SOLUTION

$$\lim_{h\to 0}\frac{f(t+h)-f(t)}{h} = \lim_{h\to 0}\frac{2(2+h)^2-1-(2\cdot 2^2-1)}{h}$$
$$= \lim_{h\to 0}\frac{8+8h+2h^2-1-7}{h}$$
$$= \lim_{h\to 0}\frac{8h+2h^2}{h}$$
$$= \lim_{h\to 0}(8+2h) = 8$$

The instantaneous rate of change of the object is 8 ft/sec.

Now Try Exercise 23.

EXAMPLE 7 Investigating Free Fall

Find the speed of the falling rock in Example 1, Section 2.1, at $t = 1$ sec.

SOLUTION

The position function of the rock is $f(t) = 16t^2$. The average speed of the rock over the interval between $t = 1$ and $t = 1 + h$ sec was

$$\frac{f(1+h)-f(1)}{h} = \frac{16(1+h)^2-16(1)^2}{h} = \frac{16(h^2+2h)}{h} = 16(h+2).$$

The rock's speed at the instant $t = 1$ was

$$\lim_{h\to 0} 16(h+2) = 32 \text{ ft/sec.}$$

Now Try Exercise 31.

Quick Review 2.4 *(For help, go to Section 1.1.)*

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

In Exercises 1 and 2, find the increments Δx and Δy from point A to point B.

1. $A(-5, 2)$, $B(3, 5)$ **2.** $A(1, 3)$, $B(a, b)$

In Exercises 3 and 4, find the slope of the line determined by the points.

3. $(-2, 3)$, $(5, -1)$ **4.** $(-3, -1)$, $(3, 3)$

In Exercises 5–9, write an equation for the specified line.

5. through $(-2, 3)$ with slope $= 3/2$

6. through $(1, 6)$ and $(4, -1)$

7. through $(1, 4)$ and parallel to $y = -\frac{3}{4}x + 2$

8. through $(1, 4)$ and perpendicular to $y = -\frac{3}{4}x + 2$

9. through $(-1, 3)$ and parallel to $2x + 3y = 5$

10. For what value of b will the slope of the line through $(2, 3)$ and $(4, b)$ be $5/3$?

Section 2.4 Exercises

In Exercises 1–6, find the average rate of change of the function over each interval.

1. $f(x) = x^3 + 1$
(a) $[2, 3]$ **(b)** $[-1, 1]$

2. $f(x) = \sqrt{4x+1}$
(a) $[0, 2]$ **(b)** $[10, 12]$

3. $f(x) = e^x$
(a) $[-2, 0]$ **(b)** $[1, 3]$

4. $f(x) = \ln x$
(a) $[1, 4]$ **(b)** $[100, 103]$

5. $f(x) = \cot x$
(a) $[\pi/4, 3\pi/4]$ **(b)** $[\pi/6, \pi/2]$

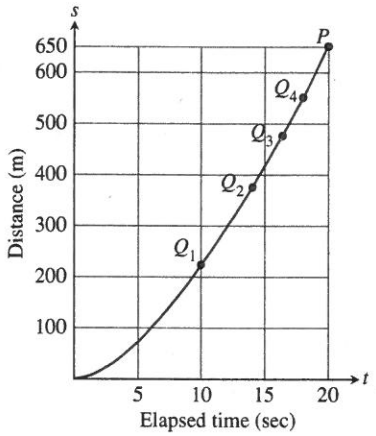
6. $f(x) = 2 + \cos x$
(a) $[0, \pi]$ **(b)** $[-\pi, \pi]$

In Exercises 7 and 8, a distance-time graph is shown.

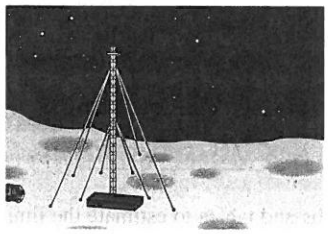
(a) Estimate the slopes of the secants PQ_1, PQ_2, PQ_3, and PQ_4, arranging them in order in a table. What is the appropriate unit for these slopes?

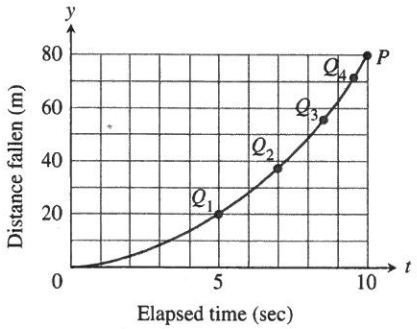
(b) Estimate the speed at point P.

7. *Accelerating from a Standstill* The figure shows the distance-time graph for a 1994 Ford® Mustang Cobra™ accelerating from a standstill.



8. *Lunar Data* The accompanying figure shows a distance-time graph for a wrench that fell from the top platform of a communication mast on the moon to the station roof 80 m below.





In Exercises 9–12, at the indicated point find

(a) the slope of the curve,

(b) an equation of the tangent, and

(c) an equation of the normal.

(d) Then draw a graph of the curve, tangent line, and normal line in the same square viewing window.

9. $y = x^2$ at $x = -2$

10. $y = x^2 - 4x$ at $x = 1$

11. $y = \dfrac{1}{x - 1}$ at $x = 2$

12. $y = x^2 - 3x - 1$ at $x = 0$

In Exercises 13 and 14, find the slope of the curve at the indicated point.

13. $f(x) = |x|$ at (a) $x = 2$ (b) $x = -3$

14. $f(x) = |x - 2|$ at $x = 1$

In Exercises 15–18, determine whether the curve has a tangent at the indicated point. If it does, give its slope. If not, explain why not.

15. $f(x) = \begin{cases} 2 - 2x - x^2, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$ at $x = 0$

16. $f(x) = \begin{cases} -x, & x < 0 \\ x^2 - x, & x \geq 0 \end{cases}$ at $x = 0$

17. $f(x) = \begin{cases} 1/x, & x \leq 2 \\ \dfrac{4 - x}{4}, & x > 2 \end{cases}$ at $x = 2$

18. $f(x) = \begin{cases} \sin x, & 0 \leq x < 3\pi/4 \\ \cos x, & 3\pi/4 \leq x \leq 2\pi \end{cases}$ at $x = 3\pi/4$

In Exercises 19–22, (a) find the slope of the curve at $x = a$.

(b) **Writing to Learn** Describe what happens to the tangent at $x = a$ as a changes.

19. $y = x^2 + 2$

20. $y = 2/x$

21. $y = \dfrac{1}{x - 1}$

22. $y = 9 - x^2$

Find the instantaneous rate of change of the position function $y = f(t)$ in feet at the given time t in seconds.

23. $f(t) = 3t - 7$, $t = 1$

24. $f(t) = 3t^2 + 2t$, $t = 3$

25. $f(t) = \dfrac{t + 1}{t}$, $t = 2$

26. $f(t) = t^3 - 1$, $t = 2$

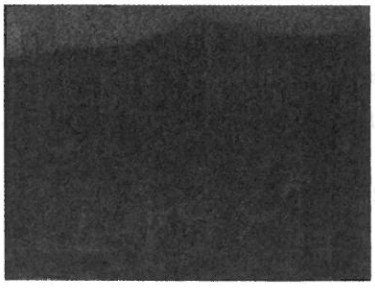
27. *Free Fall* An object is dropped from the top of a 100-m tower. Its height above ground after t sec is $100 - 4.9t^2$ m. How fast is it falling 2 sec after it is dropped?

28. *Rocket Launch* At t sec after lift-off, the height of a rocket is $3t^2$ ft. How fast is the rocket climbing after 10 sec?

29. *Area of Circle* What is the rate of change of the area of a circle with respect to the radius when the radius is $r = 3$ in.?

30. *Volume of Sphere* What is the rate of change of the volume of a sphere with respect to the radius when the radius is $r = 2$ in.?

31. *Free Fall on Mars* The equation for free fall at the surface of Mars is $s = 1.86t^2$ m with t in seconds. Assume a rock is dropped from the top of a 200-m cliff. Find the speed of the rock at $t = 1$ sec.



32. *Free Fall on Jupiter* The equation for free fall at the surface of Jupiter is $s = 11.44t^2$ m with t in seconds. Assume a rock is dropped from the top of a 500-m cliff. Find the speed of the rock at $t = 2$ sec.

33. *Horizontal Tangent* At what point is the tangent to $f(x) = x^2 + 4x - 1$ horizontal?

34. *Horizontal Tangent* At what point is the tangent to $f(x) = 3 - 4x - x^2$ horizontal?

35. *Finding Tangents and Normals*

(a) Find an equation for each tangent to the curve $y = 1/(x - 1)$ that has slope -1. (See Exercise 21.)

(b) Find an equation for each normal to the curve $y = 1/(x - 1)$ that has slope 1.

36. *Finding Tangents* Find the equations of all lines tangent to $y = 9 - x^2$ that pass through the point $(1, 12)$.

37. Table 2.2 gives the total amount of U.S. exported wheat products in metric tons for several years.

TABLE 2.2 U.S. Exported Wheat Products

Year	Exported Wheat Products (metric tons)
2000	844
2004	381
2005	313
2006	281
2007	448
2008	389

Source: U.S. Department of Agriculture, Economic Research Service, *Foreign Agricultural Trade of the United States, (FATUS)*, Table 820.

(a) Let $x = 0$ represent 2000, $x = 1$ represent 2001, and so forth. Make a scatter plot of the data.

(b) Let P represent the point corresponding to 2008, Q_1 the point corresponding to 2004, Q_2 the point corresponding to 2005, and Q_3 the point corresponding to 2007. Find the slope of the secant line PQ_i for $i = 1, 2, 3$.

38. Table 2.3 gives the amount of federal spending in billions of dollars for national defense for several years.

TABLE 2.3 National Defense Spending

Year	National Defense Spending ($ billions)
2003	404.8
2004	455.8
2005	495.3
2006	521.8
2007	551.3
2008	616.1
2009	690.3

Source: U.S. Office of Management and Budget, *Budget Authority by Function and Subfunction, Outlay by Function and Subfunction*, Table 492.

(a) Find the average rate of change in spending from 2003 to 2009.

(b) Find the average rate of change in spending from 2005 to 2008.

(c) Find the average rate of change in spending from 2008 to 2009.

(d) Writing to Learn Explain why someone might be hesitant to make predictions about the rate of change of national defense spending based on the data given in Table 2.3.

Standardized Test Questions

39. True or False If the graph of a function has a tangent line at $x = a$, then the graph also has a normal line at $x = a$. Justify your answer.

40. True or False The graph of $f(x) = |x|$ has a tangent line at $x = 0$. Justify your answer.

41. Multiple Choice If the line L tangent to the graph of a function f at the point $(2, 5)$ passes through the point $(-1, -3)$, what is the slope of L?

(A) $-3/8$ **(B)** $3/8$ **(C)** $-8/3$ **(D)** $8/3$ **(E)** undefined

42. Multiple Choice Find the average rate of change of $f(x) = x^2 + x$ over the interval $[1, 3]$.

(A) -5 **(B)** $1/5$ **(C)** $1/4$ **(D)** 4 **(E)** 5

43. Multiple Choice Which of the following is an equation of the tangent to the graph of $f(x) = 2/x$ at $x = 1$?

(A) $y = -2x$ **(B)** $y = 2x$ **(C)** $y = -2x + 4$

(D) $y = -x + 3$ **(E)** $y = x + 3$

44. Multiple Choice Which of the following is an equation of the normal to the graph of $f(x) = 2/x$ at $x = 1$?

(A) $y = \frac{1}{2}x + \frac{3}{2}$ **(B)** $y = -\frac{1}{2}x$ **(C)** $y = \frac{1}{2}x + 2$

(D) $y = -\frac{1}{2}x + 2$ **(E)** $y = 2x + 5$

Explorations

In Exercises 45 and 46, complete the following for the function.

(a) Compute the difference quotient

$$\frac{f(1 + h) - f(1)}{h}.$$

(b) Use graphs and tables to estimate the limit of the difference quotient in part (a) as $h \to 0$.

(c) Compare your estimate in part (b) with the given number.

(d) Writing to Learn Based on your computations, do you think the graph of f has a tangent at $x = 1$? If so, estimate its slope. If not, explain why not.

45. $f(x) = e^x$, e **46.** $f(x) = 2^x$, $\ln 4$

Group Activity In Exercises 47–50, the curve $y = f(x)$ has a **vertical tangent** at $x = a$ if

$$\lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \infty$$

or if

$$\lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = -\infty.$$

In each case, the right- and left-hand limits are required to be the same: both $+\infty$ or both $-\infty$.

Use graphs to investigate whether the curve has a vertical tangent at $x = 0$.

47. $y = x^{2/5}$ **48.** $y = x^{3/5}$

49. $y = x^{1/3}$ **50.** $y = x^{2/3}$