Calculus

1.8 The Squeeze Theorem

Notes

Write your questions and thoughts here!

Squeeze Theorem: a.k.a. "Sandwich Theorem" or "Pinching Theorem"

If $\quad(x) \leq \quad(x) \leq \quad(x)$

and if $\lim g(x) = \quad$ and $\lim h(x) =$

then $\lim f(x) =$

1. Find $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$

2. Let g and h be the functions defined by $g(x) = -x^2 + 2x - 3$ and $h(x) = 2x + 1$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x\to 2} f(x)$?

3. Let g and h be the functions defined by $g(x) = \cos\left(\frac{\pi}{2}x\right) + 2$ and $h(x) = x^2 + 3$. If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for $-1 \leq x \leq 5$, what is $\lim_{x\to 0} f(x)$?

Let f, g, and h be the functions defined by $f(x) = \frac{1-\cos x}{2x}$, $g(x) = x^3 \sin\left(\frac{1}{x}\right)$, and $h(x) = \frac{x}{\sin x}$ for $x \neq 0$. All of the following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as x approaches 0?

4. $-\frac{1}{2} \leq f(x) \leq x^2 + \frac{1}{2}$

5. $-x^3 \leq g(x) \leq x^3$

6. $1 - x^2 \leq h(x) \leq x^2 + 1$

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Calculus

Practice

Evaluate each limit.

1. $\lim_{x \to 0} x \cos\left(\frac{1}{x}\right)$

2. $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x^2}\right)$

3. $\lim_{x \to 0} x \sin\left(\frac{1}{x^2}\right)$

4. Let g and h be the functions defined by $g(x) = x^2 - 3x$ and $h(x) = 2 - 2x$.
   If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to 2} f(x)$?

5. Let g and h be the functions defined by $g(x) = \cos\big(\pi(x+2)\big) - 3$ and $h(x) = \frac{x^2}{2} + x - \frac{7}{2}$.
   If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for $-2 \leq x \leq 0$, what is $\lim_{x \to -1} f(x)$?

6. Let g and h be the functions defined by $g(x) = x^2 + x - 1$ and $h(x) = -x^2 - 4x - 2$.
   If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to -1} f(x)$?

7. Let g and h be the functions defined by $g(x) = -x^2 - 2x + 5$ and $h(x) = 2x^2 - x - 4$.
   If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to -2} f(x)$?

8. Let g and h be the functions defined by $g(x) = \sin\left(\frac{\pi}{2}(x+1)\right) - 1$ and $h(x) = \cos(\pi x) - 3$.
   If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for all x, what is $\lim_{x \to 2} f(x)$?

9. Let g and h be the functions defined by $g(x) = x^2$ and $h(x) = \cos(x)$.
   If f is a function that satisfies $g(x) \leq f(x) \leq h(x)$ for $-0.4 \leq x \leq 0.4$, what is $\lim_{x \to 0} f(x)$?

10. Let f and g be the functions defined by $f(x) = \frac{\sin x}{5x}$ and $g(x) = x^2 \cos\left(\frac{1}{x^3}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as x approaches 0?

a. $5\big(\sin(\pi(x+1))\big) \leq f(x) \leq \frac{1}{5}$

b. $-x^2 \leq g(x) \leq x^2$

11. Let f and g be the functions defined by $f(x) = \frac{\cos x - 1}{x^2}$ and $g(x) = x^2 \sin\left(\frac{1}{x}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as x approaches 0?

a. $\frac{1}{5} \leq f(x) \leq \frac{1}{2}$

b. $\frac{1}{2} - x^2 \leq f(x) \leq \frac{1}{2} + x^2$

c. $-x^2 \leq g(x) \leq x^2$

d. $-\frac{1}{x} \leq g(x) \leq \frac{1}{x}$

12. Let f and g be the functions defined by $f(x) = \frac{x^2 \sin x}{x}$ and $g(x) = x \cos\left(\frac{1}{|x|}\right)$ for $x \neq 0$. The following inequalities are true for $x \neq 0$. State whether each inequality can be used with the squeeze theorem to find the limit of the function as x approaches 0?

a. $-x \leq f(x) \leq x$

b. $-\frac{1}{x} \leq g(x) \leq \frac{1}{x}$

c. $2^{-x} \leq g(x) \leq 2^x$