SECTION 1.3 Exercises

In Exercises 1–12, each graph is a slight variation on the graph of one of the twelve basic functions described in this section. Match the graph to one of the twelve functions (a)–(l) and then support your answer by checking the graph on your calculator. (All graphs are shown in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.)

(a) $y = -\sin x$ (b) $y = \cos x + 1$ (c) $y = e^x - 2$

(d) $y = (x + 2)^3$ (e) $y = x^3 + 1$ (f) $y = (x - 1)^2$

(g) $y = |x| - 2$ (h) $y = -1/x$ (i) $y = -x$

(j) $y = -\sqrt{x}$ (k) $y = \text{int}(x + 1)$

(l) $y = 2 - 4/(1 + e^{-x})$

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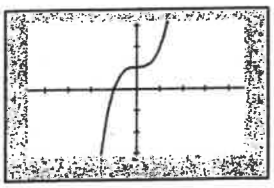

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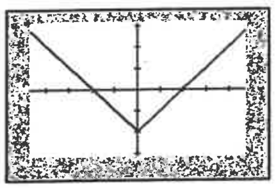

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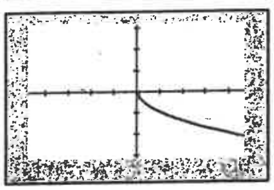

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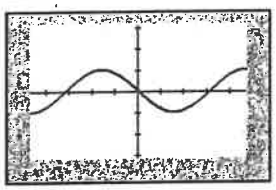

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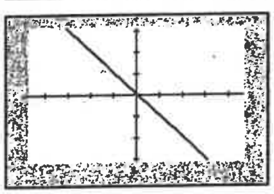

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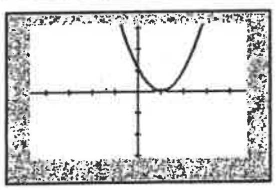

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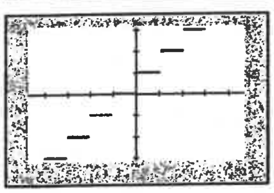

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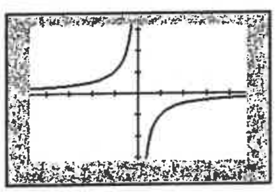

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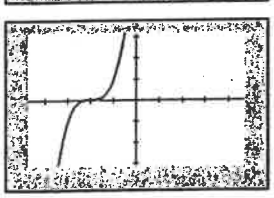

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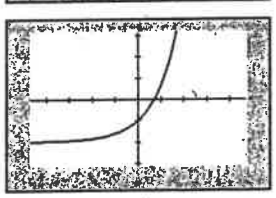

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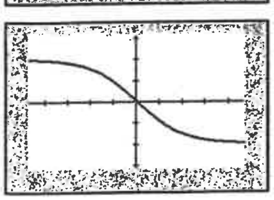

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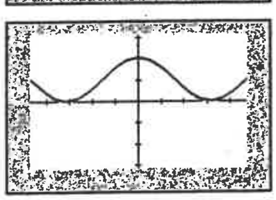

In Exercises 13–18, identify which of Exercises 1–12 display functions that fit the description given.

13. The function whose domain excludes zero

14. The function whose domain consists of all nonnegative real numbers

15. The two functions that have at least one point of discontinuity

16. The function that is not a *continuous function*

17. The six functions that are bounded below

18. The four functions that are bounded above

In Exercises 19–28, identify which of the *twelve basic functions* fit the description given.

19. The four functions that are odd

20. The six functions that are increasing on their entire domains

21. The three functions that are decreasing on the interval $(-\infty, 0)$

22. The three functions with infinitely many local extrema

23. The three functions with no zeros

24. The three functions with range {all real numbers}

25. The four functions that do *not* have end behavior $\lim_{x \to +\infty} f(x) = +\infty$

26. The three functions with end behavior $\lim_{x \to -\infty} f(x) = -\infty$

27. The four functions whose graphs look the same when turned upside-down and flipped about the y-axis

28. The two functions whose graphs are identical except for a horizontal shift

In Exercises 29–34, use your graphing calculator to produce a graph of the function. Then determine the domain and range of the function by looking at its graph.

29. $f(x) = x^2 - 5$

30. $g(x) = |x - 4|$

31. $h(x) = \ln(x + 6)$

32. $k(x) = 1/x + 3$

33. $s(x) = \text{int}(x/2)$

34. $p(x) = (x + 3)^2$

In Exercises 35–42, graph the function. Then answer the following questions:

(a) On what interval, if any, is the function increasing? Decreasing?

(b) Is the function odd, even, or neither?

(c) Give the function's extrema, if any.

(d) How is the graph related to a graph of one of the twelve basic functions?

35. $r(x) = \sqrt{x - 10}$

36. $f(x) = \sin(x) + 5$

37. $f(x) = 3/(1 + e^{-x})$

38. $q(x) = e^x + 2$

39. $h(x) = |x| - 10$

40. $g(x) = 4\cos(x)$

41. $s(x) = |x - 2|$

42. $f(x) = 5 - |x|$

43. Find the horizontal asymptotes for the graph shown in Exercise 11.

44. Find the horizontal asymptotes for the graph of $f(x)$ in Exercise 37.

In Exercises 45–50, identify which of the twelve basic functions fit the description given.

45. The two basic functions with graphs that are concave up on $(-\infty, \infty)$.

46. The two basic functions with graphs that are concave down on their entire domains.

47. The two basic functions with graphs that have a single point of inflection.

48. The two basic functions with graphs that have infinitely many points of inflection.

49. The two basic functions with graphs that are concave down on $(-\infty, 0)$ and up on $(0, \infty)$.

50. The only basic function with a graph that is concave up on $(-\infty, 0)$ and down on $(0, \infty)$.

51. **Writing to Learn** Explain why the graph of an even function that is concave up on $(-\infty, 0)$ will also be concave up on $(0, \infty)$.

52. **Writing to Learn** Explain why the graph of an odd function that is concave up on $(-\infty, 0)$ will be concave down on $(0, \infty)$ and vice versa.

53. **Writing to Learn** The function $f(x) = \sqrt{x^2}$ is one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it is.

(b) Explain algebraically why the two functions are equal.

54. **Uncovering Hidden Behavior** The function $g(x) = \sqrt{x^2 + 0.0001} - 0.01$ is *not* one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it appears to be.

(b) Verify numerically that it is not the basic function that it appears to be.

55. **Writing to Learn** The function $f(x) = \ln(e^x)$ is one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it is.

(b) Explain how the equivalence of the two functions in (a) shows that the natural logarithm function is *not* bounded above (even though it *appears* to be bounded above in Figure 7).

56. **Writing to Learn** Let $f(x)$ be the function that gives the cost, in cents, to mail a first-class package that weighs x ounces. In 2022, the cost was \$4.15 for a package that weighed up to 3 ounces, plus 20 cents for each additional ounce or portion thereof (up to 13 ounces). (*Source: United States Postal Service.*)

(a) Sketch a graph of $f(x)$.

(b) How is this function similar to the greatest integer function? How is it different?

Packages

Weight Not Over	Price
3 ounces	\$4.15
4 ounces	\$4.35
5 ounces	\$4.55
6 ounces	\$4.75
7 ounces	\$4.95
8 ounces	\$5.15
9 ounces	\$5.35
10 ounces	\$5.55
11 ounces	\$5.75
12 ounces	\$5.95
13 ounces	\$6.15

57. **Analyzing a Function** Graph the greatest integer function, $y = \text{int}(x)$, in the ZDecimal window. Then complete the following analysis.

FUNCTION SPOTLIGHT

The Greatest Integer Function

$f(x) = \text{int } x$

- Domain:
- Range:
- Continuity:
- Decreasing on:
- Increasing on:
- Concave up on:
- Concave down on:
- Points of inflection:
- Symmetry:
- Boundedness:
- Local extrema:
- Horizontal asymptotes:
- Vertical asymptotes:
- End behavior:

58. **Power Functions** For positive values of x, we wish to compare the values of the basic functions x^2, x, and $\sqrt{x}$.

(a) How would you order them from least to greatest?

(b) Graph the three functions in the viewing window $[0, 30]$ by $[0, 20]$. Does the graph confirm your response in (a)?

(c) Now graph the three functions in the viewing window $[0, 2]$ by $[0, 1.5]$.

(d) Write a careful response to the question in (a) that accounts for all positive values of x.

59. **Product Functions** There are four odd functions and three even functions in the gallery of twelve basic functions. After multiplying these functions together pairwise in different combinations and exploring the graphs of the products, make a conjecture about the symmetry of:

(a) a product of two odd functions;

(b) a product of two even functions;

(c) a product of an odd function and an even function.

60. **Pepperoni Pizzas** For a statistics project, a student counted the number of pepperoni slices on pizzas of various sizes at a local pizzeria, compiling the data shown in the table below.

Nemanja Tomic/123RF

Local Pizza Data

Type of Pizza	Radius	Pepperoni Count
Personal	4″	12
Medium	6″	27
Large	7″	37
Extra large	8″	48

(a) Explain why the pepperoni count (P) ought to be proportional to the square of the radius (r).

(b) Assuming that $P = k \cdot r^2$, use the data pair $(4, 12)$ to find the value of k.

(c) Does the algebraic model fit the rest of the data well?

(d) Some pizza places have charts showing their kitchen staff how much of each topping should be put on each size of pizza. Do you think this pizzeria uses such a chart? Explain.

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A graphing calculator is *not permitted* for solving the following problems.

61. Multiple Choice Which of the following functions proves that there can be infinitely many solutions to the equation $f(x) = 0$?

(A) $\ln x$ (B) $\cos x$ (C) $\dfrac{1}{1+e^{-x}}$ (D) $\sqrt{x}$

62. Multiple Choice Which of the following functions proves that a bounded function can be increasing on the entire interval $(-\infty, \infty)$?

(A) $\ln x$ (B) $\cos x$ (C) $\dfrac{1}{1+e^{-x}}$ (D) $\sqrt{x}$

63. Multiple Choice Which of the following functions has a graph that would have odd symmetry if it were shifted down ½ unit?

(A) $\sin x$ (B) $\cos x$ (C) $\text{int}(x)$ (D) $\dfrac{1}{1+e^{-x}}$

64. Multiple Choice Which of the following functions proves that a function can be always negative to the left of the y-axis, always positive to the right of the y-axis, and yet decreasing over every interval in its domain?

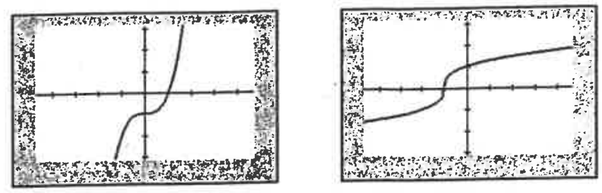
(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{1+e^{-x}}$ (C) x^3 (D) $\text{int}(x)$

65. Answer and Explain Is every x-intercept of the cosine function a point of inflection? Justify your answer.

66. Answer and Explain Are there any solutions to the equation $x = \ln x$? Justify your answer.

Extending the Ideas

67. Inverse Functions Two functions are said to be *inverses* of each other if the graph of one can be obtained from the graph of the other by reflecting it across the line $y = x$. For example, the functions with the graphs shown below are inverses of each other:



$[-4.7, 4.7]$ by $[-3.1, 3.1]$
(a)

$[-4.7, 4.7]$ by $[-3.1, 3.1]$
(b)

(a) Two of the twelve basic functions in this section are inverses of each other. Which are they?

(b) Two of the twelve basic functions in this section are their own inverses. Which are they?

(c) If you restrict the domain of one of the twelve basic functions to $[0, \infty)$, it becomes the inverse of another one. Which are they?

68. Identifying a Function by Its Properties

(a) Seven of the twelve basic functions have the property that $f(0) = 0$. Which five do not?

(b) Only one of the twelve basic functions has the property that $f(x + y) = f(x) + f(y)$ for all x and y in its domain. Which one is it?

(c) One of the twelve basic functions has the property that $f(x + y) = f(x)f(y)$ for all x and y in its domain. Which one is it?

(d) One of the twelve basic functions has the property that $f(xy) = f(x) + f(y)$ for all x and y in its domain. Which one is it?

(e) Four of the twelve basic functions have the property that $f(x) + f(-x) = 0$ for all x in their domains. Which four are they?

1.4 Building Functions from Functions

What you'll learn ...

- Combining Functions Algebraically
- Composition of Functions
- Relations and Implicitly Defined Functions
- Piecewise-Defined Functions

... and why

Most of the functions that you will encounter in calculus and in real life can be created by combining or modifying other functions.

Combining Functions Algebraically

Knowing how a function is "put together" is an important first step when applying the tools of calculus. Functions have their own algebra based on the same operations we apply to real numbers (addition, subtraction, multiplication, and division). One way to build new functions is to apply these operations, using the following definitions.

DEFINITION Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions with intersecting domains. Then for all values of x in the intersection, the algebraic combinations of f and g are defined by the following rules:

Sum: $(f + g)(x) = f(x) + g(x)$

Difference: $(f - g)(x) = f(x) - g(x)$

Product: $(fg)(x) = f(x)g(x)$

Quotient: $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$, provided $g(x) \neq 0$

In each case, the domain of the new function consists of all numbers that belong to both the domain of f and the domain of g, except that the zeros of the denominator are excluded from the domain of the quotient.