Can you find the *exact* value of c such that $f(c) = c^3 - c - 1 = 0$ and that you know exists by the application of the Intermediate Value Theorem? Discuss your answer with your fellow students and your teacher.

Quick Review 2.3 (For help, go to Sections 1.2 and 2.1.)

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

1. Find $\lim_{x\to-1} \dfrac{3x^2 - 2x + 1}{x^3 + 4}$.

2. Let $f(x) = \text{int } x$. Find each limit.

(a) $\lim_{x\to-1^-} f(x)$ (b) $\lim_{x\to-1^+} f(x)$ (c) $\lim_{x\to-1} f(x)$ (d) $f(-1)$

3. Let $f(x) = \begin{cases} x^2 - 4x + 5, & x < 2 \\ 4 - x, & x \geq 2. \end{cases}$

Find each limit.

(a) $\lim_{x\to2^-} f(x)$ (b) $\lim_{x\to2^+} f(x)$ (c) $\lim_{x\to2} f(x)$ (d) $f(2)$

In Exercises 4–6, find the remaining functions in the list of functions: $f, g, f \circ g, g \circ f$.

4. $f(x) = \dfrac{2x - 1}{x + 5}$, $g(x) = \dfrac{1}{x} + 1$

5. $f(x) = x^2$, $(g \circ f)(x) = \sin x^2$, domain of $g = [0, \infty)$

6. $g(x) = \sqrt{x - 1}$, $(g \circ f)(x) = 1/x$, $x > 0$

7. Use factoring to solve $2x^2 + 9x - 5 = 0$.

8. Use graphing to solve $x^3 + 2x - 1 = 0$.

In Exercises 9 and 10, let

$$f(x) = \begin{cases} 5 - x, & x \leq 3 \\ -x^2 + 6x - 8, & x > 3. \end{cases}$$

9. Solve the equation $f(x) = 4$.

10. Find a value of c for which the equation $f(x) = c$ has no solution.

Section 2.3 Exercises

In Exercises 1–10, find the points of continuity and the points of discontinuity of the function. Identify each type of discontinuity.

1. $y = \dfrac{1}{(x + 2)^2}$

2. $y = \dfrac{x + 1}{x^2 - 4x + 3}$

3. $y = \dfrac{1}{x^2 + 1}$

4. $y = |x - 1|$

5. $y = \sqrt{2x + 3}$

6. $y = \sqrt[3]{2x - 1}$

7. $y = |x|/x$

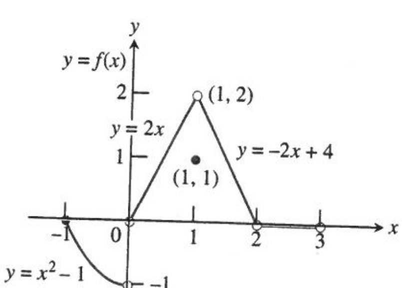
8. $y = \cot x$

9. $y = e^{1/x}$

10. $y = \ln (x + 1)$

In Exercises 11–18, use the function f defined and graphed below to answer the questions.

$$f(x) = \begin{cases} x^2 - 1, & -1 \leq x < 0 \\ 2x, & 0 < x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x < 3 \end{cases}$$



11. (a) Does $f(-1)$ exist?

(b) Does $\lim_{x\to-1^+} f(x)$ exist?

(c) Does $\lim_{x\to-1^+} f(x) = f(-1)$?

(d) Is f continuous at $x = -1$?

12. (a) Does $f(1)$ exist?

(b) Does $\lim_{x\to1} f(x)$ exist?

(c) Does $\lim_{x\to1} f(x) = f(1)$?

(d) Is f continuous at $x = 1$?

13. (a) Is f defined at $x = 2$? (Look at the definition of f.)

(b) Is f continuous at $x = 2$?

14. At what values of x is f continuous?

15. What value should be assigned to $f(2)$ to make the extended function continuous at $x = 2$?

16. What new value should be assigned to $f(1)$ to make the new function continuous at $x = 1$?

17. Writing to Learn Is it possible to extend f to be continuous at $x = 0$? If so, what value should the extended function have there? If not, why not?

18. Writing to Learn Is it possible to extend f to be continuous at $x = 3$? If so, what value should the extended function have there? If not, why not?

In Exercises 19–24, **(a)** find each point of discontinuity. **(b)** Which of the discontinuities are removable? not removable? Give reasons for your answers.

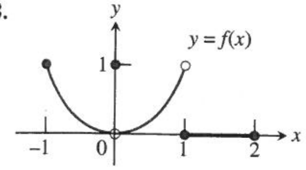
19. $f(x) = \begin{cases} 3 - x, & x < 2 \\ \frac{x}{2} + 1, & x > 2 \end{cases}$

20. $f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ x/2, & x > 2 \end{cases}$

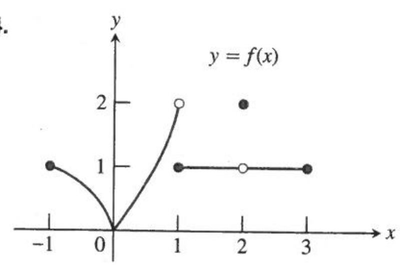
21. $f(x) = \begin{cases} \frac{1}{x - 1}, & x < 1 \\ x^3 - 2x + 5, & x \geq 1 \end{cases}$

22. $f(x) = \begin{cases} 1 - x^2, & x \neq -1 \\ 2, & x = -1 \end{cases}$

23.



24.



In Exercises 25–30, give a formula for the extended function that is continuous at the indicated point.

25. $f(x) = \frac{x^2 - 9}{x + 3}, \quad x = -3$ **26.** $f(x) = \frac{x^3 - 1}{x^2 - 1}, \quad x = 1$

27. $f(x) = \frac{\sin x}{x}, \quad x = 0$ **28.** $f(x) = \frac{\sin 4x}{x}, \quad x = 0$

29. $f(x) = \frac{x - 4}{\sqrt{x} - 2}, \quad x = 4$

30. $f(x) = \frac{x^3 - 4x^2 - 11x + 30}{x^2 - 4}, \quad x = 2$

In Exercises 31 and 32, explain why the given function is continuous.

31. $f(x) = \frac{1}{x - 3}$ **32.** $g(x) = \frac{1}{\sqrt{x - 1}}$

In Exercises 33–36, use Theorem 7 to show that the given function is continuous.

33. $f(x) = \sqrt{\left(\frac{x}{x + 1}\right)}$ **34.** $f(x) = \sin(x^2 + 1)$

35. $f(x) = \cos(\sqrt[3]{1 - x})$ **36.** $f(x) = \tan\left(\frac{x^2}{x^2 + 4}\right)$

Group Activity In Exercises 37–40, verify that the function is continuous and state its domain. Indicate which theorems you are using, and which functions you are assuming to be continuous.

37. $y = \frac{1}{\sqrt{x + 2}}$ **38.** $y = x^2 + \sqrt[3]{4 - x}$

39. $y = |x^2 - 4x|$ **40.** $y = \begin{cases} \frac{x^2 - 1}{x - 1}, & x \neq 1 \\ 2, & x = 1 \end{cases}$

In Exercises 41–44, sketch a possible graph for a function f that has the stated properties.

41. $f(3)$ exists but $\lim_{x \to 3} f(x)$ does not.

42. $f(-2)$ exists, $\lim_{x \to -2^+} f(x) = f(-2)$, but $\lim_{x \to -2} f(x)$ does not exist.

43. $f(4)$ exists, $\lim_{x \to 4} f(x)$ exists, but f is not continuous at $x = 4$.

44. $f(x)$ is continuous for all x except $x = 1$, where f has a nonremovable discontinuity.

45. ***Solving Equations*** Is any real number exactly 1 less than its fourth power? Give any such values accurate to 3 decimal places.

46. ***Solving Equations*** Is any real number exactly 2 more than its cube? Give any such values accurate to 3 decimal places.

47. ***Continuous Function*** Find a value for a so that the function

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

is continuous.

48. ***Continuous Function*** Find a value for a so that the function

$$f(x) = \begin{cases} 2x + 3, & x \leq 2 \\ ax + 1, & x > 2 \end{cases}$$

is continuous.

49. ***Continuous Function*** Find a value for a so that the function

$$f(x) = \begin{cases} 4 - x^2, & x < -1 \\ ax^2 - 1, & x \geq -1 \end{cases}$$

is continuous.

50. ***Continuous Function*** Find a value for a so that the function

$$f(x) = \begin{cases} x^2 + x + a, & x < 1 \\ x^3, & x \geq 1 \end{cases}$$

is continuous.

51. ***Writing to Learn*** Explain why the equation $e^{-x} = x$ has at least one solution.

52. ***Salary Negotiation*** A welder's contract promises a 3.5% salary increase each year for 4 years and Luisa has an initial salary of \$36,500.

(a) Show that Luisa's salary is given by

$$y = 36{,}500(1.035)^{\text{int } t},$$

where t is the time, measured in years, since Luisa signed the contract.

(b) Graph Luisa's salary function. At what values of t is it continuous?