

Name \_\_\_\_\_

Either scan your answers or send a picture using remind or email

Quiz 1: determinants of 3 x 3 matrices and inverses of 2 x 2 matrices

Show ALL the steps in determining the determinant of a 3 x 3 Matrix and state the determinant of  $\begin{bmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & -7 \end{bmatrix}$

$$\begin{bmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & -7 \end{bmatrix} \left| \begin{array}{l} 2 \\ 1 \\ 6 \end{array} \right| \begin{array}{l} 2 \\ 0 \\ 3 \end{array} \text{ this leads to } (2 \cdot 0 \cdot -7 + 2 \cdot 5 \cdot 6 + -3 \cdot 1 \cdot 3) - (6 \cdot 0 \cdot -3 + 3 \cdot 5 \cdot 2 + -7 \cdot 1 \cdot 2) \\ = (0 + 60 - 9) - (0 + 30 - 14) = 51 - 16 = 35$$

$$\det \left( \begin{bmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & -7 \end{bmatrix} \right) = 35 \text{ Which means } \begin{bmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & -7 \end{bmatrix} \text{ is a matrix with an inverse}$$

State the value of n that would mean that the following matrix would NOT have an inverse  $\begin{bmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & n \end{bmatrix}$

$$\begin{bmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & n \end{bmatrix} \left| \begin{array}{l} 2 \\ 1 \\ 6 \end{array} \right| \begin{array}{l} 2 \\ 0 \\ 3 \end{array} \text{ this leads to } (2 \cdot 0 \cdot n + 2 \cdot 5 \cdot 6 + -3 \cdot 1 \cdot 3) - (6 \cdot 0 \cdot -3 + 3 \cdot 5 \cdot 2 + n \cdot 1 \cdot 2) \\ = (0 + 60 - 9) - (0 + 30 + 2n) = 51 - 30 - 2n = 21 - 2n \\ \text{So for this matrix to have an inverse } 21 - 2n \neq 0 \\ \text{Or for this matrix to NOT have an inverse } n = 10.5$$

Related TINSPIRE PAGES problems 1 and 2

The image shows a TI-84 Plus calculator screen with the following content:

- Row 1:  $\det \begin{pmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & -7 \end{pmatrix}$  followed by the value 35.
- Row 2:  $\det \begin{pmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & 10.5 \end{pmatrix}$  followed by the value 0.
- Row 3:  $\begin{bmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & -7 \end{bmatrix}^{-1}$  followed by the inverse matrix  $\begin{bmatrix} \frac{-3}{35} & \frac{1}{35} & \frac{2}{35} \\ \frac{7}{35} & \frac{4}{35} & \frac{-13}{35} \\ \frac{3}{35} & \frac{6}{35} & \frac{-2}{35} \end{bmatrix}$ .
- Row 4:  $\begin{bmatrix} 2 & 2 & -3 \\ 1 & 0 & 5 \\ 6 & 3 & 10.5 \end{bmatrix}^{-1}$  followed by the error message "Error: Singular matrix".

State the inverse matrix of  $A = \begin{bmatrix} 2 & 5 \\ 8 & -6 \end{bmatrix}$

$\begin{bmatrix} 2 & 5 \\ 8 & -6 \end{bmatrix}$  has a determinant of  $(2)(-6) - (8)(5) = -12 - 40 = -52$

$$\text{So this leads to } A^{-1} = \frac{1}{-52} \begin{bmatrix} -6 & -5 \\ -8 & 2 \end{bmatrix} = \begin{bmatrix} \frac{-6}{-52} & \frac{-5}{-52} \\ \frac{-8}{-52} & \frac{2}{-52} \end{bmatrix} = \begin{bmatrix} \frac{3}{26} & \frac{5}{52} \\ \frac{2}{13} & \frac{-1}{26} \end{bmatrix}$$

Related TINSPiRE PAGES problem 3

TI-Nspire CAS screenshot showing the calculation of the inverse of a 2x2 matrix. The determinant of the matrix  $\begin{bmatrix} 2 & 5 \\ 8 & -6 \end{bmatrix}$  is calculated as  $-52$ . The inverse is then found by multiplying the adjugate matrix  $\begin{bmatrix} 3 & 5 \\ 2 & -1 \end{bmatrix}$  by the reciprocal of the determinant,  $\frac{1}{-52}$ , resulting in the inverse matrix  $\begin{bmatrix} \frac{3}{26} & \frac{5}{52} \\ \frac{2}{13} & \frac{-1}{26} \end{bmatrix}$ .

State the value of  $n$  that would mean that the following matrix would NOT have an inverse  $\begin{bmatrix} 2 & n \\ 8 & -6 \end{bmatrix}$

$$\det \left( \begin{bmatrix} 2 & n \\ 8 & -6 \end{bmatrix} \right) = (2)(-6) - 8n = -12 - 8n$$

$\begin{bmatrix} 2 & n \\ 8 & -6 \end{bmatrix}$  has an inverse for all  $n$  such that  $-12 - 8n \neq 0$

$\begin{bmatrix} 2 & n \\ 8 & -6 \end{bmatrix}$  does NOT have an inverse if  $n = \frac{-12}{8} = \frac{-3}{2} = -1.5$

Related TINSPiRE PAGES problem 4

TI-Nspire CAS screenshot showing the expansion of the determinant of a 2x2 matrix with  $n$ . The determinant is expanded to  $-8n - 12$ . When  $n$  is set to  $-1.5$ , the determinant is calculated as  $0$ , and the inverse calculation fails with the error message "Error: Singular matrix".