Name____Daily Quiz Using Determinant to solve a 2x2 and a 3x3 system

Use Cramer's Rule and determinants to solve the given 2x2 and 3x3 systems, if not possible, then state why not

System 1
$$\begin{aligned}9x-7y&=46\\-5x+8y&=-42\end{aligned}$$

$$\begin{bmatrix}9 & -7\\-5 & 8\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}46\\-42\end{bmatrix}$$

$9\cdot x-7\cdot y=46$

$-5\cdot x+8\cdot y=-42$

Coefficient matrix= $\begin{bmatrix}9 & -7\\-5 & 8\end{bmatrix}$ Coefficient matrix determinant $\det\left(\begin{bmatrix}9 & -7\\-5 & 8\end{bmatrix}\right) ▸ 37$

Replace x matrix = $\begin{bmatrix}46 & -7\\-42 & 8\end{bmatrix}$ Replace x determinant $\det\left(\begin{bmatrix}46 & -7\\-42 & 8\end{bmatrix}\right) ▸ 74$

Replace y matrix = $\begin{bmatrix}9 & 46\\-5 & -42\end{bmatrix}$ Replace y matrix determinant $\det\left(\begin{bmatrix}9 & 46\\-5 & -42\end{bmatrix}\right) ▸ -148$

Solutions to the matrix equation and the 2 x 2 system of equations

$$x=\frac{\det\left(\begin{bmatrix}46 & -7\\-42 & 8\end{bmatrix}\right)}{\det\left(\begin{bmatrix}9 & -7\\-5 & 8\end{bmatrix}\right)}=\frac{74}{37} ▸ 2 \qquad y=\frac{\det\left(\begin{bmatrix}9 & 46\\-5 & -42\end{bmatrix}\right)}{\det\left(\begin{bmatrix}9 & -7\\-5 & 8\end{bmatrix}\right)}=\frac{-148}{37} ▸ -4$$

$$\begin{bmatrix}9 & -7\\-5 & 8\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}46\\-42\end{bmatrix}$$

& $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\-4\end{bmatrix}$ imply

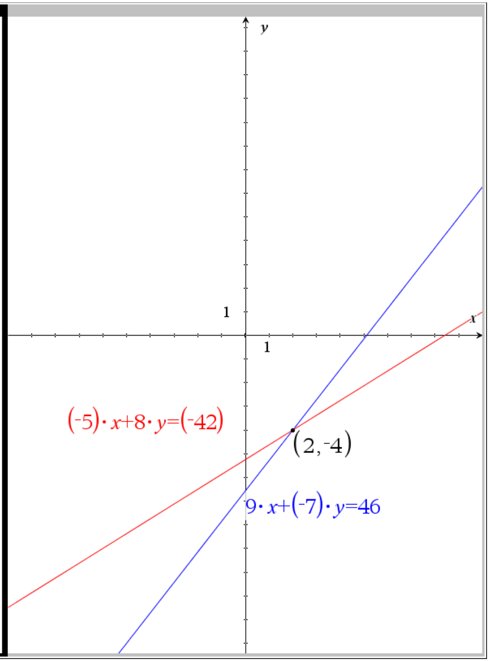
$$\begin{bmatrix}9 & -7\\-5 & 8\end{bmatrix}\cdot\begin{bmatrix}2\\-4\end{bmatrix} ▸ \begin{bmatrix}46\\-42\end{bmatrix}$$

This checks so the solution to

$9\cdot x-7\cdot y=46$

$-5\cdot x+8\cdot y=-42$

is (x, y) = (2, −4)



System 2 $\begin{aligned}5x-3y&=13\\-20x+12y&=-52\end{aligned}$

$$\begin{bmatrix}5 & -3\\-20 & 12\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}13\\-52\end{bmatrix}$$

$9\cdot x-7\cdot y=46$

$^{-}5\cdot x+8\cdot y={}^{-}42$

Coefficient matrix= $\begin{bmatrix}5 & ^{-}3\\^{-}20 & 12\end{bmatrix}$ Coefficient matrix determinant $\det\left(\begin{bmatrix}5 & ^{-}3\\^{-}20 & 12\end{bmatrix}\right) \blacktriangleright 0$

Recall if a matrix has a determinant of 0, then there is no invcrse and if there is no inverse, then there is no single solution to a matrix equation

1) because there is no point of intersection for the lines or the planes

or

2) there are infinitely many solutions because there are coinciding lines (as in this case) , coinciding planes, or planes that intersect in a line

$$\begin{bmatrix}5 & -3\\-20 & 12\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}13\\-52\end{bmatrix}$$

has no single solution

(x,y) = $\left(x, \frac{13-5x}{-3}\right)$ or $\left(\frac{13+3y}{5}, y\right)$

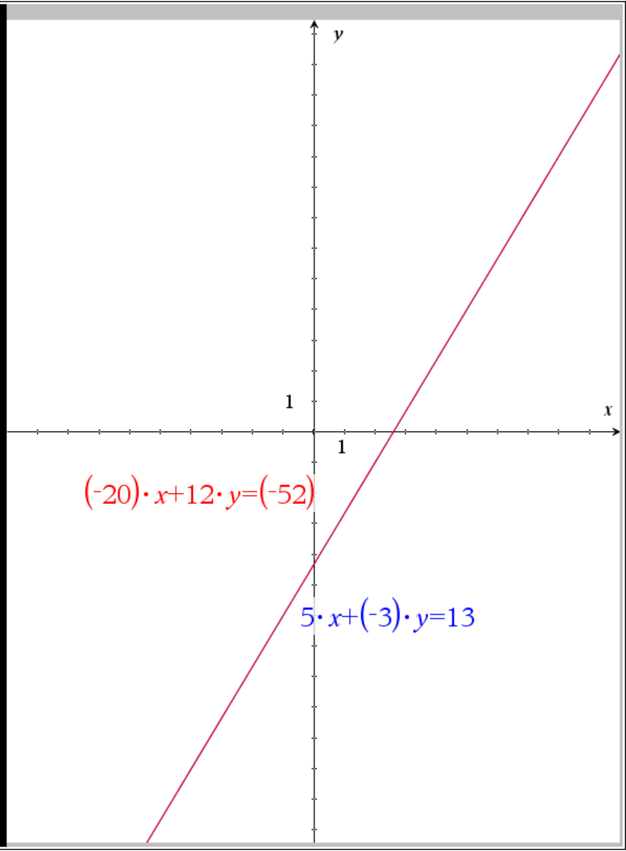
(x,y) = $\left(x, \frac{-52+20x}{12}\right)$ or $\left(\frac{-52-12y}{-20}, y\right)$

There are infinitely many solutions

$9\cdot x-7\cdot y=46$

$^{-}5\cdot x+8\cdot y={}^{-}42$

any point on the line is a solution



$$\text{System 3}\quad \begin{aligned} 2x+5y&=-18\\ x+3y-2z&=-25\\ 5x+y+4z&=29 \end{aligned}$$

https://www.geogebra.org/3d/ayq5yqzp

$$\begin{bmatrix}2&5&0\\1&3&-2\\5&1&4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-18\\-25\\29\end{bmatrix}$$

2x+5y=−18

x+3y−2z=−25

5x+y+4z=29

Coefficient matrix= $\begin{bmatrix}2&5&0\\1&3&-2\\5&1&4\end{bmatrix}$ Coefficient matrix determinant $\det\left(\begin{bmatrix}2&5&0\\1&3&-2\\5&1&4\end{bmatrix}\right) \blacktriangleright -42$

Replace x matrix $=\begin{bmatrix}-18&5&0\\-25&3&-2\\29&1&4\end{bmatrix}$ Replace x determinant $\det\left(\begin{bmatrix}-18&5&0\\-25&3&-2\\29&1&4\end{bmatrix}\right) \blacktriangleright -42$

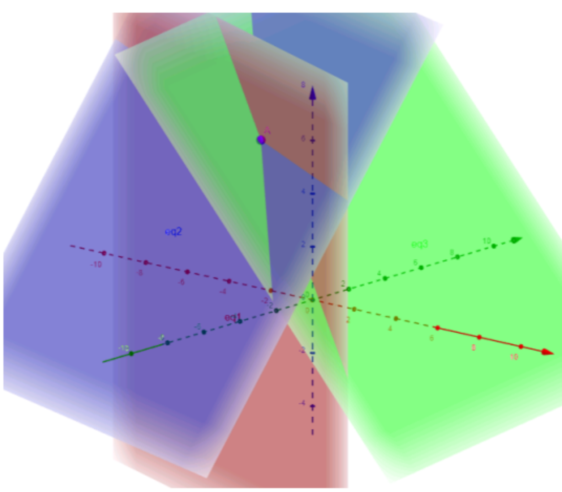
Replace y matrix $=\begin{bmatrix}2&-18&0\\1&-25&-2\\5&29&4\end{bmatrix}$ Replace y matrix determinant $\det\left(\begin{bmatrix}2&-18&0\\1&-25&-2\\5&29&4\end{bmatrix}\right) \blacktriangleright 168$

Replace z matrix $=\begin{bmatrix}2&5&-18\\1&3&-25\\5&1&29\end{bmatrix}$ Replace z matrix determinant $\det\left(\begin{bmatrix}2&5&-18\\1&3&-25\\5&1&29\end{bmatrix}\right) \blacktriangleright -294$

$x=\frac{-42}{-42} \blacktriangleright 1$ $y=\frac{168}{-42} \blacktriangleright -4$ $z=\frac{-294}{-42} \blacktriangleright 7$ solution $(1, -4, 7)$

https://www.geogebra.org/3d/ayq5yqzp

The purple dot is the solution to the system of planes



2x+5y=−18

x+3y−2z=−25

5x+y+4z=29

$\begin{bmatrix}2&5&0\\1&3&-2\\5&1&4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-18\\-25\\29\end{bmatrix}$ has solution $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-4\\7\end{bmatrix}$

$$\begin{bmatrix}2&5&0\\1&3&-2\\5&1&4\end{bmatrix}\cdot\begin{bmatrix}1\\-4\\7\end{bmatrix} \blacktriangleright \begin{bmatrix}-18\\-25\\29\end{bmatrix}$$

$$\text{System 4}\quad \begin{aligned} 4x+1z&=2\\ 3x+2y+5z&=-16\\ 6x+1.5z&=3 \end{aligned}$$

https://www.geogebra.org/3d/kz7hhr4z

$$\begin{bmatrix}4&0&1\\3&2&5\\6&0&1.5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\-16\\3\end{bmatrix}$$

4x+1z=2

3x+2y+5z=−16

6x+1.5z=3

Coefficient matrix= $\begin{bmatrix}4&0&1\\3&2&5\\6&0&1.5\end{bmatrix}$

Coefficient matrix determinant $\det\left(\begin{bmatrix}4&0&1\\3&2&5\\6&0&1.5\end{bmatrix}\right) \blacktriangleright 0$

Recall if a matrix has a determinant of 0, then there is no inverse and if there is no inverse, then there is no single solution to a matrix equation

1) because there is no point of intersection for the lines or the planes

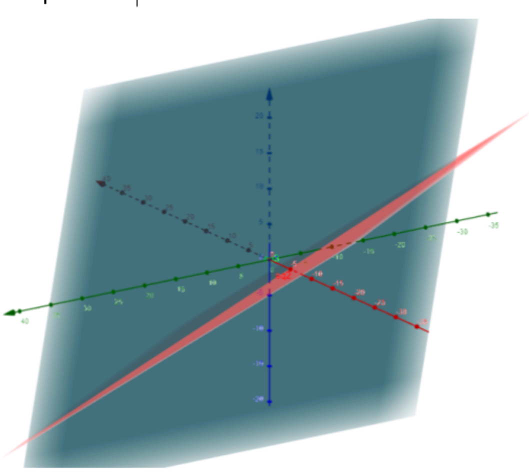
or

2) there are infinitely many solutions because there are coinciding lines, coinciding planes, or planes that intersect in a line $\left(\frac{-z+2}{4}, \frac{-17z-70}{8}, z\right)$ This is the line the two planes intersect in

$\text{solve}\left(\begin{cases}4\cdot x+1\cdot z=2\\ 3\cdot x+2\cdot y+5\cdot z=-16\\ 6\cdot x+\frac{3}{2}\cdot z=3\end{cases},\{x,y,z\}\right) \blacktriangleright x=\frac{-(c2-2)}{4}$ and $y=\frac{-(17\cdot c2+70)}{8}$ and $z=c2$ Neat trick a CAS calculator can do!

https://www.geogebra.org/3d/kz7hhr4z

Intersection of the two planes is the line on both planes.



4x+1z=2

3x+2y+5z=−16

6x+1.5z=3

$$\begin{bmatrix}4&0&1\\3&2&5\\6&0&1.5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-18\\-25\\29\end{bmatrix}$$

has solution $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}\frac{-z+2}{4}\\ \frac{-17z-70}{8}\\ z\end{bmatrix}$

$$\begin{bmatrix}4&0&1\\3&2&5\\6&0&1.5\end{bmatrix}\cdot\begin{bmatrix}\frac{-z+2}{4}\\ \frac{-17\cdot z-70}{8}\\ z\end{bmatrix} \blacktriangleright \begin{bmatrix}2\\-16\\3.\end{bmatrix}$$