

Finding any Determinant and using technology to find inverse matrices and solve matrix equations

Helpful link <https://www.desmos.com/matrix>

Find the determinant and the inverse matrix of each of the following matrices using technology

Page 1

$$\begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix} \text{ has determinant } \det \begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix} = 10$$

$$\begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix} \text{ has inverse } \begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{-7}{5} & \frac{-16}{5} & \frac{12}{5} \\ \frac{11}{5} & \frac{23}{5} & \frac{-16}{5} \\ \frac{-3}{10} & \frac{-2}{5} & \frac{3}{10} \end{bmatrix}$$

Given Matrix Equation $\begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 85 \\ -62 \\ -31 \end{bmatrix}$

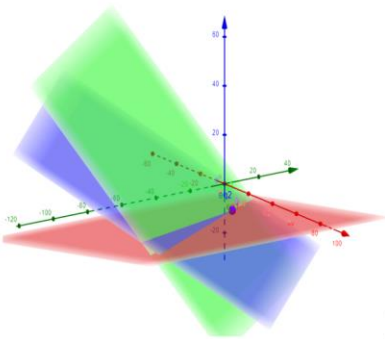
Apply the Inverse $\begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 85 \\ -62 \\ -31 \end{bmatrix}$

This results in Identity Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -10 \end{bmatrix}$

This is the solution $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -10 \end{bmatrix}$

This is the double check of the solution $\begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 1 \\ -10 \end{bmatrix} = \begin{bmatrix} 85 \\ -62 \\ -31 \end{bmatrix}$

The purple point (5, 1, -10) is the solution to $\begin{bmatrix} 1 & 0 & -8 \\ 3 & 3 & 8 \\ 5 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 85 \\ -62 \\ -31 \end{bmatrix}$



<https://www.geogebra.org/3d/frn58cam>

Find the determinant and the inverse matrix of each of the following matrices using technology

Page 2

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & 4 & 5 & -2 \\ 5 & 2 & -3 & 4 \\ -8 & 3 & 2 & 41 \end{bmatrix} \text{ has determinant } \det \begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & 4 & 5 & -2 \\ 5 & 2 & -3 & 4 \\ -8 & 3 & 2 & 41 \end{bmatrix} = -30$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & 4 & 5 & -2 \\ 5 & 2 & -3 & 4 \\ -8 & 3 & 2 & 41 \end{bmatrix} \text{ has inverse } \begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & 4 & 5 & -2 \\ 5 & 2 & -3 & 4 \\ -8 & 3 & 2 & 41 \end{bmatrix}^{-1} = \begin{bmatrix} 30 & \frac{7}{3} & \frac{-17}{3} & \frac{2}{3} \\ -473 & -53 & 55 & -16 \\ 10 & 15 & 6 & 15 \\ 29 & \frac{7}{3} & \frac{-17}{3} & \frac{2}{3} \\ \frac{79}{10} & \frac{3}{5} & \frac{-3}{2} & \frac{1}{5} \\ 10 & 5 & 2 & 5 \end{bmatrix}$$

$$\text{Given Matrix Equation } \begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & 4 & 5 & -2 \\ 5 & 2 & -3 & 4 \\ -8 & 3 & 2 & 41 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -6 \\ -21 \\ -18 \\ 195 \end{bmatrix}$$

$$\text{Apply the Inverse } \begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & 4 & 5 & -2 \\ 5 & 2 & -3 & 4 \\ -8 & 3 & 2 & 41 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & 4 & 5 & -2 \\ 5 & 2 & -3 & 4 \\ -8 & 3 & 2 & 41 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & 4 & 5 & -2 \\ 5 & 2 & -3 & 4 \\ -8 & 3 & 2 & 41 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -6 \\ -21 \\ -18 \\ 195 \end{bmatrix}$$

$$\text{This results in the Identity Matrix } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 3 \\ -15 \\ 9 \\ 6 \end{bmatrix}$$

$$\text{This is the solution } \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 3 \\ -15 \\ 9 \\ 6 \end{bmatrix}$$

$$\text{This is the double check of the solution } \begin{bmatrix} 1 & 0 & -1 & 0 \\ 2 & 4 & 5 & -2 \\ 5 & 2 & -3 & 4 \\ -8 & 3 & 2 & 41 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -15 \\ 9 \\ 6 \end{bmatrix} = \begin{bmatrix} -6 \\ -21 \\ -18 \\ 195 \end{bmatrix}$$