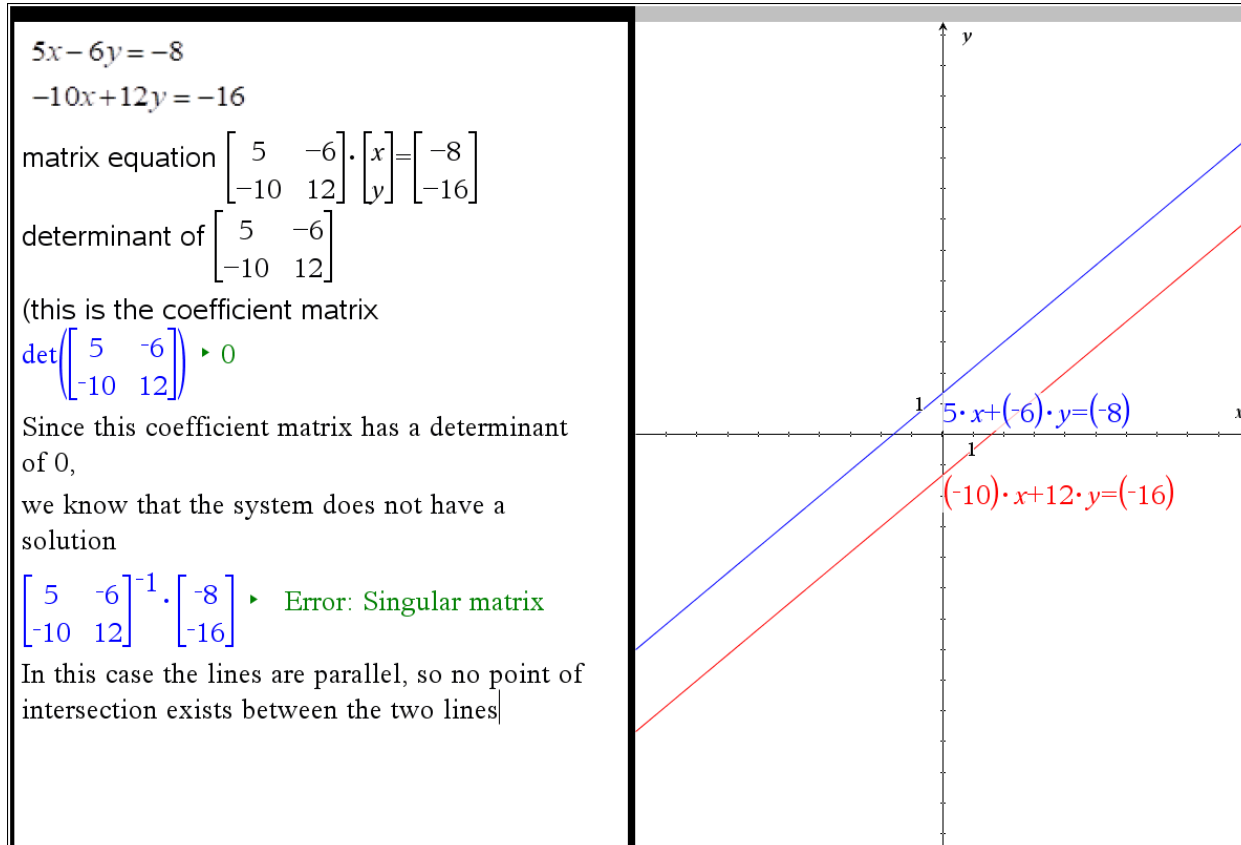


Use Cramer's Rule and determinants to solve the given 2x2 and 3x3 systems, if not possible, then state why not

$$\text{System 1} \quad \begin{aligned} 5x - 6y &= -8 \\ -10x + 12y &= -16 \end{aligned}$$



System 2

$$6x - 8y = 30$$

$$-5x + 2y = -11$$

$6x - 8y = 30$

$-5x + 2y = -11$

matrix equation $\begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30 \\ -11 \end{bmatrix}$

Find determinant of $\begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix}$

(this is the coefficient matrix) $\det\left(\begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix}\right) = -28$

Since this coefficient matrix has a determinant of -28 , we know that the system does have a solution

Find determinant of replace x matrix $\begin{bmatrix} 30 & -8 \\ -11 & 2 \end{bmatrix}$

$\det\left(\begin{bmatrix} 30 & -8 \\ -11 & 2 \end{bmatrix}\right) = -28$

Find determinant of replace Y matrix $\begin{bmatrix} 6 & 30 \\ -5 & -11 \end{bmatrix}$

$\det\left(\begin{bmatrix} 6 & 30 \\ -5 & -11 \end{bmatrix}\right) = 84$

$x = \frac{\det\left(\begin{bmatrix} 30 & -8 \\ -11 & 2 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix}\right)} = 1$ $y = \frac{\det\left(\begin{bmatrix} 6 & 30 \\ -5 & -11 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix}\right)} = -3$

$6 \cdot x + (-8) \cdot y = (30)$

$(-5) \cdot x + 2 \cdot y = (-11)$

$(1, -3)$

$6x - 8y = 30$

$-5x + 2y = -11$

matrix equation

$\begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 30 \\ -11 \end{bmatrix}$

$\begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 30 \\ -11 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

The solution matrix is

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

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$\begin{bmatrix} 6 & -8 \\ -5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 30 \\ -11 \end{bmatrix}$

$6 \cdot x + (-8) \cdot y = (30)$

$(-5) \cdot x + 2 \cdot y = (-11)$

$(1, -3)$

$$2x - y + 4z = -3$$

$$\text{System 3 } 5x + 3y - 2z = 1$$

$$2x + 8y - 7z = -15$$

$$2x - y + 4z = -3$$

$$5x + 3y - 2z = 1$$

$$2x + 8y - 7z = -15 \quad |$$

$$\text{related matrix equation } \begin{bmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -15 \end{bmatrix}$$

Find the determinant of coefficient matrix

$$\det \begin{pmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{pmatrix}$$

Find the determinant of the replace x matrix

$$\det \begin{pmatrix} -3 & -1 & 4 \\ 1 & 3 & -2 \\ -15 & 8 & -7 \end{pmatrix}$$

Find the determinant of the replace y matrix

$$\det \begin{pmatrix} 2 & -3 & 4 \\ 5 & 1 & -2 \\ 2 & -15 & -7 \end{pmatrix}$$

Find the determinant of the replace z matrix

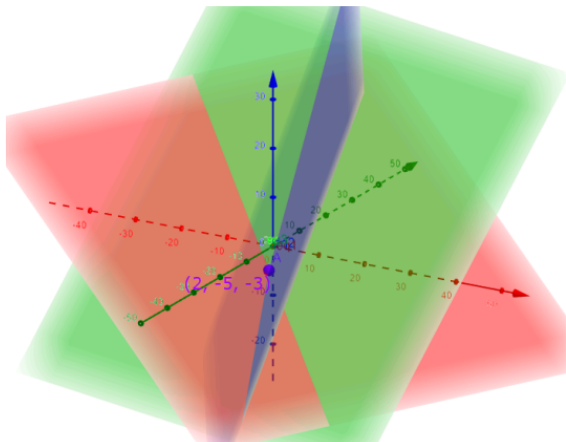
$$\det \begin{pmatrix} 2 & -1 & -3 \\ 5 & 3 & 1 \\ 2 & 8 & -15 \end{pmatrix}$$

$$\text{Solution } x = \frac{\det \begin{pmatrix} -3 & -1 & 4 \\ 1 & 3 & -2 \\ -15 & 8 & -7 \end{pmatrix}}{\det \begin{pmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{pmatrix}} \quad y = \frac{\det \begin{pmatrix} 2 & -3 & 4 \\ 5 & 1 & -2 \\ 2 & -15 & -7 \end{pmatrix}}{\det \begin{pmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{pmatrix}} \quad z = \frac{\det \begin{pmatrix} 2 & -1 & -3 \\ 5 & 3 & 1 \\ 2 & 8 & -15 \end{pmatrix}}{\det \begin{pmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{pmatrix}}$$

$$\text{Solution to } \begin{bmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}$$

The solution is the purple point



$$2x - y + 4z = -3$$

$$5x + 3y - 2z = 1$$

$$2x + 8y - 7z = -15$$

$$\text{related matrix equation } \begin{bmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -3 \\ 1 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}$$

$$\checkmark \checkmark \begin{bmatrix} 2 & -1 & 4 \\ 5 & 3 & -2 \\ 2 & 8 & -7 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ -15 \end{bmatrix}$$

$$8x + 3y - 7z = -135$$

$$\text{System 3 } 3x + 4y - z = -46$$

$$-4x + 6y + 9z = 101$$

$$8x + 3y - 7z = -135$$

$$3x + 4y - z = -46$$

$$-4x + 6y + 9z = 101$$

$$\text{related matrix equation } \begin{bmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -135 \\ -46 \\ 101 \end{bmatrix}$$

Find the determinant of coefficient matrix

$$\det \begin{pmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{pmatrix} \rightarrow 29$$

Find the determinant of the replace x matrix

$$\det \begin{pmatrix} -135 & 3 & -7 \\ -46 & 4 & -1 \\ 101 & 6 & 9 \end{pmatrix} \rightarrow 29$$

Find the determinant of the replace y matrix

$$\det \begin{pmatrix} 8 & -135 & -7 \\ 3 & -46 & -1 \\ -4 & 101 & 9 \end{pmatrix} \rightarrow -232$$

Find the determinant of the replace z matrix

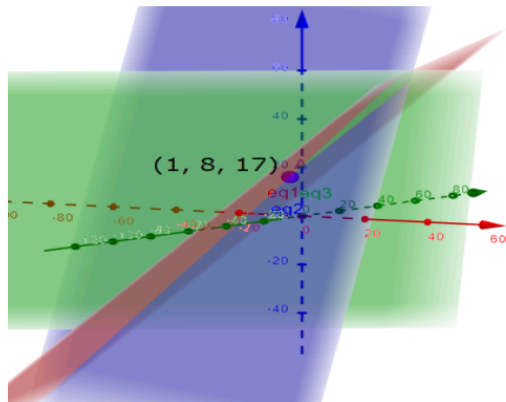
$$\det \begin{pmatrix} 8 & 3 & -135 \\ 3 & 4 & -46 \\ -4 & 6 & 101 \end{pmatrix} \rightarrow 493$$

$$\text{Solution } x = \frac{\det \begin{pmatrix} -135 & 3 & -7 \\ -46 & 4 & -1 \\ 101 & 6 & 9 \end{pmatrix}}{\det \begin{pmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{pmatrix}} = 1 \quad y = \frac{\det \begin{pmatrix} 8 & -135 & -7 \\ 3 & -46 & -1 \\ -4 & 101 & 9 \end{pmatrix}}{\det \begin{pmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{pmatrix}} = -8 \quad z = \frac{\det \begin{pmatrix} 8 & 3 & -135 \\ 3 & 4 & -46 \\ -4 & 6 & 101 \end{pmatrix}}{\det \begin{pmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{pmatrix}} = 17$$

$$\text{Solution to } \begin{bmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -135 \\ -46 \\ 101 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \\ 17 \end{bmatrix}$$

The solution is the purple point



$$8x + 3y - 7z = -135$$

$$3x + 4y - z = -46$$

$$-4x + 6y + 9z = 101$$

$$\text{related matrix equation } \begin{bmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -135 \\ -46 \\ 101 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -135 \\ -46 \\ 101 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \\ 17 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -8 \\ 17 \end{bmatrix}$$

$$\checkmark \begin{bmatrix} 8 & 3 & -7 \\ 3 & 4 & -1 \\ -4 & 6 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -8 \\ 17 \end{bmatrix} = \begin{bmatrix} -135 \\ -46 \\ 101 \end{bmatrix}$$