Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

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Geometry Overview

Congruence

- Experiment with transformations in the plane

- Understand congruence in terms of rigid motions

- Prove geometric theorems

- Make geometric constructions

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations

- Prove theorems involving similarity

- Define trigonometric ratios and solve problems involving right triangles

- Apply trigonometry to general triangles

Circles

- Understand and apply theorems about circles

- Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section

- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems

- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry

- Apply geometric concepts in modeling situations

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Congruence   HSG-CO

A. Experiment with transformations in the plane

1. HSG-CO.A.1
   Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

2. HSG-CO.A.2
   Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

3. HSG-CO.A.3
   Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

4. HSG-CO.A.4
   Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

5. HSG-CO.A.5
   Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

B. Understand congruence in terms of rigid motions

6. HSG-CO.B.6
   Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

7. HSG-CO.B.7
   Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

8. HSG-CO.B.8
   Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

C. Prove geometric theorems

9. HSG-CO.C.9
   Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

10. HSG-CO.C.10
    Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

11. HSG-CO.C.11
    Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

D. Make geometric constructions

12. HSG-CO.D.12
    Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

13. HSG-CO.D.13
    Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry — HSG-SRT

A. Understand similarity in terms of similarity transformations

1. HSG-SRT.A.1
   Verify experimentally the properties of dilations given by a center and a scale factor:

   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

2. HSG-SRT.A.2
   Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

3. HSG-SRT.A.3
   Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

B. Prove theorems involving similarity

4. HSG-SRT.B.4
   Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

5. HSG-SRT.B.5
   Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

C. Define trigonometric ratios and solve problems involving right triangles

6. HSG-SRT.C.6
   Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

7. HSG-SRT.C.7
   Explain and use the relationship between the sine and cosine of complementary angles.

8. HSG-SRT.C.8
   Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

D. Apply trigonometry to general triangles

9. HSG-SRT.D.9
   (+) Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

10. HSG-SRT.D.10
    (+) Prove the Laws of Sines and Cosines and use them to solve problems.

11. HSG-SRT.D.11
    (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles — HSG-C

A. Understand and apply theorems about circles

1. HSG-C.A.1
   Prove that all circles are similar.

2. HSG-C.A.2
   Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

3. HSG-C.A.3
   Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

4. HSG-C.A.4
   (+) Construct a tangent line from a point outside a given circle to the circle.

B. Find arc lengths and areas of sectors of circles

5. HSG-C.B.5

   Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations   HSG-GPE

A. Translate between the geometric description and the equation for a conic section

1. HSG-GPE.A.1

   Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

2. HSG-GPE.A.2

   Derive the equation of a parabola given a focus and directrix.

3. HSG-GPE.A.3

   (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

B. Use coordinates to prove simple geometric theorems algebraically

4. HSG-GPE.B.4

   Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

5. HSG-GPE.B.5

   Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

6. HSG-GPE.B.6

   Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

7. HSG-GPE.B.7

   Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

Geometric Measurement and Dimension   HSG-GMD

A. Explain volume formulas and use them to solve problems

1. HSG-GMD.A.1

   Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

2. HSG-GMD.A.2

   (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

3. HSG-GMD.A.3

   Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

B. Visualize relationships between two-dimensional and three-dimensional objects

4. HSG-GMD.B.4

   Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry   HSG-MG

A. Apply geometric concepts in modeling situations

1. HSG-MG.A.1

   Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

2. HSG-MG.A.2

   Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

3. HSG-MG.A.3

   Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*