Section 2.2 Exercises

In Exercises 1–8, use graphs and tables to find **(a)** $\lim_{x\to\infty} f(x)$ and **(b)** $\lim_{x\to-\infty} f(x)$. **(c)** Identify all horizontal asymptotes.

1. $f(x) = \cos\left(\dfrac{1}{x}\right)$

2. $f(x) = \dfrac{\sin 2x}{x}$

3. $f(x) = \dfrac{e^{-x}}{x}$

4. $f(x) = \dfrac{3x^3 - x + 1}{x + 3}$

5. $f(x) = \dfrac{3x + 1}{|x| + 2}$

6. $f(x) = \dfrac{2x - 1}{|x| - 3}$

7. $f(x) = \dfrac{x}{|x|}$

8. $f(x) = \dfrac{|x|}{|x| + 1}$

In Exercises 9–12, find the limit and confirm your answer using the Sandwich Theorem.

9. $\displaystyle\lim_{x\to\infty} \frac{1 - \cos x}{x^2}$

10. $\displaystyle\lim_{x\to-\infty} \frac{1 - \cos x}{x^2}$

11. $\displaystyle\lim_{x\to-\infty} \frac{\sin x}{x}$

12. $\displaystyle\lim_{x\to\infty} \frac{\sin (x^2)}{x}$

In Exercises 13–20, use graphs and tables to find the limits.

13. $\displaystyle\lim_{x\to2^+} \frac{1}{x - 2}$

14. $\displaystyle\lim_{x\to2^-} \frac{x}{x - 2}$

15. $\displaystyle\lim_{x\to-3^-} \frac{1}{x + 3}$

16. $\displaystyle\lim_{x\to-3^+} \frac{x}{x + 3}$

17. $\displaystyle\lim_{x\to0^+} \frac{\text{int } x}{x}$

18. $\displaystyle\lim_{x\to0^-} \frac{\text{int } x}{x}$

19. $\displaystyle\lim_{x\to0^+} \csc x$

20. $\displaystyle\lim_{x\to(\pi/2)^+} \sec x$

In Exercises 21–26, find $\lim_{x\to\infty} y$ and $\lim_{x\to-\infty} y$.

21. $y = \left(2 - \dfrac{x}{x + 1}\right)\left(\dfrac{x^2}{5 + x^2}\right)$

22. $y = \left(\dfrac{2}{x} + 1\right)\left(\dfrac{5x^2 - 1}{x^2}\right)$

23. $y = \dfrac{\cos (1/x)}{1 + (1/x)}$

24. $y = \dfrac{2x + \sin x}{x}$

25. $y = \dfrac{\sin x}{2x^2 + x}$

26. $y = \dfrac{x \sin x + 2 \sin x}{2x^2}$

In Exercises 27–34, **(a)** find the vertical asymptotes of the graph of $f(x)$. **(b)** Describe the behavior of $f(x)$ to the left and right of each vertical asymptote.

27. $f(x) = \dfrac{1}{x^2 - 4}$

28. $f(x) = \dfrac{x^2 - 1}{2x + 4}$

29. $f(x) = \dfrac{x^2 - 2x}{x + 1}$

30. $f(x) = \dfrac{1 - x}{2x^2 - 5x - 3}$

31. $f(x) = \cot x$

32. $f(x) = \sec x$

33. $f(x) = \dfrac{\tan x}{\sin x}$

34. $f(x) = \dfrac{\cot x}{\cos x}$

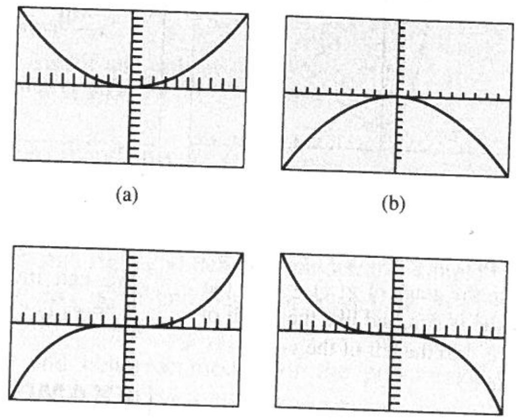
In Exercises 35–38, match the function with the graph of its end behavior model.

35. $y = \dfrac{2x^3 - 3x^2 + 1}{x + 3}$

36. $y = \dfrac{x^5 - x^4 + x + 1}{2x^2 + x - 3}$

37. $y = \dfrac{2x^4 - x^3 + x^2 - 1}{2 - x}$

38. $y = \dfrac{x^4 - 3x^3 + x^2 - 1}{1 - x^2}$



(a) (b) (c) (d)

In Exercises 39–44, **(a)** find a power function end behavior model for f. **(b)** Identify any horizontal asymptotes.

39. $f(x) = 3x^2 - 2x + 1$

40. $f(x) = -4x^3 + x^2 - 2x - 1$

41. $f(x) = \dfrac{x - 2}{2x^2 + 3x - 5}$

42. $f(x) = \dfrac{3x^2 - x + 5}{x^2 - 4}$

43. $f(x) = \dfrac{4x^3 - 2x + 1}{x - 2}$

44. $f(x) = \dfrac{-x^4 + 2x^2 + x - 3}{x^2 - 4}$

In Exercises 45–48, find **(a)** a simple basic function as a right end behavior model and **(b)** a simple basic function as a left end behavior model for the function.

45. $y = e^x - 2x$

46. $y = x^2 + e^{-x}$

47. $y = x + \ln |x|$

48. $y = x^2 + \sin x$

In Exercises 49–52, use the graph of $y = f(1/x)$ to find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

49. $f(x) = xe^x$

50. $f(x) = x^2e^{-x}$

51. $f(x) = \dfrac{\ln |x|}{x}$

52. $f(x) = x \sin \dfrac{1}{x}$

In Exercises 53 and 54, find the limit of $f(x)$ as **(a)** $x \to -\infty$, **(b)** $x \to \infty$, **(c)** $x \to 0^-$, and **(d)** $x \to 0^+$.

53. $f(x) = \begin{cases} 1/x, & x < 0 \\ -1, & x \ge 0 \end{cases}$

54. $f(x) = \begin{cases} \dfrac{x - 2}{x - 1}, & x \le 0 \\ 1/x^2, & x > 0 \end{cases}$

Group Activity In Exercises 55 and 56, sketch a graph of a function $y = f(x)$ that satisfies the stated conditions. Include any asymptotes.

55. $\displaystyle\lim_{x\to1} f(x) = 2, \quad \lim_{x\to5^-} f(x) = \infty, \quad \lim_{x\to5^+} f(x) = \infty,$
$\displaystyle\lim_{x\to\infty} f(x) = -1, \quad \lim_{x\to-2^+} f(x) = -\infty,$
$\displaystyle\lim_{x\to-2^-} f(x) = \infty, \quad \lim_{x\to-\infty} f(x) = 0$

56. $\displaystyle\lim_{x\to2} f(x) = -1, \quad \lim_{x\to4^+} f(x) = -\infty, \quad \lim_{x\to4^-} f(x) = \infty,$
$\displaystyle\lim_{x\to\infty} f(x) = \infty, \quad \lim_{x\to-\infty} f(x) = 2$

57. Group Activity *End Behavior Models* Suppose that $g_1(x)$ is a right end behavior model for $f_1(x)$ and that $g_2(x)$ is a right end behavior model for $f_2(x)$. Explain why this makes $g_1(x)/g_2(x)$ a right end behavior model for $f_1(x)/f_2(x)$.

58. Writing to Learn Let L be a real number, $\lim_{x\to c} f(x) = L$, and $\lim_{x\to c} g(x) = \infty$ or $-\infty$. Can $\lim_{x\to c} (f(x) + g(x))$ be determined? Explain.

Standardized Test Questions

59. True or False It is possible for a function to have more than one horizontal asymptote. Justify your answer.

60. True or False If $f(x)$ has a vertical asymptote at $x = c$, then either $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = \infty$ or $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = -\infty$. Justify your answer.

61. Multiple Choice $\lim_{x\to 2^-} \dfrac{x}{x-2} =$

(A) $-\infty$ **(B)** ∞ **(C)** 1 **(D)** $-1/2$ **(E)** -1

You may use a graphing calculator to solve the following problems.

62. Multiple Choice $\lim_{x\to 0} \dfrac{\cos(2x)}{x} =$

(A) 1/2 **(B)** 1 **(C)** 2 **(D)** cos 2 **(E)** does not exist

63. Multiple Choice $\lim_{x\to 0} \dfrac{\sin(3x)}{x} =$

(A) 1/3 **(B)** 1 **(C)** 3 **(D)** sin 3 **(E)** does not exist

64. Multiple Choice Which of the following is an end behavior for

$$f(x) = \frac{2x^3 - x^2 + x + 1}{x^3 - 1}?$$

(A) x^3 **(B)** $2x^3$ **(C)** $1/x^3$ **(D)** 2 **(E)** 1/2

Exploration

65. *Exploring Properties of Limits* Find the limits of f, g, and fg as $x \to c$.

(a) $f(x) = \dfrac{1}{x}$, $g(x) = x$, $c = 0$

(b) $f(x) = -\dfrac{2}{x^3}$, $g(x) = 4x^3$, $c = 0$

(c) $f(x) = \dfrac{3}{x-2}$, $g(x) = (x-2)^3$, $c = 2$

(d) $f(x) = \dfrac{5}{(3-x)^4}$, $g(x) = (x-3)^2$, $c = 3$

(e) Writing to Learn Suppose that $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c} g(x) = \infty$. Based on your observations in parts (a)–(d), what can you say about $\lim_{x\to c} (f(x)\cdot g(x))$?

Extending the Ideas

66. *The Greatest Integer Function*

(a) Show that

$$\frac{x-1}{x} < \frac{\text{int } x}{x} \le 1 \ (x > 0) \text{ and } \frac{x-1}{x} > \frac{\text{int } x}{x} \ge 1 \ (x < 0).$$

(b) Determine $\lim_{x\to\infty} \dfrac{\text{int } x}{x}$.

(c) Determine $\lim_{x\to-\infty} \dfrac{\text{int } x}{x}$.

67. *Sandwich Theorem* Use the Sandwich Theorem to confirm the limit as $x \to \infty$ found in Exercise 3.

68. Writing to Learn Explain why there is no value L for which $\lim_{x\to\infty} \sin x = L$.

In Exercises 69–71, find the limit. Give a convincing argument that the value is correct.

69. $\lim_{x\to\infty} \dfrac{\ln x^2}{\ln x}$

70. $\lim_{x\to\infty} \dfrac{\ln x}{\log x}$

71. $\lim_{x\to\infty} \dfrac{\ln(x+1)}{\ln x}$

Quick Quiz for AP* Preparation: Sections 2.1 and 2.2

1. Multiple Choice Find $\lim_{x\to 3} \dfrac{x^2 - x - 6}{x - 3}$, if it exists.

(A) -1 **(B)** 1 **(C)** 2 **(D)** 5 **(E)** does not exist

2. Multiple Choice Find $\lim_{x\to 2^+} f(x)$, if it exists, where

$$f(x) = \begin{cases} 3x + 1, & x < 2 \\ \dfrac{5}{x+1}, & x \ge 2 \end{cases}$$

(A) 5/3 **(B)** 13/3 **(C)** 7 **(D)** ∞ **(E)** does not exist

3. Multiple Choice Which of the following lines is a horizontal asymptote for

$$f(x) = \frac{3x^3 - x^2 + x - 7}{2x^3 + 4x - 5}?$$

(A) $y = \frac{3}{2}x$ **(B)** $y = 0$ **(C)** $y = 2/3$ **(D)** $y = 7/5$ **(E)** $y = 3/2$

4. Free Response Let $f(x) = \dfrac{\cos x}{x}$.

(a) Find the domain and range of f.

(b) Is f even, odd, or neither? Justify your answer.

(c) Find $\lim_{x\to\infty} f(x)$.

(d) Use the Sandwich Theorem to justify your answer to part (c).