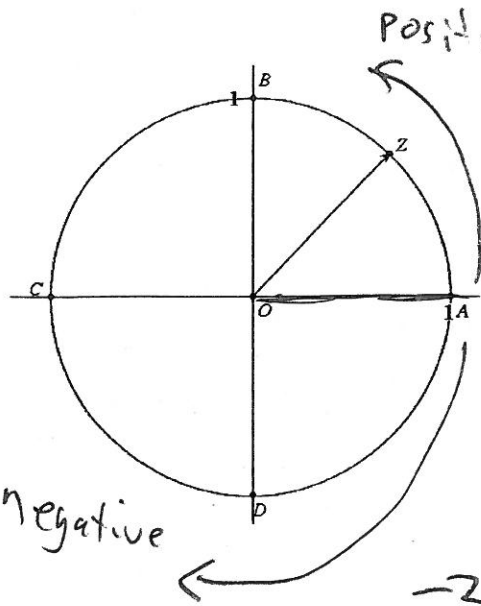
Counting by  $\frac{\pi}{6}$  radians is like counting by 30 degrees

Counting by  $\frac{\pi}{4}$  radians is like counting by 45 degrees

Counting by  $\frac{\pi}{3}$  radians is like counting by 60 degrees

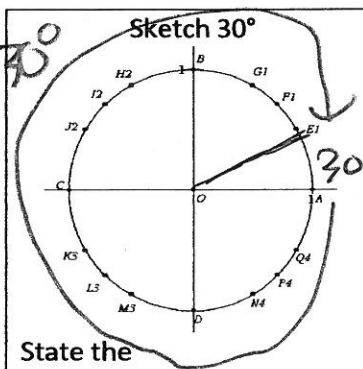
Counting by  $\frac{\pi}{2}$  radians is like counting by 90 degrees

Counting by  $\pi$  radians is like counting by 180 degrees

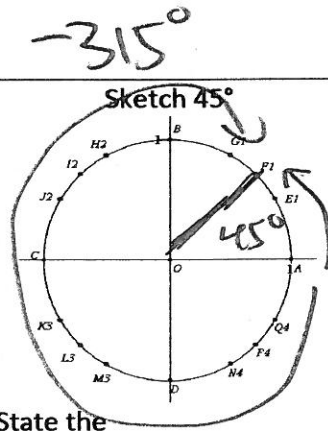


If we measure an angle using positive measurements then the initial side of the angle is AO (name segment) and the angle is measured from this initial side by measuring the angle in a counter-clockwise manner (clockwise or counterclockwise)

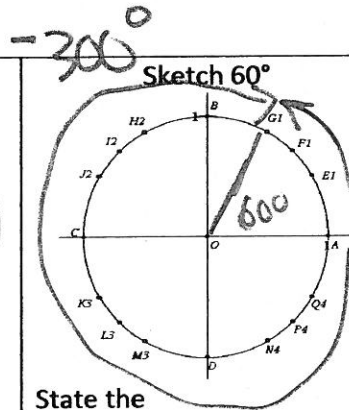
If we measure an angle using negative measurements then the initial side of the angle is AO (name segment) and the angle is measured from this initial side by measuring the angle in a clockwise manner (clockwise or counterclockwise)



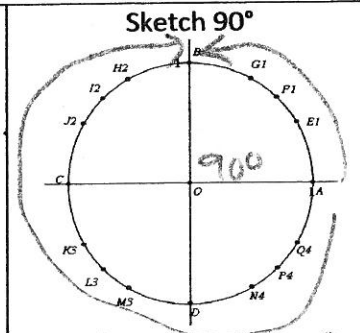
State the equivalent negative angle measurement *-370°*



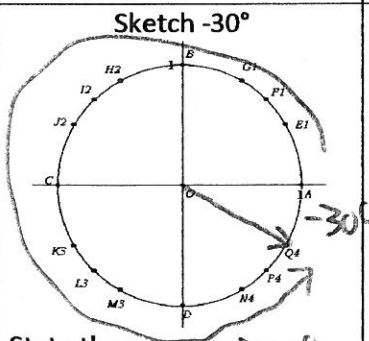
State the equivalent negative angle measurement *-315°*



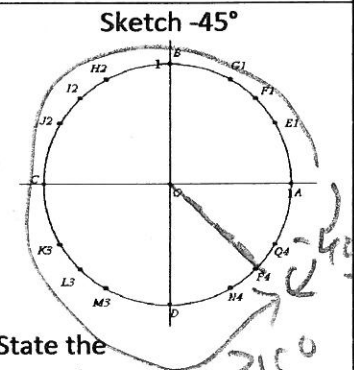
State the equivalent negative angle measurement *-300°*



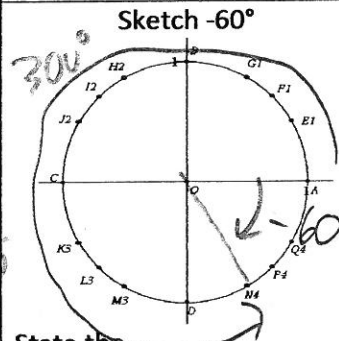
State the equivalent negative angle measurement *-270°*



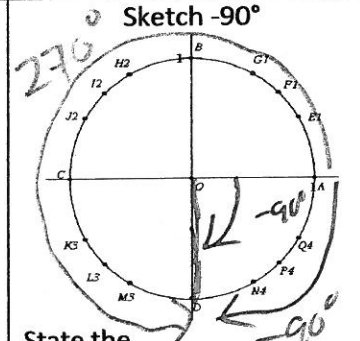
State the equivalent positive angle measurement *330°*



State the equivalent positive angle measurement *315°*



State the equivalent positive angle measurement *300°*



State the equivalent positive angle measurement *270°*

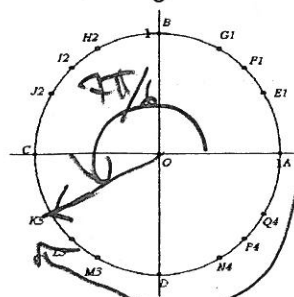
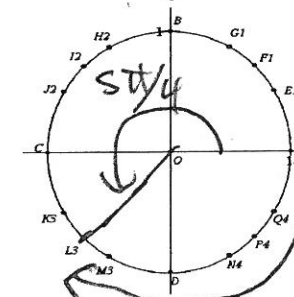
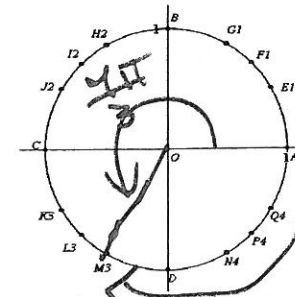
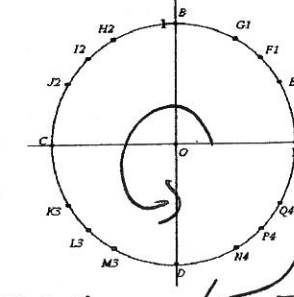
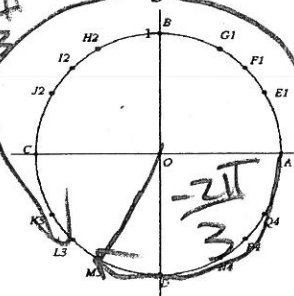
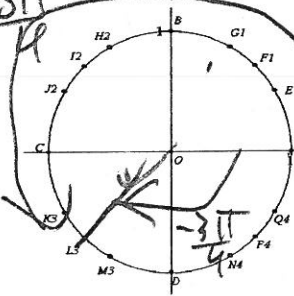
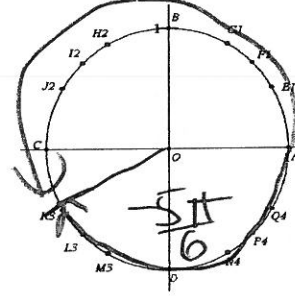
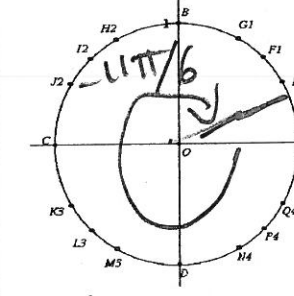
State how to convert positive degree measurement to negative degree measurement

$positive + |negative| = 360$   
 $positive - 360°$

State how to convert negative degree measurement to positive degree measurement

$positive + |negative| = 36$   
 $negative + 360°$

Angles in Trigonometry Guided Notes 2

<p>Sketch <math>\frac{7\pi}{6}</math> radians</p>  <p>State the equivalent negative exact radian measurement</p> <p><math>-\frac{5\pi}{6}</math></p>	<p>Sketch <math>\frac{5\pi}{4}</math> radians</p>  <p>State the equivalent negative exact radian measurement</p> <p><math>-\frac{3\pi}{4}</math></p>	<p>Sketch <math>\frac{4\pi}{3}</math> radians</p>  <p>State the equivalent negative exact radian measurement</p> <p><math>-\frac{2\pi}{3}</math></p>	<p>Sketch <math>\frac{3\pi}{2}</math> radians</p>  <p>State the equivalent negative exact radian measurement</p> <p><math>-\frac{\pi}{2}</math></p>
<p>Sketch <math>-\frac{2\pi}{3}</math> radians</p>  <p>State the equivalent positive exact radian measurement</p> <p><math>\frac{4\pi}{3}</math></p>	<p>Sketch <math>-\frac{3\pi}{4}</math> radians</p>  <p>State the equivalent positive exact radian measurement</p> <p><math>\frac{5\pi}{4}</math></p>	<p>Sketch <math>-\frac{5\pi}{6}</math> radians</p>  <p>State the equivalent positive exact radian measurement</p> <p><math>\frac{7\pi}{6}</math></p>	<p>Sketch <math>-\frac{11\pi}{6}</math> radians</p>  <p>State the equivalent positive exact radian measurement</p> <p><math>\frac{\pi}{6}</math></p>

<p>State how to convert positive exact radian measurement to negative exact radian measurement</p> <p>positive <math>-2\pi</math> if angle <math>[0, 2\pi)</math></p>	<p>State how to convert negative exact radian measurement to positive exact radian measurement</p> <p>negative <math>+2\pi</math> if angle <math>[-2\pi, 0]</math></p>
<p>What do you do to sketch angles that have a measurement that is greater than <math>360^\circ</math>?</p> <p>① find coterminal <math>\angle</math>'s ② find rotations</p>	<p>What do you do to sketch angles that have a measurement that is less than <math>-360^\circ</math>?</p> <p>① Find coterminal <math>\angle</math>'s ② find rotations</p>
<p>What do you do to sketch angles that have a measurement that is greater than <math>2\pi</math> radians?</p> <p>① find coterminal <math>\angle</math>'s ② find rotations</p>	<p>What do you do to sketch angles that have a measurement that is less than <math>-2\pi</math> radians?</p> <p>① find coterminal <math>\angle</math>'s ② find rotations</p>

State the degree measurements related to the first six rotations of a circle

1 rotation	2 rotations	3 rotations	4 rotations	5 rotations	6 rotations
360	720	1080	1440	1800	2160

Let  $n$  be an integer

Write a formula in terms of  $n$  that will give you the total number of rotations an angle measure represents in degrees

$$\frac{\text{angle}}{360}$$

State the radian measurements related to the first six rotations of a circle

1 rotation	2 rotations	3 rotations	4 rotations	5 rotations	6 rotations
$2\pi$	$4\pi$	$6\pi$	$8\pi$	$10\pi$	$12\pi$

Let  $n$  be an integer

Write a formula in terms of  $n$  that will give you the total number of rotations a radian measure represents in exact radians

$$\frac{\text{angle}}{2\pi}$$

How do you convert a positive angle greater than  $360^\circ$  to its smallest positive coterminal angle that you can plot?

$$360 \overline{) \text{angle}} \begin{matrix} n \\ r \\ w \end{matrix}$$

Plot "w" remainder

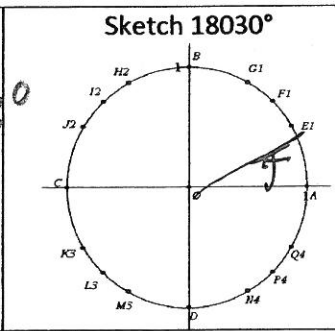
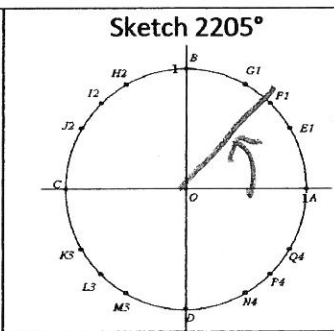
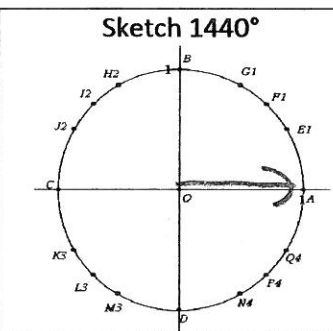
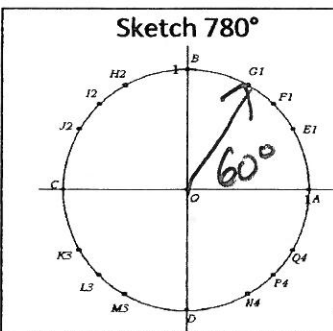
How do you convert a positive angle greater than  $360^\circ$  to the negative angle that is closest to  $0^\circ$  that you can plot?

$$360 \overline{) \text{angle}} \begin{matrix} n \\ r \\ w \end{matrix}$$

plot w "remainder"

How do you convert a positive angle greater than  $360^\circ$  to its number of revolutions?

$$\frac{\text{angle}}{360^\circ}$$



State the smallest equivalent positive coterminal angle

$$60^\circ$$

State the smallest equivalent positive coterminal angle

$$0^\circ$$

State the smallest equivalent positive coterminal angle

$$45^\circ$$

State the smallest equivalent positive coterminal angle

$$30^\circ$$

State the equivalent negative coterminal angle measurement closest to  $0^\circ$

$$-300^\circ$$

State the equivalent negative coterminal angle measurement closest to  $0^\circ$

$$0^\circ$$

State the equivalent negative coterminal angle measurement closest to  $0^\circ$

$$-315^\circ$$

State the equivalent negative coterminal angle measurement closest to  $0^\circ$

$$-330^\circ$$

State the number of revolutions related to this angle measurement

$$\frac{780}{360}$$

State the number of revolutions related to this angle measurement

$$4$$

State the number of revolutions related to this angle measurement

$$6 \frac{1}{8}$$

State the number of revolutions related to this angle measurement

$$50 \frac{1}{12}$$

$$2 \frac{60}{360} = 2 \frac{1}{6} \quad \frac{1440}{360}$$

$$\frac{2205}{360} = \frac{49}{8} = 6 \frac{1}{8}$$

$$\frac{18030}{360} = \frac{601}{12}$$

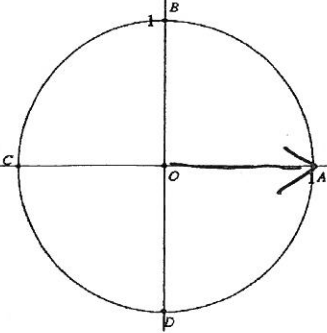
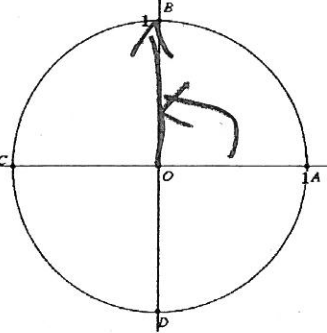
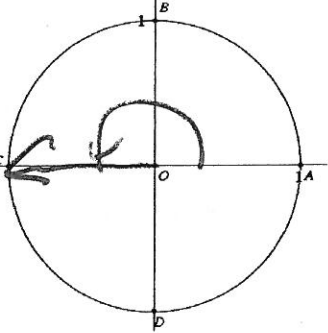
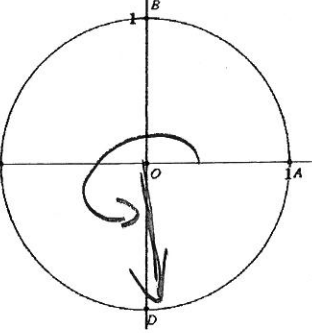


Angles in Trigonometry Guided Notes 3

<p>Sketch <math>-840^\circ</math></p>	<p>Sketch <math>-1920^\circ</math></p>	<p>Sketch <math>-900^\circ</math></p>	<p>Sketch <math>-36135^\circ</math></p>
<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>240^\circ</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>240^\circ</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>180^\circ</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>225^\circ</math></p>
<p>State the equivalent negative coterminal angle measurement closest to <math>0^\circ</math></p> <p style="text-align: center;"><math>-120^\circ</math></p>	<p>State the equivalent negative coterminal angle measurement closest to <math>0^\circ</math></p> <p style="text-align: center;"><math>-120^\circ</math></p>	<p>State the equivalent negative coterminal angle measurement closest to <math>0^\circ</math></p> <p style="text-align: center;"><math>-180^\circ</math></p>	<p>State the equivalent negative coterminal angle measurement closest to <math>0^\circ</math></p> <p style="text-align: center;"><math>-135^\circ</math></p>
<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>\frac{7}{3} = 2\frac{1}{3}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>5\frac{1}{3} = \frac{16}{3}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>2\frac{1}{2}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>100\frac{3}{8}</math></p>
<p>Sketch <math>\frac{73\pi}{6}</math> radians</p>	<p>Sketch <math>-\frac{73\pi}{4}</math> radians</p>	<p>Sketch <math>\frac{1003\pi}{2}</math> radians</p>	<p>Sketch <math>-\frac{91004\pi}{3}</math> radians</p>
<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>\frac{\pi}{6}</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>\frac{7\pi}{4}</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>\frac{\pi}{2}</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>5\pi</math></p>
<p>State the equivalent negative coterminal angle measurement closest to 0 radians</p> <p style="text-align: center;"><math>-\frac{11\pi}{6}</math></p>	<p>State the equivalent negative coterminal angle measurement closest to 0 radians</p> <p style="text-align: center;"><math>-\frac{\pi}{4}</math></p>	<p>State the equivalent negative coterminal angle measurement closest to 0 radians</p> <p style="text-align: center;"><math>-\frac{3\pi}{2}</math></p>	<p>State the equivalent negative coterminal angle measurement closest to 0 radians</p> <p style="text-align: center;"><math>-\frac{\pi}{3}</math></p>
<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>6\frac{1}{2}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>9\frac{1}{8}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>250\frac{3}{4}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>15167\frac{1}{3}</math></p>
<p>How do you convert a positive angle greater than <math>2\pi</math> radians to its smallest positive coterminal angle that you can plot?</p> <p style="text-align: center;">① <math>\frac{\text{angle}}{2\pi}</math></p>	<p>How do you convert a positive angle greater than <math>2\pi</math> radians to the negative angle that is closest to 0 radians that you can plot?</p> <p style="text-align: center;">① <math>\frac{\text{angle}}{2\pi}</math></p>	<p>How do you convert a positive angle greater than <math>2\pi</math> radians to its number of revolutions?</p> <p style="text-align: center;"><math>\frac{\text{angle}}{2\pi}</math></p>	

- ② Keep remainder
- ③ Get remainder
- ④ remainder  $-2\pi$

State the four quadrantal angles in mathematics in degrees and radians

Quadrantal Angle between Q1 and Q4	Quadrantal Angle between Q1 and Q2	Quadrantal Angle between Q2 and Q3	Quadrantal Angle between Q3 and Q4
			
State using degrees	State using positive degrees	State using positive degrees	State using positive degrees
$0^\circ$	$90^\circ$	$180^\circ$	$270^\circ$
State using radians	State using positive and exact radians	State using positive and exact radians	State using positive and exact radians
$0 \text{ rad.}$	$\frac{\pi}{2} = \frac{1}{2}\pi$	$1\pi$	$\frac{3\pi}{2} = 1.5\pi$
State using approximate radians	State using approximate radians (round to two decimals)	State using approximate radians (round to two decimals)	State using approximate radians (round to two decimals)
NOT APPLICABLE	1.57	3.14	4.71
State using negative degrees	State using negative degrees	State using negative degrees	State using negative degrees
NOT APPLICABLE	$-270^\circ$	$-180^\circ$	$-90^\circ$
State using negative radians	State using negative and exact radians	State using negative and exact radians	State using negative and exact radians
NOT APPLICABLE	$-\frac{3\pi}{2}$ (negative)	$-\pi$ (negative)	$-\frac{\pi}{2}$ (negative)
State using approximate radians	State using approximate radians (round to two decimals)	State using approximate radians (round to two decimals)	State using approximate radians (round to two decimals)
NOT APPLICABLE	-4.71	-3.14	-1.57

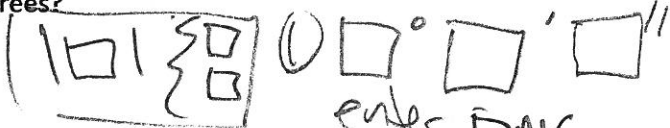
What is the difference between DMS notation and decimal degree notation?

$$90^\circ 1' 2'' = 90^\circ \text{ 1 minute 2 sec} \approx 90.0175^\circ$$

How do you use a TI Nspire to convert decimal degrees to DMS?

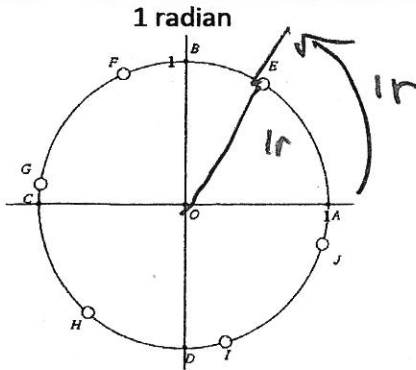
- ① (angle entry)
- ② 2nd Back button (angle)  $\blacktriangleright$  DMS

How do you a TI Nspire to convert DMS to decimal degrees?

- 
- ①  $\square^\circ \square' \square''$  enter DMS
  - ② 2nd Back button  $\square^\circ \square' \square'' \blacktriangleright$  DD

Angles in Trigonometry Guided Notes 4

Radian Measurements



State using positive degrees  
(round to two decimals)

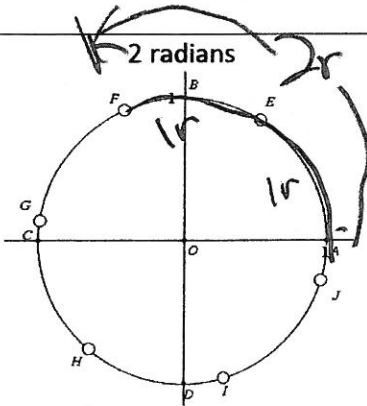
$$57.30^\circ$$

State using negative degrees  
(round to two decimals)

$$-302.70^\circ$$

State using negative radians  
(round to two decimals)

$$-5.28 \text{ rad}$$



State using positive degrees  
(round to two decimals)

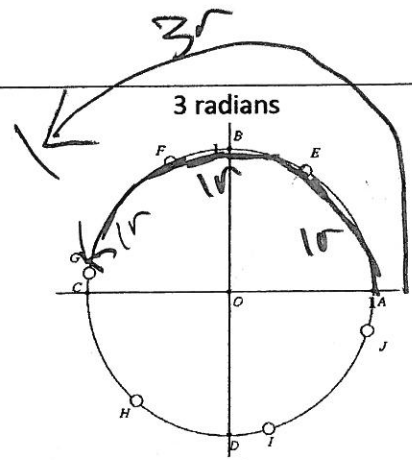
$$114.59^\circ$$

State using negative degrees  
(round to two decimals)

$$245.41^\circ$$

State using negative radians  
(round to two decimals)

$$-4.28 \text{ rad}$$



State using positive degrees  
(round to two decimals)

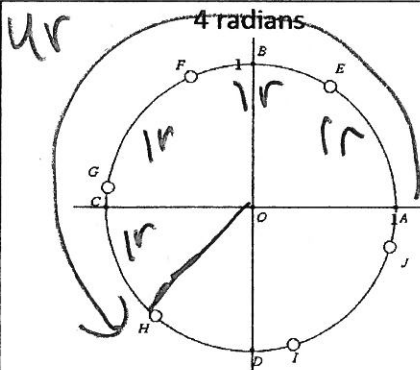
$$171.89^\circ$$

State using negative degrees  
(round to two decimals)

$$188.11^\circ$$

State using negative radians  
(round to two decimals)

$$-3.28 \text{ rad}$$



State using positive degrees  
(round to two decimals)

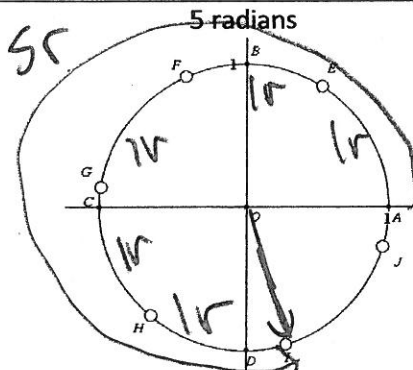
$$229.18^\circ$$

State using negative degrees  
(round to two decimals)

$$-130.82^\circ$$

State using negative radians  
(round to two decimals)

$$-2.28 \text{ rad}$$



State using positive degrees  
(round to two decimals)

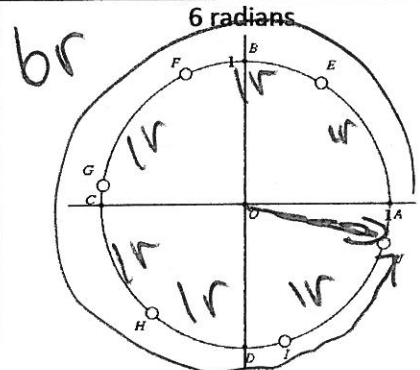
$$286.48$$

State using negative degrees  
(round to two decimals)

$$-73.52$$

State using negative radians  
(round to two decimals)

$$-1.28 \text{ rad}$$



State using positive degrees  
(round to two decimals)

$$343.78$$

State using negative degrees  
(round to two decimals)

$$-16.23$$

State using negative radians  
(round to two decimals)

$$-0.28 \text{ rad}$$

Radian Measurements

**-1 radian**

State using positive degrees  
(round to two decimals)

$302.70^\circ$

State using negative degrees  
(round to two decimals)

$-57.30^\circ$

State using positive radians  
(round to two decimals)

$5.28 \text{ rad.}$

**-2 radians**

State using positive degrees  
(round to two decimals)

$245.41^\circ$

State using negative degrees  
(round to two decimals)

$-114.59^\circ$

State using positive radians  
(round to two decimals)

$4.28 \text{ rad}$

**-3 radians**

State using positive degrees  
(round to two decimals)

$188.11^\circ$

State using negative degrees  
(round to two decimals)

$171.89^\circ$

State using positive radians  
(round to two decimals)

$3.28 \text{ rad}$

**-4 radians**

State using positive degrees  
(round to two decimals)

$130.82^\circ$

State using negative degrees  
(round to two decimals)

$229.18^\circ$

State using positive radians  
(round to two decimals)

$2.28 \text{ rad}$

**-5 radians**

State using positive degrees  
(round to two decimals)

$73.52^\circ$

State using negative degrees  
(round to two decimals)

$-286.48^\circ$

State using positive radians  
(round to two decimals)

$1.28 \text{ rad}$

**-6 radians**

State using positive degrees  
(round to two decimals)

$16.23^\circ$

State using negative degrees  
(round to two decimals)

$-343.78^\circ$

State using positive radians  
(round to two decimals)

$0.28 \text{ rad}$

$$\frac{180}{\pi}$$

What is a radian?  
 $\approx 57.3^\circ$   $\leftarrow$  formed by 1 radius on  $\odot$

What is the approximate degree measure of a radian?  
 $\frac{180}{\pi} \approx 57.3^\circ$

Draw the radius OE  
 Shade the arc EA

How do you convert degrees to radians?  
 multiply by  $\frac{\pi}{180^\circ}$

How do you convert radians to degrees?  
 multiply by  $\frac{180^\circ}{\pi}$

What is the difference between exact radian measure and approximate radian measure?  
 $\pi = \text{exact}$   
 $\approx 3.1415 \text{ approx}$   
 1 radian  $\approx 57.3^\circ$

Explain the difference between 1 radian and  $1\pi$  radians

$3.14$   
 1 radian  $\approx 57.3^\circ$   
 $\pi$  radians  $\approx 180^\circ$

It is suggested that each of these exact radian measurements' conversion to degrees is important enough to commit to memory

Exact Radian Measure								
0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
Degree Measure								
0	30	45	60	90	120	135	150	180
Exact Radian Measure								
$\pi$	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$2\pi$
Degree Measure								
180	210	225	240	270	300	315	330	360

Counting by  $\frac{\pi}{6}$  radians is like counting by 30 degrees

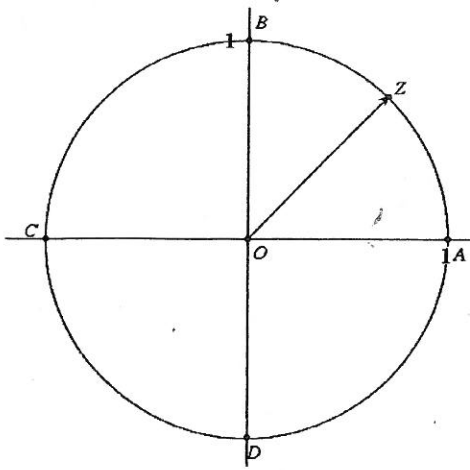
Counting by  $\frac{\pi}{4}$  radians is like counting by 45 degrees

Counting by  $\frac{\pi}{3}$  radians is like counting by 60 degrees

Counting by  $\frac{\pi}{2}$  radians is like counting by 90 degrees

Counting by  $\pi$  radians is like counting by 180 degrees





If we measure an angle using positive measurements then the initial side of the angle is AO (name segment) and the angle is measured from this initial side by measuring the angle in a \_\_\_\_\_ manner (clockwise or counterclockwise)

If we measure an angle using negative measurements then the initial side of the angle is AO (name segment) and the angle is measured from this initial side by measuring the angle in a \_\_\_\_\_ manner (clockwise or counterclockwise)

<p>Sketch <math>30^\circ</math></p> <p>State the equivalent negative angle measurement <math>-330^\circ</math></p>	<p>Sketch <math>45^\circ</math></p> <p>State the equivalent negative angle measurement <math>-315^\circ</math></p>	<p>Sketch <math>60^\circ</math></p> <p>State the equivalent negative angle measurement <math>-300^\circ</math></p>	<p>Sketch <math>90^\circ</math></p> <p>State the equivalent negative angle measurement <math>-270^\circ</math></p>
<p>Sketch <math>-30^\circ</math></p> <p>State the equivalent positive angle measurement <math>330^\circ</math></p>	<p>Sketch <math>-45^\circ</math></p> <p>State the equivalent positive angle measurement <math>315^\circ</math></p>	<p>Sketch <math>-60^\circ</math></p> <p>State the equivalent positive angle measurement <math>300^\circ</math></p>	<p>Sketch <math>-90^\circ</math></p> <p>State the equivalent positive angle measurement <math>270^\circ</math></p>

State how to convert positive degree measurement to negative degree measurement

$$p - 360 = n$$

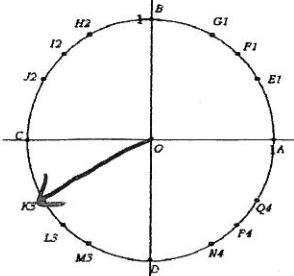
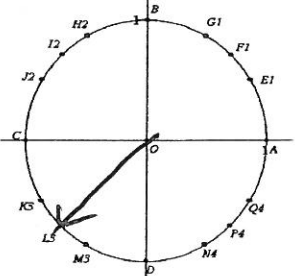
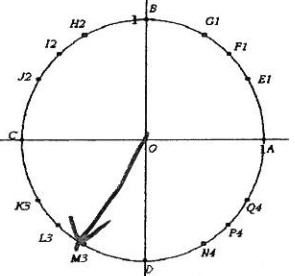
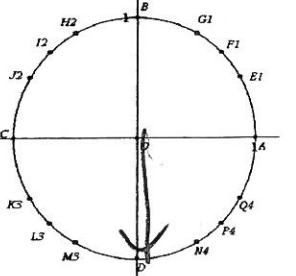
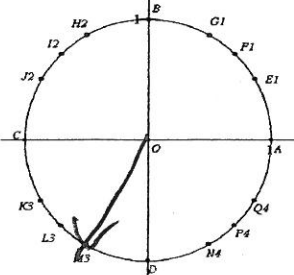
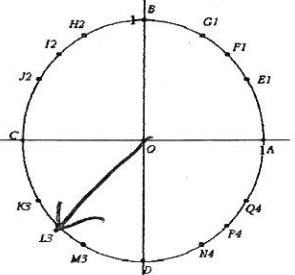
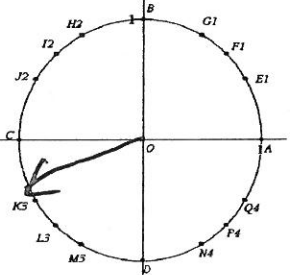
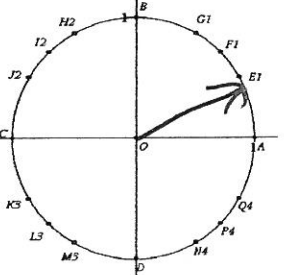
$$p \in [0, 360)$$

State how to convert negative degree measurement to positive degree measurement

$$360 + n = p$$

$$n \in (-360, 0]$$

Angles in Trigonometry Guided Notes 2

<p>Sketch <math>\frac{7\pi}{6}</math> radians</p>  <p>State the equivalent negative exact radian measurement</p> <p><math>-\frac{5\pi}{6}</math></p>	<p>Sketch <math>\frac{5\pi}{4}</math> radians</p>  <p>State the equivalent negative exact radian measurement</p> <p><math>-\frac{3\pi}{4}</math></p>	<p>Sketch <math>\frac{4\pi}{3}</math> radians</p>  <p>State the equivalent negative exact radian measurement</p> <p><math>-\frac{2\pi}{3}</math></p>	<p>Sketch <math>\frac{3\pi}{2}</math> radians</p>  <p>State the equivalent negative exact radian measurement</p> <p><math>-\frac{\pi}{2}</math></p>
<p>Sketch <math>-\frac{2\pi}{3}</math> radians</p>  <p>State the equivalent positive exact radian measurement</p> <p><math>\frac{4\pi}{3}</math></p>	<p>Sketch <math>-\frac{3\pi}{4}</math> radians</p>  <p>State the equivalent positive exact radian measurement</p> <p><math>\frac{5\pi}{4}</math></p>	<p>Sketch <math>-\frac{5\pi}{6}</math> radians</p>  <p>State the equivalent positive exact radian measurement</p> <p><math>\frac{7\pi}{6}</math></p>	<p>Sketch <math>-\frac{11\pi}{6}</math> radians</p>  <p>State the equivalent positive exact radian measurement</p> <p><math>\frac{\pi}{6}</math></p>

<p>State how to convert positive exact radian measurement to negative exact radian measurement</p> <p><math>p - 2\pi</math>     <math>p \in [0, 2\pi)</math></p>	<p>State how to convert negative exact radian measurement to positive exact radian measurement</p> <p><math>n + 2\pi</math>     <math>n \in (-2\pi, 0]</math></p>
--	---

What do you do to sketch angles that have a measurement that is greater than  $360^\circ$ ?

① Remove Rotations  
② Plot remainder

What do you do to sketch angles that have a measurement that is less than  $-360^\circ$ ?

① Remove negative Rotations  
② Plot Negative Remainder

What do you do to sketch angles that have a measurement that is greater than  $2\pi$  radians?

① Remove Rotations  
② Plot remainder

What do you do to sketch angles that have a measurement that is less than  $-2\pi$  radians?

① Remove negative Rotations  
② Plot negative Remainder

State the degree measurements related to the first six rotations of a circle						Let $n$ be an integer Write a formula in terms of $n$ that will give you the total number of rotations an angle measure represents in degrees $\frac{\text{angle}}{360n}$ $n \in \mathbb{Z}$
1 rotation	2 rotations	3 rotations	4 rotations	5 rotations	6 rotations	
$360^\circ$	$720^\circ$	$1080^\circ$	$1440^\circ$	$1800^\circ$	$2160^\circ$	

State the radian measurements related to the first six rotations of a circle						Let $n$ be an integer Write a formula in terms of $n$ that will give you the total number of rotations a radian measure represents in exact radians $\frac{\text{angle}}{2\pi}$ $n \in \mathbb{Z}$
1 rotation	2 rotations	3 rotations	4 rotations	5 rotations	6 rotations	
$2\pi$	$4\pi$	$6\pi$	$8\pi$	$10\pi$	$12\pi$	

$n \in \mathbb{Z}$

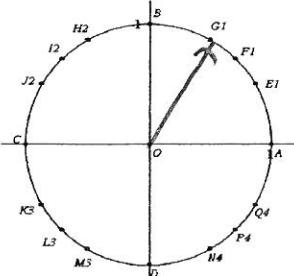
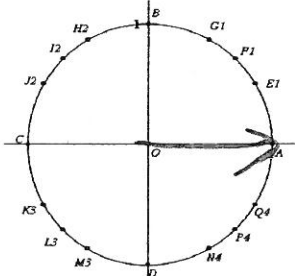
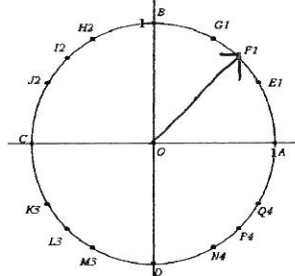
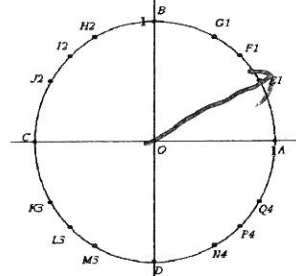
How do you convert a positive angle greater than $360^\circ$ to its smallest positive coterminal angle that you can plot? ① Divide by $360^\circ$ ② Plot remainder $P = L - 360n$	How do you convert a positive angle greater than $360^\circ$ to the negative angle that is closest to $0^\circ$ that you can plot? ① find $p$ (see previous) ② $p - 360$	How do you convert a positive angle greater than $360^\circ$ to its number of revolutions? $\frac{L}{360} = n$ $n$ $\in$ $\mathbb{Z}$ Rotations
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$720^\circ + 60^\circ$

$1440^\circ + 0^\circ$

$2160^\circ + 45^\circ$

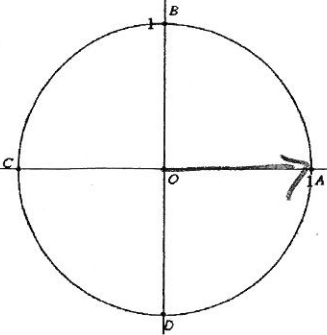
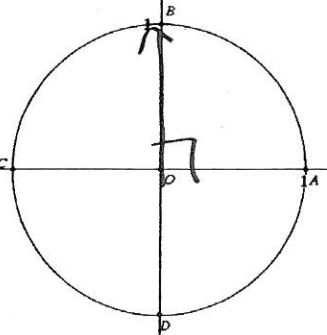
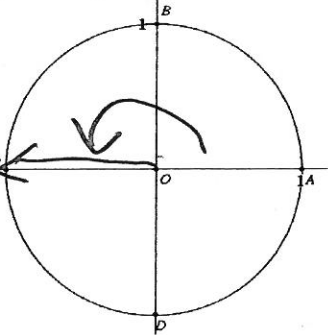
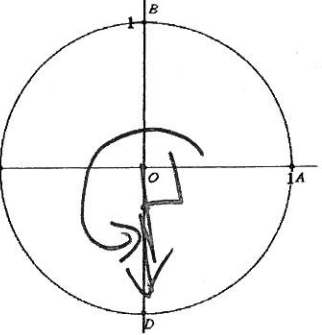
$1800^\circ + 30^\circ$

Sketch $780^\circ$	Sketch $1440^\circ$	Sketch $2205^\circ$	Sketch $18030^\circ$
			
State the smallest equivalent positive coterminal angle $60^\circ$	State the smallest equivalent positive coterminal angle $0^\circ$	State the smallest equivalent positive coterminal angle $45^\circ$	State the smallest equivalent positive coterminal angle $30^\circ$
State the equivalent negative coterminal angle measurement closest to $0^\circ$ $-300^\circ$	State the equivalent negative coterminal angle measurement closest to $0^\circ$ $-360^\circ$	State the equivalent negative coterminal angle measurement closest to $0^\circ$ $-315^\circ$	State the equivalent negative coterminal angle measurement closest to $0^\circ$ $-330^\circ$
State the number of revolutions related to this angle measurement $2\frac{1}{6}$	State the number of revolutions related to this angle measurement $4$	State the number of revolutions related to this angle measurement $6\frac{1}{8}$	State the number of revolutions related to this angle measurement $50\frac{1}{12}$

Angles in Trigonometry Guided Notes 3

<p>Sketch <math>-840^\circ</math></p>	<p>Sketch <math>-1920^\circ</math></p>	<p>Sketch <math>-900^\circ</math></p>	<p>Sketch <math>-36135^\circ</math></p>
<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>240^\circ</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>240^\circ</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>180^\circ</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>225^\circ</math></p>
<p>State the equivalent negative coterminal angle measurement closest to <math>0^\circ</math></p> <p style="text-align: center;"><math>-120^\circ</math></p>	<p>State the equivalent negative coterminal angle measurement closest to <math>0^\circ</math></p> <p style="text-align: center;"><math>-120^\circ</math></p>	<p>State the equivalent negative coterminal angle measurement closest to <math>0^\circ</math></p> <p style="text-align: center;"><math>-180^\circ</math></p>	<p>State the equivalent negative coterminal angle measurement closest to <math>0^\circ</math></p> <p style="text-align: center;"><math>-135^\circ</math></p>
<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>2\frac{1}{3}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>5\frac{1}{3}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>2\frac{1}{2}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>100\frac{3}{8}</math></p>
<p>Sketch <math>\frac{73\pi}{6}</math> radians</p>	<p>Sketch <math>-\frac{73\pi}{4}</math> radians</p>	<p>Sketch <math>\frac{1003\pi}{2}</math> radians</p>	<p>Sketch <math>-\frac{91004\pi}{3}</math> radians</p>
<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>\frac{\pi}{6}</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>\frac{7\pi}{4}</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>\frac{\pi}{2}</math></p>	<p>State the smallest equivalent positive coterminal angle</p> <p style="text-align: center;"><math>\frac{4\pi}{3}</math></p>
<p>State the equivalent negative coterminal angle measurement closest to 0 radians</p> <p style="text-align: center;"><math>-\frac{4\pi}{6}</math></p>	<p>State the equivalent negative coterminal angle measurement closest to 0 radians</p> <p style="text-align: center;"><math>-\frac{\pi}{4}</math></p>	<p>State the equivalent negative coterminal angle measurement closest to 0 radians</p> <p style="text-align: center;"><math>-\frac{3\pi}{2}</math></p>	<p>State the equivalent negative coterminal angle measurement closest to 0 radians</p> <p style="text-align: center;"><math>-\frac{2\pi}{3}</math></p>
<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>6\frac{1}{12}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>9\frac{1}{8}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>125\frac{3}{8}</math></p>	<p>State the number of revolutions related to this angle measurement</p> <p style="text-align: center;"><math>15167\frac{1}{3}</math></p>
<p>How do you convert a positive angle greater than <math>2\pi</math> radians to its smallest positive coterminal angle that you can plot?</p> <p>① Divide by <math>2\pi</math></p> <p>② Remainder</p> <p>is <math>\angle</math></p>	<p>How do you convert a positive angle greater than <math>2\pi</math> radians to the negative angle that is closest to 0 radians that you can plot?</p> <p>① Divide by <math>2\pi</math></p> <p>② Remainder is angle</p> <p>③ angle <math>-2\pi</math></p>	<p>How do you convert a positive angle greater than <math>2\pi</math> radians to its number of revolutions?</p> <p>① Divide by <math>2\pi</math></p>	

State the four quadrantal angles in mathematics in degrees and radians

Quadrantal Angle between Q1 and Q4	Quadrantal Angle between Q1 and Q2	Quadrantal Angle between Q2 and Q3	Quadrantal Angle between Q3 and Q4
			
State using degrees	State using positive degrees	State using positive degrees	State using positive degrees
$0^\circ$	$90^\circ$	$180^\circ$	$270^\circ$
State using radians	State using positive and exact radians	State using positive and exact radians	State using positive and exact radians
0 radian	$\frac{\pi}{2}$ rad	$\pi$ radians	$3\frac{\pi}{2}$ rad.
State using approximate radians	State using approximate radians (round to two decimals)	State using approximate radians (round to two decimals)	State using approximate radians (round to two decimals)
NOT APPLICABLE	1.57	3.14 rad.	4.71 rad.
State using negative degrees	State using negative degrees	State using negative degrees	State using negative degrees
NOT APPLICABLE	$-270^\circ$	$-180^\circ$	$-90^\circ$
State using negative radians	State using negative and exact radians	State using negative and exact radians	State using negative and exact radians
NOT APPLICABLE	$-\frac{3\pi}{2}$ rad	$-\pi$ rad.	$-\frac{\pi}{2}$
State using approximate radians	State using approximate radians (round to two decimals)	State using approximate radians (round to two decimals)	State using approximate radians (round to two decimals)
NOT APPLICABLE	$-4.71$ radian	$-3.14$	$-1.57$ rad

What is the difference between DMS notation and decimal degree notation?

$10 = 30'$        $1^\circ = 3600''$        $1'' = 60'$        $1.645^\circ$        $1 + 64.5\%$  of  $1^\circ$

How do you use a TI Nspire to convert decimal degrees to DMS?

$32.123^\circ \rightarrow$  DMS

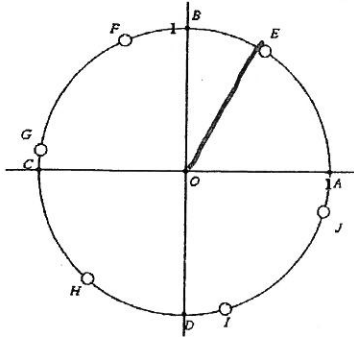
How do you a TI Nspire to convert DMS to decimal degrees?

$16^\circ 15' 45'' \rightarrow$  DD



Radian Measurements

1 radian



State using positive degrees  
(round to two decimals)

$$\approx 57.30^\circ$$

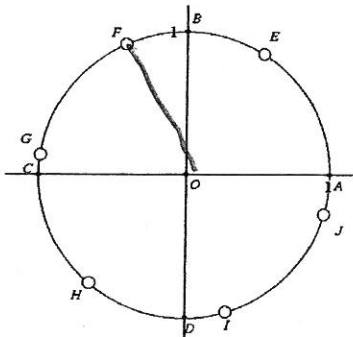
State using negative degrees  
(round to two decimals)

$$-302.70^\circ$$

State using negative radians  
(round to two decimals)

$$-5.28 \text{ rad.}$$

2 radians



State using positive degrees  
(round to two decimals)

$$114.59^\circ$$

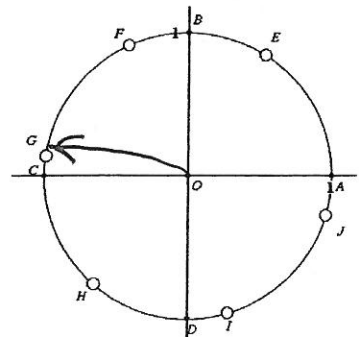
State using negative degrees  
(round to two decimals)

$$245.41^\circ$$

State using negative radians  
(round to two decimals)

$$-4.28 \text{ rad.}$$

3 radians



State using positive degrees  
(round to two decimals)

$$171.89^\circ$$

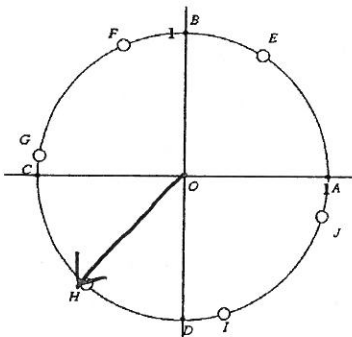
State using negative degrees  
(round to two decimals)

$$-188.11^\circ$$

State using negative radians  
(round to two decimals)

$$-3.28 \text{ radians}$$

4 radians



State using positive degrees  
(round to two decimals)

$$229.18^\circ$$

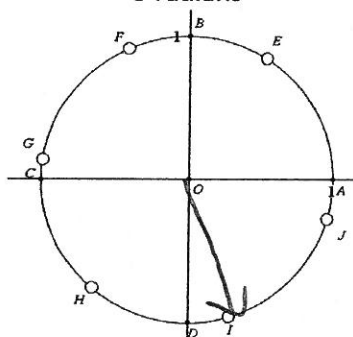
State using negative degrees  
(round to two decimals)

$$-130.82^\circ$$

State using negative radians  
(round to two decimals)

$$-2.28 \text{ radian}$$

5 radians



State using positive degrees  
(round to two decimals)

$$286.48^\circ$$

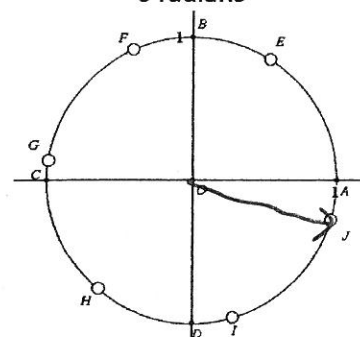
State using negative degrees  
(round to two decimals)

$$-73.52^\circ$$

State using negative radians  
(round to two decimals)

$$-1.28 \text{ radian}$$

6 radians



State using positive degrees  
(round to two decimals)

$$343.77^\circ$$

State using negative degrees  
(round to two decimals)

$$-16.23^\circ$$

State using negative radians  
(round to two decimals)

$$-0.28 \text{ radians}$$

Radian Measurements

**-1 radian**

State using positive degrees  
(round to two decimals)  
 $302.70^\circ$

State using negative degrees  
(round to two decimals)  
 $-57.29^\circ$

State using positive radians  
(round to two decimals)  
 $5.28 \text{ rad.}$

**-2 radians**

State using positive degrees  
(round to two decimals)  
 $245.41^\circ$

State using negative degrees  
(round to two decimals)  
 $-114.59^\circ$

State using positive radians  
(round to two decimals)  
 $4.28 \text{ rad}$

**-3 radians**

State using positive degrees  
(round to two decimals)  
 $188.11^\circ$

State using negative degrees  
(round to two decimals)  
 $-171.89^\circ$

State using positive radians  
(round to two decimals)  
 $3.28 \text{ rad}$

**-4 radians**

State using positive degrees  
(round to two decimals)  
 $130.82^\circ$

State using negative degrees  
(round to two decimals)  
 $-229.18^\circ$

State using positive radians  
(round to two decimals)  
 $2.28 \text{ rad.}$

**-5 radians**

State using positive degrees  
(round to two decimals)  
 $73.52^\circ$

State using negative degrees  
(round to two decimals)  
 $-286.48^\circ$

State using positive radians  
(round to two decimals)  
 $1.28 \text{ rad}$

**-6 radians**

State using positive degrees  
(round to two decimals)  
 $16.22^\circ$

State using negative degrees  
(round to two decimals)  
 $-343.77^\circ$

State using positive radians  
(round to two decimals)  
 $0.28 \text{ rad.}$

# Angles in Trigonometry Notes (cont)

How do you convert radians to degrees

$$\frac{\pi}{5} \text{ radians} \rightarrow \frac{\pi \text{ rad}}{5} \cdot \frac{180^\circ}{\pi \text{ rad}} = \frac{180}{5} \cdot \frac{\pi}{\pi}$$

$$\boxed{\frac{\pi}{5} \text{ rad} = 36^\circ}$$

$$\frac{7\pi}{10} \text{ radians} \rightarrow \frac{7\pi \text{ rad}}{10} \cdot \frac{180^\circ}{\pi \text{ radians}}$$

$$= \frac{7 \cdot 180}{10} \cdot \frac{\pi}{\pi}$$

$$= \frac{1260}{10} = \boxed{126^\circ}$$

$$\frac{13\pi}{8} \text{ radians} \rightarrow \frac{13\pi \text{ radian}}{8} \cdot \frac{180^\circ}{\pi \text{ rad}}$$

$$= \frac{13 \cdot 180}{8} \frac{\pi}{\pi} = \frac{2340}{8} \frac{\pi}{\pi}$$

$$= \boxed{292.5^\circ}$$

# Angles in Trigonometry Notes ①

What is the difference between exact radian measure and approximate radian measure?

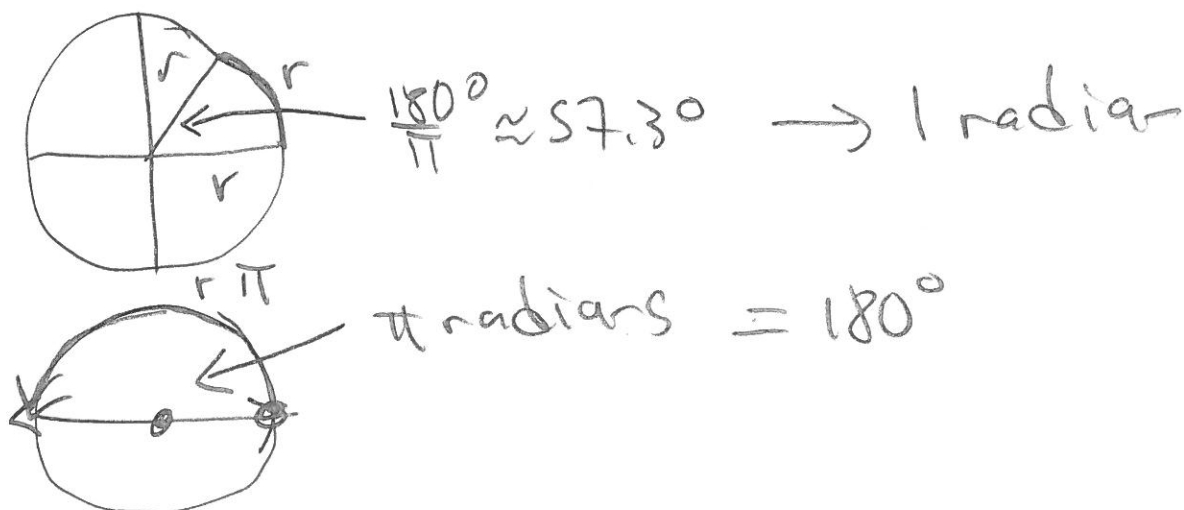
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$\pi$  radians       $\approx$  3.14 radians  
exact                      approx

1 radian       $1.5\pi$  radian       $\approx$  4.71 radian  
exact                      exact                      approx.

typically exact radians will use  $\pi$  in radian or give integer radian measurement.

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# Angles in Trig Notes ① cont.

radians

$$0$$

$$\frac{\pi}{6} = 30^\circ$$

$$\frac{\pi}{4} = 45^\circ$$

$$\frac{\pi}{3} = 60^\circ$$

$$\frac{\pi}{2} = 90^\circ$$

$$\frac{2\pi}{3} = 120^\circ$$

$$\frac{3\pi}{4} = 135^\circ$$

$$\frac{5\pi}{6} = 150^\circ$$

$$\pi = 180^\circ$$

work

$$\frac{\pi}{6} \cdot \frac{180}{\pi} = \frac{180}{6} = 30^\circ$$

$$\frac{\pi}{4} \cdot \frac{180}{\pi} = \frac{180}{4} = 45^\circ$$

$$\frac{\pi}{3} \cdot \frac{180}{\pi} = \frac{180}{3} = 60^\circ$$

$$\frac{\pi}{2} \cdot \frac{180}{\pi} = \frac{180}{2} = 90^\circ$$

$$\begin{aligned} \frac{2\pi}{3} \cdot \frac{180}{\pi} &= \frac{2}{3} \cdot 180 \\ &= \frac{360}{3} = 120^\circ \end{aligned}$$

$$\frac{3\pi}{4} \cdot \frac{180}{\pi} = \frac{540}{4} = 135^\circ$$

$$\frac{5\pi}{6} \cdot \frac{180}{\pi} = \frac{900}{6} = 150^\circ$$

$$\pi \cdot \frac{180}{\pi} = 180^\circ$$



# Angles in Trig Notes ① cont

$$\pi = 180^\circ$$

$$\frac{2\pi}{6} = 210^\circ$$

$$\frac{5\pi}{4} = 225^\circ$$

$$\frac{4\pi}{3} = 240^\circ$$

$$\frac{3\pi}{2} = 270^\circ$$

$$\frac{5\pi}{3} = 300^\circ$$

$$\frac{7\pi}{4} = 315^\circ$$

$$\frac{11\pi}{6} = 330^\circ$$

$$2\pi = 360^\circ$$

$$\pi \cdot \frac{180}{\pi} = 180^\circ$$

$$\frac{7\pi}{6} \cdot \frac{180}{\pi} = \frac{7}{6} \cdot 180 = \frac{1260}{6} = 210^\circ$$

$$\frac{5\pi}{4} \cdot \frac{180}{\pi} = \frac{5}{4} \cdot \frac{180}{1} = \frac{900}{4} = 225^\circ$$

$$\frac{4\pi}{3} \cdot \frac{180}{\pi} = \frac{4}{3} \cdot 180 = \frac{720}{3} = 240^\circ$$

$$\frac{3\pi}{2} \cdot \frac{180}{\pi} = \frac{540}{2} = 270^\circ$$

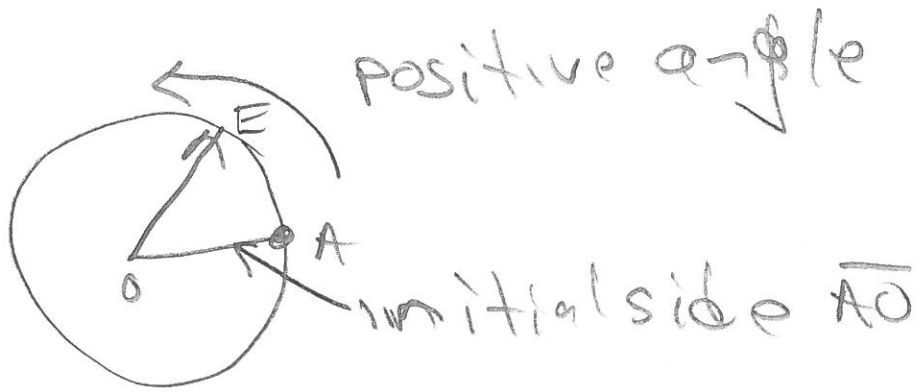
$$\frac{5\pi}{3} \cdot \frac{180}{\pi} = \frac{5}{3} \cdot 180 = \frac{900}{3} = 300^\circ$$

$$\frac{7\pi}{4} \cdot \frac{180}{\pi} = \frac{7}{4} \cdot 180 = \frac{1260}{4} = 315^\circ$$

$$\frac{11\pi}{6} \cdot \frac{180}{\pi} = \frac{1980}{6} = 330^\circ$$

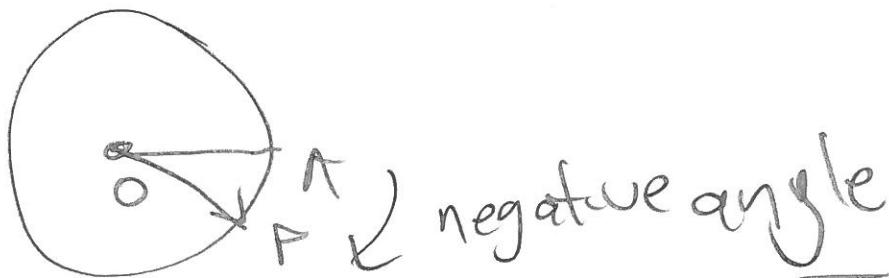
$$2\pi \cdot \frac{180}{\pi} = 2 \cdot 180 = 360^\circ$$

# Angles in trigonometry ① Notes cont



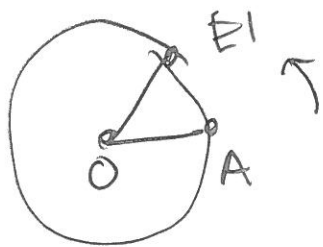
If  $\vec{EO}$  revolves from  $\overline{AO}$  counter-clockwise  
then  $\angle AOE = \text{positive angle}$

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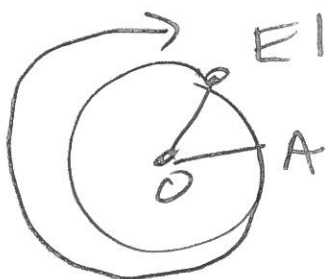


If  $\vec{PO}$  revolves from  $\overline{AO}$  clockwise  
then  $\angle AOP$  is negative angle

# Positive Angles



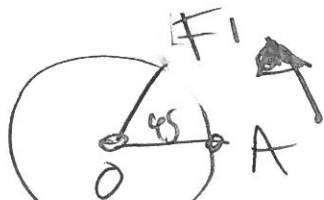
$30^\circ$



$-330^\circ$

$m\angle AOE = 30^\circ$   
Reflex  $\angle = 330^\circ$   
negative angle

$$30 - 360 = -330^\circ$$



$45^\circ$



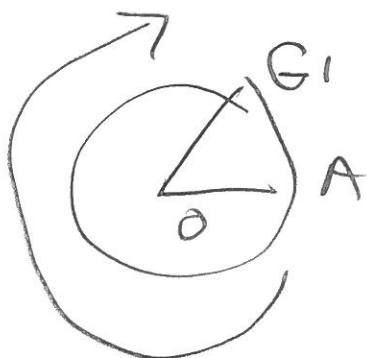
$-315^\circ$

$m\angle AOF = 45^\circ$   
Reflex  $\angle = 315^\circ$   
Negative  $\angle$

$$45 - 360 = -315^\circ$$



$60^\circ$

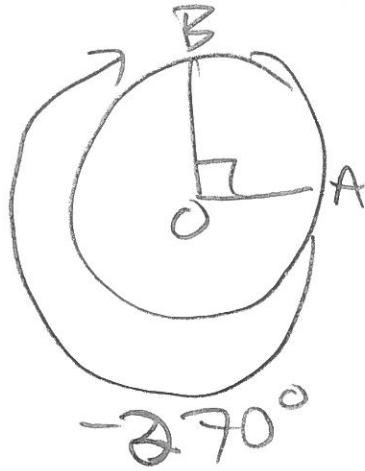
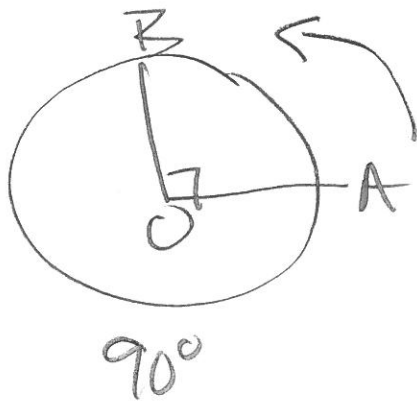


$-300^\circ$

$m\angle AOG = 60^\circ$   
Reflex  $\angle = 300^\circ$   
Negative  $\angle$

$$60 - 360 = -300^\circ$$

# Angles in Trigonometry Notes ①



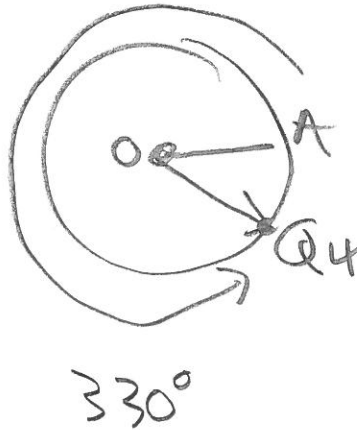
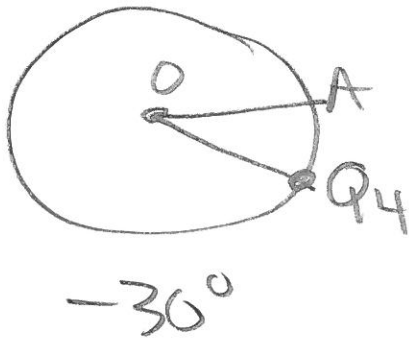
$$m\angle BOA = 90^\circ$$

$$\text{Reflex } \angle = 270^\circ$$

Negative  $\angle$

$$90^\circ - 360^\circ = -270^\circ$$

## Negative $\angle$ s



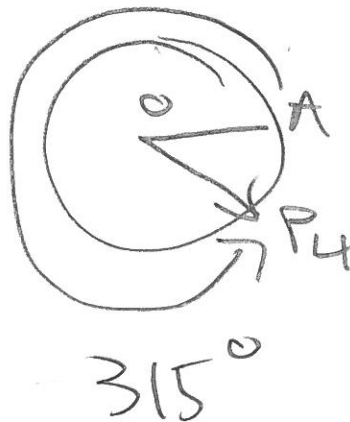
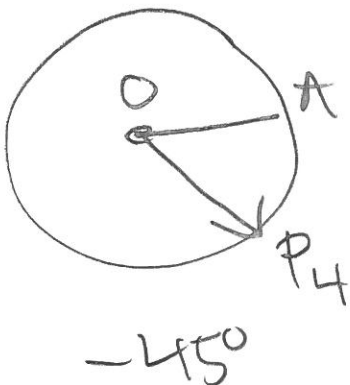
$$m\angle AOQ_4 = 30^\circ$$

negative  $\angle$  -30°

Reflex 330°

positive 330°

$$-30 + 360 = 330^\circ$$



$$m\angle AOP_4 = 45^\circ$$

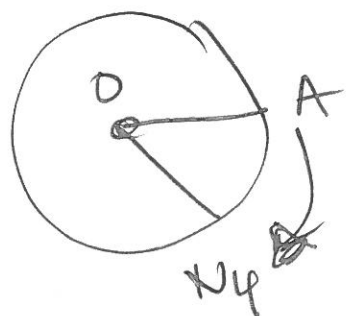
negative  $\angle$  -45°

Reflex = 315°

$$-45 + 360 = 315^\circ$$

Positive  $\angle$  = 315°

# Angles in trigonometry Notes ①



$-60^\circ$



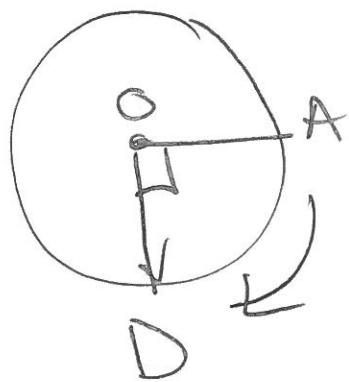
$300^\circ$

$$m\angle AON_4 = 60^\circ$$

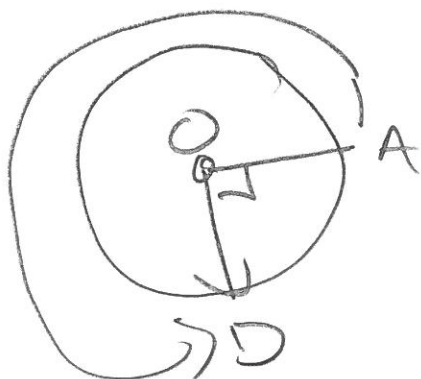
$$\text{Reflex } \angle = 300^\circ$$

$$-60 + 360 = 300^\circ$$

$$\text{Positive } \angle = 300^\circ$$



$-90^\circ$



$270^\circ$

$$m\angle AOD = 90^\circ$$

$$\text{Reflex } \angle = 270^\circ$$

$$-90^\circ + 360^\circ = 270^\circ$$

$$\text{Positive } \angle = 270^\circ$$

When  $x \in [0, 360)$

this means  $x$  is less than 1 rotation

positive to negative

$y = \text{negative } \angle$   
related

$$x - 360 = y$$

to  $x$



# Angles in trigonometry Notes ①

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When  $w \in (-360^\circ, 0^\circ]$

this means  $w$  is less than 1 negative rotation

$360 + w = v$  where  $v$  is positive equivalent of  $w$

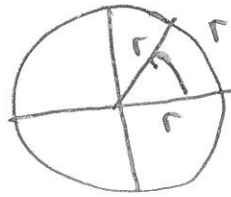
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If  $\angle$  is more than one rotation, then we must use modulo to find smallest positive coterminal  $\angle$ 's & negative coterminal  $\angle$  closest to  $0^\circ$  (none on modulo later)

# Angles in trigonometry Notes ①

What is a radian?

method to measure  
an angle based on



$$1 \text{ radian} = \frac{180}{\pi} \\ \approx 57.3^\circ$$

How do you convert degrees to radians?

$$36^\circ \rightarrow 36^\circ \cdot \frac{\pi \text{ radian}}{180^\circ} = \frac{36}{180} \pi = \frac{1}{5} \pi \text{ radian} \\ = \boxed{\frac{\pi}{5} \text{ radians}}$$

$$100^\circ \rightarrow 100^\circ \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{100}{180} \pi \text{ radians} \\ = \frac{5}{9} \pi = \boxed{\frac{5\pi}{9} \text{ radians}}$$

$$240^\circ \rightarrow 240^\circ \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{240}{180} \pi \text{ radians} \\ = \boxed{\frac{4}{3} \pi \text{ radians}}$$

$$320^\circ \rightarrow 320^\circ \cdot \frac{\pi \text{ radians}}{180^\circ} = \frac{320}{180} \pi \text{ radians} \\ = \boxed{\frac{16}{9} \pi \text{ radians}}$$