CHAPTER 7 Polynomial Functions

7.1 PAGE 429 GUIDED SKILLS PRACTICE

4. $x^4 + 3x^3 - x^2$
The greatest exponent of x is 4 and the polynomial has three terms. The polynomial is a quartic trinomial.

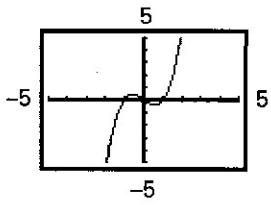
5. $x^5 + 3x^4 - 4x^2 + x - 1$
The greatest exponent of x is 5 and the polynomial has five terms. The polynomial is a quintic polynomial.

6. $x^3 + 2x^2 - x + 1$
$= 2^3 + 2(2)^2 - 2 + 1$
$= 8 + 8 - 2 + 1$
$= 15$

7. $(2x^3 + 3x^2 - x + 2) + (-3x^2 + 4x + 5) = (2x^3) + (3x^2 - 3x^2) + (-x + 4x) + (2 + 5)$
$= 2x^3 + 3x + 7$

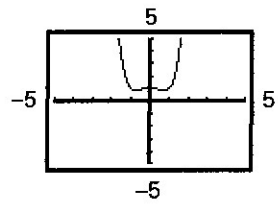
8. $(6x^3 - 5x^2 + 14x + 3) - (3x^3 - 2x^2 + 7x - 2) = (6x^3 - 3x^3) + (-5x^2 + 2x^2) + (14x - 7x) + (3 + 2)$
$= 3x^3 - 3x^2 + 7x + 5$

9. $f(x) = x^3 - x$



The graph of this cubic function has an S-shape with 2 turns.

10. $f(x) = x^4 - x^2 + 1$



The graph of this quartic function has a W-shape with 3 turns.

PAGES 429–430 PRACTICE AND APPLY

11. $5x^3 + 4x + 2x^2 + 1 = 5x^3 + 2x^2 + 4x + 1$

12. $4x^4 + x^2 + x^3 + x + 1 = 4x^4 + x^3 + x^2 + x + 1$

13. $2.7x^3 + 3.3x^8 + 4.1x^2 = 3.3x^8 + 2.7x^3 + 4.1x^2$

14. $9.1x^2 + 5.4x^5 + 3.3x^2 + 2.1 = 5.4x^5 + (9.1x^2 + 3.3x^2) + 2.1$
$= 5.4x^5 + 12.4x^2 + 2.1$

15. $\frac{x^7}{13} + \frac{x^9}{7} - \frac{2}{3} = \frac{x^9}{7} + \frac{x^7}{13} - \frac{2}{3}$

16. $\frac{13}{15}x^4 + \frac{5}{7}x^3 + \frac{3}{5}x^5 + \frac{1}{2} = \frac{3}{5}x^5 + \frac{13}{15}x^4 + \frac{5}{7}x^3 + \frac{1}{2}$

$^3 - 2x + 4$ is a polynomial.
xponent of x is 5 and the polynomial
it is a quintic polynomial.

$x - 7$ is not a polynomial because
contain x.

$2x^2 + 3.8x$ is a polynomial.
xponent of x is 4 and the polynomial
it is a quartic trinomial.

18. Yes, $-4x^2 + 3x^3 - 5x^6 + 4$ is a polynomial. The greatest exponent of x is 6 and the polynomial has 4 terms so it is a polynomial of degree 6.

20. No, $4^{2x} + 5^x - x + 1$ is not a polynomial because the exponents contain x.

22. Yes, $7.81x^4 + 8.9x^3 + 2.5x^2$ is a polynomial. The greatest exponent of x is 4 and the polynomial has 3 terms so it is a quartic trinomial.

Optimized using trial version www.balesio.com

23. No, $\frac{3}{x^2} + \frac{5}{x} + 6$ is not a polynomial because written as $3x^{-2} + 5x^{-1} + 6$, the variable is not raised to a whole-number power.

24. No, $\frac{8}{x^3} - \frac{7}{x^2} + x$ is not a polynomial because written as $8x^{-3} - 7x^{-2} + x$, the variable is not raised to a whole-number power.

25. Yes, $\frac{5}{7}x^6 + \frac{2}{3}x^4 + 5$ is a polynomial.
The greatest exponent of x is 6 and the polynomial has 3 terms so it is a trinomial of degree 6.

26. Yes, $\frac{x^5}{5} - \frac{x^3}{3}$ is a polynomial.
The greatest exponent of x is 5 and the polynomial has 2 terms so it is a quintic binomial.

27. No, $\sqrt{x} - 1$ is not a polynomial because written as $x^{\frac{1}{2}} - 1$, the variable is not raised to a whole-number power.

28. No, $7\sqrt{x} + 4$ is not a polynomial because written as $7x^{\frac{1}{2}} + 4$, the variable is not raised to a whole-number power.

29. $x^3 + x^2 + 1 = (-3)^3 + (-3)^2 + 1$
$= -27 + 9 + 1$
$= -17$

30. $x^4 + 2x^3 + 2 = (-2)^4 + 2(-2)^3 + 2$
$= 16 - 16 + 2$
$= 2$

31. $-2x^3 - 3x + 2 = -2(4)^3 - 3(4) + 2$
$= -128 - 12 + 2$
$= -138$

32. $-4x^3 + 1 + x = -4(3)^3 + 1 + 3$
$= -108 + 1 + 3$
$= -104$

33. $3x^3 + x^2 + 2x + 4 = 3(5)^3 + 5^2 + 2(5) + 4$
$= 375 + 25 + 10 + 4$
$= 414$

34. $5x^3 + 2x^2 - 5x + 2 = 5(6)^3 + 2(6)^2 - 5(6) + 2$
$= 1080 + 72 - 30 + 2$
$= 1124$

35. $\frac{1}{4}x^4 + \frac{1}{8}x^3 + \frac{3}{8}x^2 + \frac{5}{8}x + \frac{7}{8} = \frac{1}{4}(2)^4 + \frac{1}{8}(2)^3 + \frac{3}{8}(2)^2 + \frac{5}{8}(2) + \frac{7}{8}$
$= 4 + 1 + \frac{3}{2} + \frac{5}{4} + \frac{7}{8}$
$= \frac{32}{8} + \frac{8}{8} + \frac{12}{8} + \frac{10}{8} + \frac{7}{8}$
$= \frac{69}{8}$
$= 8\frac{5}{8}$

36. $\frac{3}{10}x^3 + \frac{7}{10}x^2 + \frac{1}{10}x + \frac{9}{10} = \frac{3}{10}(10)^3 + \frac{7}{10}(10)^2 + \frac{1}{10}(10) + \frac{9}{10}$
$= \frac{3000}{10} + \frac{700}{10} + \frac{10}{10} + \frac{9}{10}$
$= \frac{3719}{10}$
$= 371\frac{9}{10}$

37. $1 + x^2 - 3x^3 = 1 + (2.5)^2 - 3(2.5)^3$
$= 1 + 6.25 - 46.875$
$= -39.625$

38. $5x^3 + 4x + 2x^2 + 1$
$= 5(3.8)^3 + 4(3.8) + 2(3.8)^2 + 1$
$= 274.36 + 15.2 + 28.88 + 1$
$= 319.44$

39. $(x^3 + x^2 + x + 1) + (2x^3 + 3x^2 + x + 3) = (x^3 + 2x^3) + (x^2 + 3x^2) + (x + x) + (1 + 3)$
$= 3x^3 + 4x^2 + 2x + 4$
This is a cubic polynomial.

40. $(x^5 + x^3 + x) + (x^4 + x^2 + 1) = x^5 + x^4 + x^3 + x^2 + x + 1$
This is a quintic polynomial.

41. $(1 - 5x + x^3) - (2x^4 + 5x^3 - 10x^2) = -2x^4 + (x^3 - 5x^3) + 10x^2 - 5x + 1$
$= -2x^4 - 4x^3 + 10x^2 - 5x + 1$
This is a quartic polynomial.

$2) - (2x^2 + 4x + 7) = 5x^3 + (3x^2 - 2x^2) + (8x - 4x) + (2 - 7)$
$= 5x^3 + x^2 + 4x - 5$

nomial.

$4x^3 + 6x^2 - 2x + 5) = 4x^3 + (2x^2 + 6x^2) + (-5x - 2x) + (3 + 5)$
$= 4x^3 + 8x^2 - 7x + 8$

nomial.

$5x + x^3 + 3x^2) = (-5x^3 + x^3) + (x^2 + 3x^2) + 6x + 7$
$= -4x^3 + 4x^2 + 6x + 7$

nomial.

Optimized using
trial version
www.balesio.com

45. $(x^4 + 5x^2 + x) - (x^4 + 2x^3 + x - 4) = (x^4 - x^4) - 2x^3 + 5x^2 + (x - x) + 4$
$= -2x^3 + 5x^2 + 4$

This is a cubic trinomial.

46. $(8x^2 + x^3 + 1 - 3x) + (2x^3 + 11x^4) = 11x^4 + (x^3 + 2x^3) + 8x^2 - 3x + 1$
$= 11x^4 + 3x^3 + 8x^2 - 3x + 1$

This is a quartic polynomial.

47. $\left(\frac{2}{3}x + \frac{2}{3}x^3 + 1\right) + \left(\frac{2}{3} + \frac{1}{3}x^2 + \frac{1}{3}x\right) = \frac{2}{3}x^3 + \frac{1}{3}x^2 + \left(\frac{2}{3}x + \frac{1}{3}x\right) + \left(1 + \frac{2}{3}\right)$
$= \frac{2}{3}x^3 + \frac{1}{3}x^2 + x + \frac{5}{3}$

This is a cubic polynomial.

48. $\left(\frac{2}{7}x^2 + \frac{1}{7}x + \frac{3}{7}\right) - \left(\frac{4}{7}x^3 + \frac{6}{7}x^2 + \frac{2}{7}\right) = -\frac{4}{7}x^3 + \left(\frac{2}{7}x^2 - \frac{6}{7}x^2\right) + \frac{1}{7}x + \left(\frac{3}{7} - \frac{2}{7}\right)$
$= -\frac{4}{7}x^3 - \frac{4}{7}x^2 + \frac{1}{7}x + \frac{1}{7}$

This is cubic polynomial.

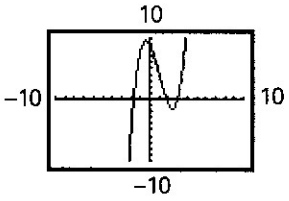
49. $(-3.2x^2 + 2.7x^3 + 7.8x) + (4.9x^3 + 2.5x^4) = 2.5x^4 + (2.7x^3 + 4.9x^3) - 3.2x^2 + 7.8x$
$= 2.5x^4 + 7.6x^3 - 3.2x^2 + 7.8x$

This is a quartic polynomial.

50. $(4.1x^2 + 5.6x + 7.8) - (x^4 + 7.6x^2 + 9.8x) = -x^4 + (4.1x^2 - 7.6x^2) + (5.6x - 9.8x) + 7.8$
$= -x^4 - 3.5x^2 - 4.2x + 7.8$

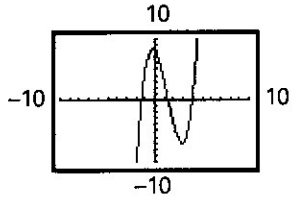
This is a quartic polynomial.

51. $f(x) = x^3 - 3x^2 - 3x + 9$



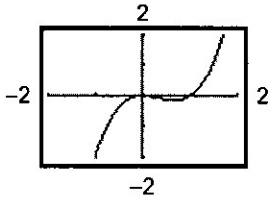
The graph of this cubic function is an S-shape with 2 turns.

52. $b(x) = x^3 - 4x^2 - 2x + 8$



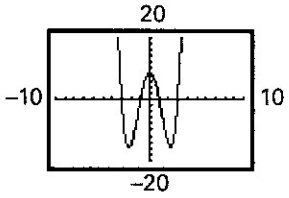
The graph of this cubic function is an S-shape with 2 turns.

53. $k(x) = x^3 - x^2$



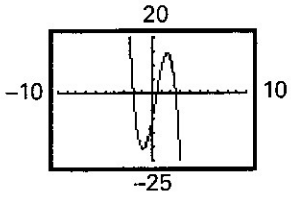
The graph of this cubic function is an S-shape with 2 turns.

54. $m(x) = x^4 - 10x^2 + 9$



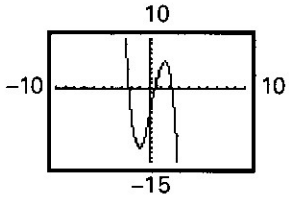
The graph of this quartic function is a W-shape with 3 turns.

55. $r(x) = -4x^3 + 4x^2 + 19x - 10$



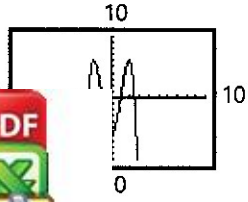
The graph of this cubic function is an S-shape with 2 turns.

56. $s(x) = -2x^3 + x^2 + 10x - 5$



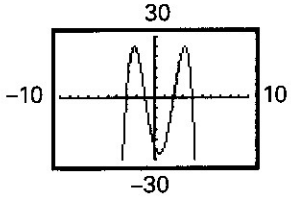
The graph of this cubic function is an S-shape with 2 turns.

57. $j(x) = -x^4 + 7x^2 - 6$



his quartic
M-shape

58. $k(x) = -x^4 + 2x^3 + 13x^2 - 14x - 24$



The graph of this quartic function is an M-shape with 3 turns.

Optimized using
trial version
www.balesio.com

59. $(11x^3 + ax^2 - x + b) - (4x^3 - 3ax + 5) = cx^3 - 2x^2 + dx - 1$
$(11x^3 - 4x^3) + ax^2 + (-x + 3ax) + (b - 5) = cx^3 - 2x^2 + dx - 1$
$7x^3 + ax^2 + (3a - 1)x + (b - 5) = cx^3 - 2x^2 + dx - 1$
$7 = c \qquad a = -2 \qquad 3a - 1 = d \qquad b - 5 = -1$
$3(-2) - 1 = d \qquad b = 4$
$-7 = d$
$a = -2, b = 4, c = 7, d = -7$

60. $(ax^3 + 2x^2 + cx + 1) - (3x^3 + bx^2 + d - 7x) = 5x^3 - 3$
$(ax^3 - 3x^3) + (2x^2 - bx^2) + (cx + 7x) + (1 - d) = 5x^3 - 3$
$(a - 3)x^3 + (2 - b)x^2 + (c + 7)x + (1 - d) = 5x^3 - 3$
$a - 3 = 5 \qquad 2 - b = 0 \qquad c + 7 = 0 \qquad 1 - d = -3$
$a = 8 \qquad b = 2 \qquad c = -7 \qquad d = 4$
$a = 8, b = 2, c = -7, d = 4$

61. $2(3)(2x) + 2(3)(7x) + 2(2x)(7x) = 12x + 42x + 28x^2$
$= 28x^2 + 54x$

62. $C(x) = x^3 - 15x + 15$
$C(10) = (10)^3 - 15(10) + 15$
$= 1000 - 150 + 15$
$= 865$
The cost is \$865 to manufacture 10,000 units.

63. $C(x) = 3x^3 - 18x + 45$
$C(200) = 3(200)^3 - 18(200) + 45$
$= 24{,}000{,}000 - 3600 + 45$
$= 23{,}996{,}445$
The cost is \$23,996,445 to manufacture 20,000 units.

PAGES 430-431, LOOK BACK

64. $A - B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix}$
$= \begin{bmatrix} 1 + 1 & -1 - 4 \\ 0 - 5 & 2 - 3 \end{bmatrix}$
$= \begin{bmatrix} 2 & -5 \\ -5 & -1 \end{bmatrix}$

65. $B - A = \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$
$= \begin{bmatrix} -1 - 1 & 4 + 1 \\ 5 - 0 & 3 - 2 \end{bmatrix}$
$= \begin{bmatrix} -2 & 5 \\ 5 & 1 \end{bmatrix}$

66. $2A = 2\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$
$= \begin{bmatrix} 2(1) & 2(-1) \\ 2(0) & 2(2) \end{bmatrix}$
$= \begin{bmatrix} 2 & -2 \\ 0 & 4 \end{bmatrix}$

67. $2B = 2\begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix}$
$= \begin{bmatrix} 2(-1) & 2(4) \\ 2(5) & 2(3) \end{bmatrix}$
$= \begin{bmatrix} -2 & 8 \\ 10 & 6 \end{bmatrix}$

68. $AB = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix}$
$= \begin{bmatrix} 1(-1) + (-1)(5) & 1(4) + (-1)(3) \\ 0(-1) + 2(5) & 0(4) + 2(3) \end{bmatrix}$
$= \begin{bmatrix} -6 & 1 \\ 10 & 6 \end{bmatrix}$

69. $BA = \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$
$= \begin{bmatrix} -1(1) + 4(0) & -1(-1) + 4(2) \\ 5(1) + 3(0) & 5(-1) + 3(2) \end{bmatrix}$
$= \begin{bmatrix} -1 & 9 \\ 5 & 1 \end{bmatrix}$

Optimized using
trial version
www.balesio.com

70. $A - 4B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} - 4\begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} -4 & 16 \\ 20 & 12 \end{bmatrix}$

$= \begin{bmatrix} 1+4 & -1-16 \\ 0-20 & 2-12 \end{bmatrix}$

$= \begin{bmatrix} 5 & -17 \\ -20 & -10 \end{bmatrix}$

71. $B - 3A = \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix}$

$= \begin{bmatrix} -1-3 & 4+3 \\ 5-0 & 3-6 \end{bmatrix}$

$= \begin{bmatrix} -4 & 7 \\ 5 & -3 \end{bmatrix}$

72. $x^2 + 12x + 11 = 0$

$(x + 11)(x + 1) = 0$

$x + 11 = 0$ or $x + 1 = 0$

$x = -11$ or $x = -1$

73. $x^2 - 2x - 15 = 0$

$(x + 3)(x - 5) = 0$

$x + 3 = 0$ or $x - 5 = 0$

$x = -3$ or $x = 5$

74. $x^2 + 14x + 48 = 0$

$(x + 8)(x + 6) = 0$

$x + 8 = 0$ or $x + 6 = 0$

$x = -8$ or $x = -6$

75. $x^2 + 17x + 72 = 0$

$(x + 9)(x + 8) = 0$

$x + 9 = 0$ or $x + 8 = 0$

$x = -9$ or $x = -8$

76. Answers may vary; should be of the form $x^2 \pm 2ax + a^2 = (x \pm a)^2$, where a is any real number and $a \neq 0$. For example, $x^2 + 4x + 4 = (x + 2)^2$.

77. $3x^2 + 7x + 2 = 0$

$a = 3, b = 7, c = 2$

$x = \dfrac{-7 \pm \sqrt{7^2 - 4(3)(2)}}{2(3)}$

$x = \dfrac{-7 \pm \sqrt{25}}{6}$

$x = \dfrac{-7 \pm 5}{6}$

$x = \dfrac{-7 - 5}{6}$ or $x = \dfrac{-7 + 5}{6}$

$x = -2$ or $x = -\dfrac{1}{3}$

78. $-2x^2 + 4x + 5 = 0$

$a = -2, b = 4, c = 5$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(-2)(5)}}{2(-2)}$

$x = \dfrac{-4 \pm \sqrt{56}}{-4}$

$x = \dfrac{-4 \pm 2\sqrt{14}}{-4}$

$x = \dfrac{-2(2 \pm \sqrt{14})}{-2(2)}$

$x = \dfrac{2 \pm \sqrt{14}}{2}$

$x = \dfrac{2 - \sqrt{14}}{2}$ or $x = \dfrac{2 + \sqrt{14}}{2}$

79. $5x^2 + 2x + 4 = 0$

$a = 5, b = 2, c = 4$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(5)(4)}}{2(5)}$

$x = \dfrac{-2 \pm \sqrt{-76}}{10}$

$x = \dfrac{-2 \pm 2i\sqrt{19}}{10}$

$x = \dfrac{2(-1 \pm i\sqrt{19})}{10}$

$x = \dfrac{-1 \pm i\sqrt{19}}{5}$

$x = \dfrac{-1 - i\sqrt{19}}{5}$ or $x = \dfrac{-1 + i\sqrt{19}}{5}$

80. $-6x^2 + 5x - 4 = 0$

$a = -6, b = 5, c = -4$

$x = \dfrac{-5 \pm \sqrt{5^2 - 4(-6)(-4)}}{2(-6)}$

$x = \dfrac{-5 \pm \sqrt{-71}}{-12}$

$x = \dfrac{-5 \pm i\sqrt{71}}{-12}$

$x = \dfrac{-1(5 \pm i\sqrt{71})}{-1(12)}$

$x = \dfrac{5 \pm i\sqrt{71}}{12}$

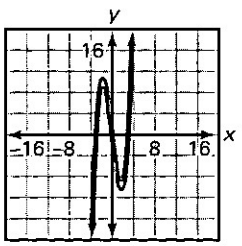
$x = \dfrac{5 - i\sqrt{71}}{12}$ or $x = \dfrac{5 + i\sqrt{71}}{12}$

Optimized using
trial version
www.balesio.com

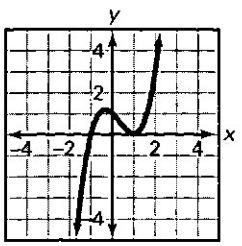
PAGE 431, LOOK BEYOND

81. Answers may vary. Possible answers:

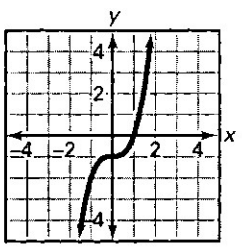
a. $x(x + 3)(x - 3) = x^3 - 9x$



b. $(x + 1)(x - 1)^2 = x^3 - x^2 - x + 1$



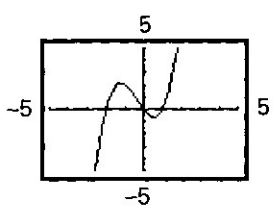
c. $x^3 - 1$



d. Impossible

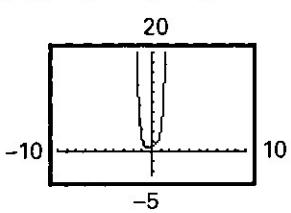
7.2 PAGE 437, GUIDED SKILLS PRACTICE

5. $P(x) = x^3 + x^2 - 2x$



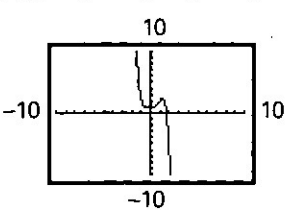
Local maximum of about 2.1
Local minimum of about -0.6
The function increases for all values of x except over the interval of approximately $-1.2 < x < 0.5$, where it decreases.

6. $P(x) = x^6 + x^4 + x + 1$



The graph rises on the left and the right.

7. $P(x) = x^4 + 1 + x^3 - x^5$

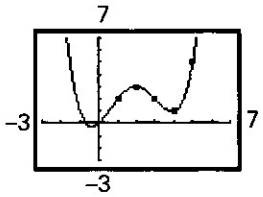


The graph rises on the left and falls on the right.

Optimized using
trial version
www.balesio.com

8. Make a scatter plot of the data points, and find the quartic regression model.

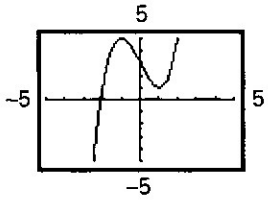
x	1	2	3	4	5
y	2	3	2	1	5



The data in the table can be modeled by the quartic function:
$f(x) \approx 0.125x^4 - 0.917x^3 + 1.375x^2 + 1.417x$

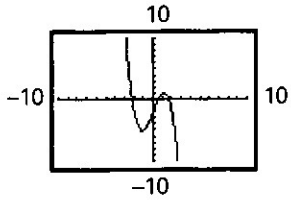
PAGES 438–439, PRACTICE AND APPLY

9. $P(x) = x^3 - 3x + 3$



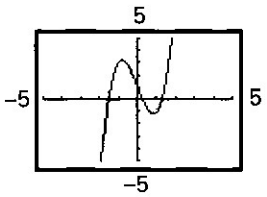
Maximum of ≈ 5.0
Minimum of ≈ 1.0

10. $P(x) = -x^3 + 4x - 2$



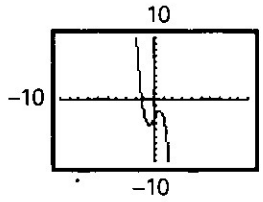
Maximum of ≈ 1.1
Minimum of ≈ -5.1

11. $P(x) = 2x^3 - 4x + 1$



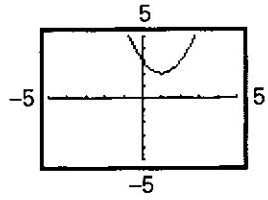
Maximum of ≈ 3.2
Minimum of ≈ -1.2

12. $P(x) = 3x - 3 - 3x^3$



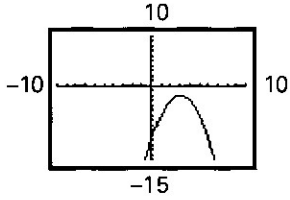
Maximum of ≈ -1.8
Minimum of ≈ -4.2

13. $P(x) = -2x + 3 + x^2$



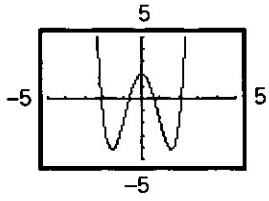
Minimum of ≈ 2
No maxima

14. $P(x) = -x^2 + 6x - 11$



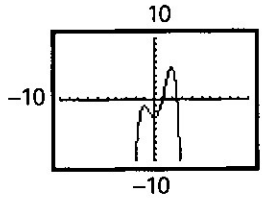
Maximum of ≈ -2.0
No minima

15. $P(x) = x^4 - 5x^2 + 2$



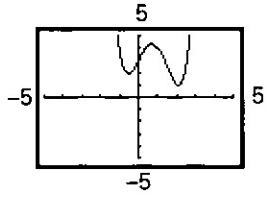
Maximum of ≈ 2.0
Two minima of ≈ -4.3

16. $P(x) = -x^4 + x^3 + 4x^2 - 3$



Maxima of ≈ -1.0 and ≈ 5.3
Minimum of ≈ -3.0

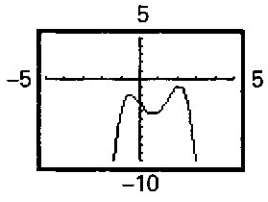
17. $P(x) = -3x^3 + 3x + x^4 + 3$



Maximum of ≈ 4.3
Minima of ≈ 1.9 and ≈ 1.0

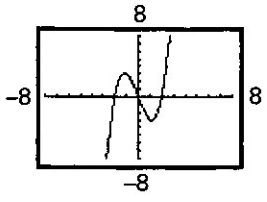
Optimized using
trial version
www.balesio.com

18. $P(x) = 3x^3 - x^4 - 3x - 3$



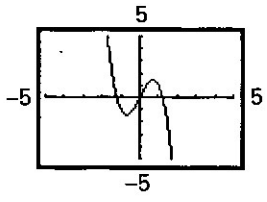
Maxima of ≈ 1.9 and ≈ -1.0
Minimum of ≈ -4.3

19. $P(x) = x^3 - 4x$



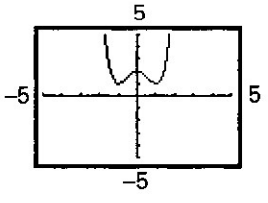
Local maximum of ≈ 3.1
Local minimum of ≈ -3.1
The function increases for all values of x except over the interval of approximately $-1.2 < x < 1.2$, where it decreases.

20. $P(x) = -2x^3 + 3x$



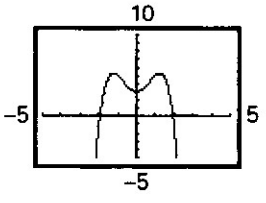
Local maximum of ≈ 1.4
Local minimum of ≈ -1.4
The function decreases for all values of x except over the interval of approximately $-0.7 < x < 0.7$, where it increases.

21. $P(x) = x^4 - 2x^2 + 2$



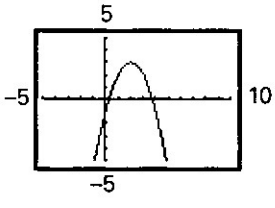
Local maximum of ≈ 2.0
Two local minima of ≈ 1.0
The function decreases for $x < -1.0$, $0.0 < x < 1.0$ and increases for $-1.0 < x < 0.0$, $1.0 < x$.

22. $P(x) = -x^4 + 3x^2 + 3$



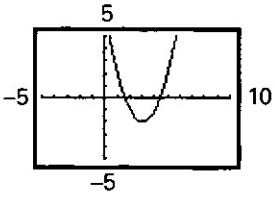
Two local maxima of ≈ 5.3
Local minimum of ≈ 3.0
The function increases for $x < -1.2$, $0.0 < x < 1.2$ and decreases for $-1.2 < x < 0.0$, $1.2 < x$.

23. $P(x) = -x^2 + 4x - 1$



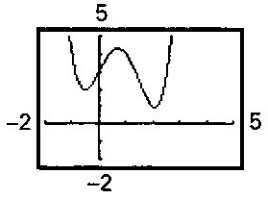
Local maximum of ≈ 3.0
No local minimum
The function increases for $x < 2.0$ and decreases for $2.0 < x$.

24. $P(x) = x^2 - 6x + 7$



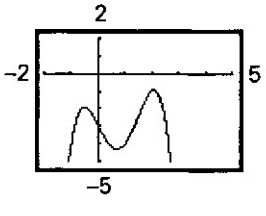
Local minimum of ≈ -2.0
No local maximum
The function decreases for $x < 3.0$ and increases for $3.0 < x$.

25. $P(x) = x^4 - 3x^3 + 3x + 3$



ıd ≈ 1.0
r $x < -0.5$,
es for $-0.5 < x < 0.7$,

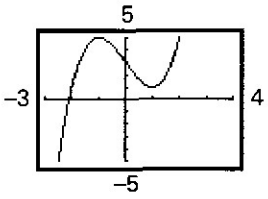
26. $P(x) = -x^4 + 3x^3 - 3x - 3$



Local maxima of ≈ -1.9 and -1.0
Local minimum of ≈ -4.3
The function decreases for $-0.5 < x < 0.7$, $2.1 < x$ and increases for $x < -0.5$, $0.7 < x < 2.1$.

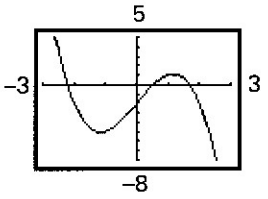
Optimized using
trial version
www.balesio.com

27. $P(x) = x^3 - 3x + 3$



Local maximum of ≈ 5.0
Local minimum of ≈ 1.0
The function increases for all values of x except over the interval of approximately $-1.0 < x < 1.0$, where it decreases.

28. $P(x) = -x^3 + 4x - 2$



Local maximum of ≈ 1.1
Local minimum of ≈ -5.1
The function decreases for all values of x except over the interval of approximately $-1.2 < x < 1.2$, where it increases.

29. $P(x) = 2x^3 + x^2 + 3x + 2$
The degree of the function is odd and the leading coefficient is positive. Therefore, the end behavior of the function is that the graph falls on the left and rises on the right.

30. $P(x) = -3x^3 + 5x^2 + x + 2$
The degree of the function is odd and the leading coefficient is negative. Therefore, the end behavior of the function is that the graph rises on the left and falls on the right.

31. $P(x) = 6x + 1 - x^2$ or $P(x) = -x^2 + 6x + 1$
The degree of the function is even and the leading coefficient is negative. Therefore, the end behavior of the function is that the graph falls on the left and the right.

32. $P(x) = x^2 - 8x + 3$
The degree of the function is even and the leading coefficient is positive. Therefore, the end behavior of the function is that the graph rises on the left and the right.

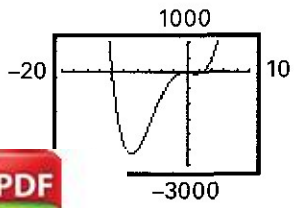
33. $P(x) = 4x^4 + x^5 + 1 + 3x^3$
or $P(x) = x^5 + 4x^4 + 3x^3 + 1$
The degree of the function is odd and the leading coefficient is positive. Therefore, the end behavior of the function is that the graph falls on the left and rises on the right.

34. $P(x) = x^6 + x^4 + 3x^2 + 2$
The degree of the function is even and the leading coefficient is positive. Therefore, the end behavior of the function is that the graph rises on the left and the right.

35. $P(x) = 7x^3 + 2 - 8x^5$ or $P(x) = -8x^5 + 7x^3 + 2$
The degree of the function is odd and the leading coefficient is negative. Therefore, the end behavior of the function is that the graph rises on the left and falls on the right.

36. $P(x) = 5x^4 - 6x^6 + 3x + 2$
or $P(x) = -6x^6 + 5x^4 + 3x + 2$
The degree of the function is even and the leading coefficient is negative. Therefore, the end behavior of the function is that the graph falls on the left and the right.

37. Since the function $y = x^4 + 10x^3 - 25x^2 - 10x + 24$ has an even degree and a positive leading coefficient, the end behavior is that the graph should rise to the left and the right. The graph shown falls to the left. A more appropriate window may be x: $[-20, 10]$ and y:$[-3000, 1000]$ as shown here:



Optimized using
trial version
www.balesio.com

38. $(0, 0)$: $a_3(0)^3 + a_2(0)^2 + a_1(0) + a_0 = 0$
$(1, 1)$: $a_3(1)^3 + a_2(1)^2 + a_1(1) + a_0 = 1$
$(2, 0)$: $a_3(2)^3 + a_2(2)^2 + a_1(2) + a_0 = 0$
$(3, 2)$: $a_3(3)^3 + a_2(3)^2 + a_1(3) + a_0 = 2$

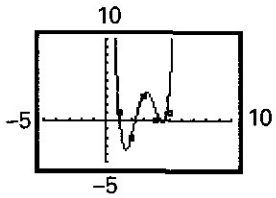
The system of equations is:

$$\begin{aligned} a_0 &= 0 \\ a_3 + a_2 + a_1 + a_0 &= 0 \\ 8a_3 + 4a_2 + 2a_1 + a_0 &= 0 \\ 27a_3 + 9a_2 + 3a_1 + a_0 &= 2 \end{aligned}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} \approx \begin{bmatrix} 0.83 \\ -3.50 \\ 3.67 \\ 0 \end{bmatrix}$$

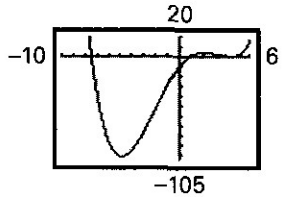
Thus $P(x) \approx 0.83x^3 - 3.5x^2 + 3.67x$.

39. Make a scatter plot of the data points and find the quartic regression model.



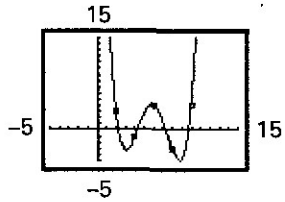
The data in the table can be modeled by the quartic function
$f(x) \approx 1.17x^4 - 14.33x^3 + 60.83x^2 - 102.67x + 56.$

40. Make a scatter plot of the data points and find the quartic regression model.



The data in the table can be modeled by the quartic function
$f(x) \approx 0.08x^4 - 0.17x^3 - 3.58x^2 + 14.67x - 11.$

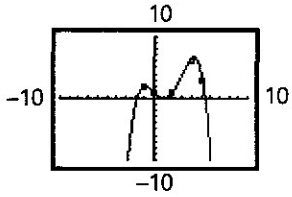
41. Make a scatter plot of the data points and find the quartic regression model.



The data in the table can be modeled by the

$x^3 + 23.51x^2 - 76.96x + 84.$

42. Make a scatter plot of the data points and find the quartic regression model.



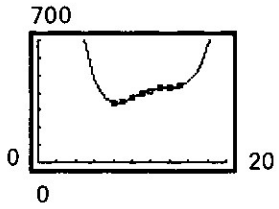
The data in the table can be modeled by the quartic function
$f(x) \approx -0.09x^4 + 0.52x^3 + 0.09x^2 - 1.52x + 1.$

Optimized using
trial version
www.balesio.com

43.

year	1988	1989	1990	1991	1992	1993	1994	1995	1996
x	8	9	10	11	12	13	14	15	16
y	343	346	371	398	411	430	429	438	444

Make a scatter plot of the data points and find the quartic regression model.

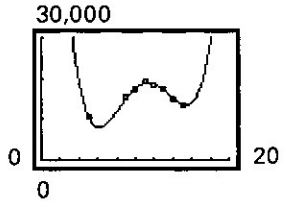


The data in the table can be modeled by the quartic function
$f(x) \approx 0.176x^4 - 8.677x^3 + 156.139x^2 - 1199.023x + 3662.963$

44.

year	1985	1989	1990	1991	1992	1993	1994	1995
x	5	9	10	11	12	13	14	15
y	10,721	15,325	17,263	19,113	18,596	17,293	14,970	13,668

Make a scatter plot of the data points and find the quartic regression model.



The data in the table can be modeled by the quartic function
$f(x) \approx 17.41x^4 - 761.72x^3 + 11{,}751.83x^2 - 74{,}233.04x + 172{,}429.44$

PAGE 439, LOOK BACK

45. There is no inverse, because the determinant of $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is 0.

46. The inverse of $\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$ is $\begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$.

47. The inverse of $\begin{bmatrix} 2 & 5 \\ 4 & 11 \end{bmatrix}$ is $\begin{bmatrix} 5.5 & -2.5 \\ -2 & 1 \end{bmatrix}$.

48. There is no inverse, because the determinant of $\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$ is 0.

49.
$$x^2 - 50 = 0$$
$$x^2 = 50$$
$$x = \pm\sqrt{50}$$
$$x = \pm 5\sqrt{2}$$

50.
$$(x + 3)^2 - 36 = 0$$
$$(x + 3)^2 = 36$$
$$x + 3 = \pm\sqrt{36}$$
$$x + 3 = \pm 6$$
$$x + 3 = -6 \quad \text{or} \quad x + 3 = 6$$
$$x = -9 \qquad\qquad x = 3$$

51. $3(x - 1)^2 = 12$
4
$\pm\sqrt{4}$
± 2
or $x - 1 = 2$
$x = 3$

Optimized using
trial version
www.balesio.com

52. $\log 1000 = x$
$10^x = 1000$
$10^x = 10^3$
$x = 3$

53. $\log_x 8 = 3$
$x^3 = 8$
$x^3 = 2^3$
$x = 2$

54. $\log_3 x = 2$
$x = 3^2$
$x = 9$

55. $\log_x 1 = 0$
$x^0 = 1$
$x =$ any positive number

PAGE 439, LOOK BEYOND

56. $(x + y)(x^2 - xy + y^2)$
$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$
$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$
$= x^3 + y^3$

57. $(x - y)(x^2 + xy + y^2)$
$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$
$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$
$= x^3 - y^3$

7.3 PAGE 445, GUIDED SKILLS PRACTICE

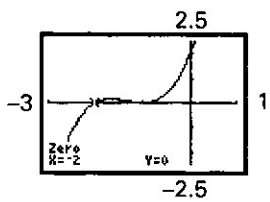
4. $P(x) = x(10 - x)(2 + x)$
$= x(20 + 10x - 2x - x^2)$
$= 20x + 10x^2 - 2x^2 - x^3$
$= -x^3 + 8x^2 + 20x$

5. $x^3 - 5x^2 + 6x = x(x^2 - 5x + 6)$
$= x(x - 2)(x - 3)$

6. $x^3 + 5x^2 + 3x + 15 = (x^3 + 5x^2) + (3x + 15$
$= x^2(x + 5) + 3(x + 5)$
$= (x^2 + 3)(x + 5)$

7. $x^3 - 216 = x^3 - 6^3$
$= (x - 6)(x^2 + 6x + 36)$

8. Let $f(x) = x^3 + 4x^2 + 5x + 2$.
$x + 2 = x - (-2)$
$f(-2) = (-2)^3 + 4(-2)^2 + 5(-2) + 2$
$= -8 + 16 - 10 + 2$
$= 0$
Yes, $x + 2$ is a factor of $x^3 + 4x^2 + 5x + 2$.



9.
$$x^2 + x + 1 \overline{)x^3 + 4x^2 + 4x + 3}$$
$x + 3$
$-(x^3 + x^2 + x)$
$3x^2 + 3x + 3$
$-(3x^2 + 3x + 3)$
0

10.
$$x + 3 \overline{)x^3 + 0x^2 - 14x - 15}$$
$x^2 - 3x - 5$
$-(x^3 + 3x^2)$
$-3x^2 - 14x$
$-(-3x^2 - 9x)$
$-5x - 15$
$-(-5x - 15)$
0

11.

−3	1	0	−14	−15
		−3	9	15
	1	−5		0

12

4	1	
	14	36
	18	37

Optimized using trial version www.balesio.com

13. $P(x) = 2x^3 + 3x^2 + 4x + 1$
$P(2) = 2(2)^3 + 3(2)^2 + 4(2) + 1$
$= 16 + 12 + 8 + 1$
$= 37$

PAGES 445–447, PRACTICE AND APPLY

14. $3x^2(4x^3 - 2x^2 + 5x + 2)$
$= 12x^5 - 6x^4 + 15x^3 + 6x^2$

15. $2x^3(4x^3 - 2x^2 + x + 3)$
$= 8x^6 - 4x^5 + 2x^4 + 6x^3$

16. $(2x - 3)(x + 4) = 2x^2 + 8x - 3x - 12$
$= 2x^2 + 5x - 12$

17. $(x + 7)(5x - 3) = 5x^2 - 3x + 35x - 21$
$= 5x^2 + 32x - 21$

18. $(x + 2)(x^2 + 4x + 1) = (x + 2)(x^2) + (x + 2)(4x) + (x + 2)(1)$
$= x^3 + 2x^2 + 4x^2 + 8x + x + 2$
$= x^3 + 6x^2 + 9x + 2$

19. $(x + 3)(2x^3 + 3x^2 + 1) = (x + 3)(2x^3) + (x + 3)(3x^2) + (x + 3)(1)$
$= 2x^4 + 6x^3 + 3x^3 + 9x^2 + x + 3$
$= 2x^4 + 9x^3 + 9x^2 + x + 3$

20. $(2x + 3)(x^3 - 5x^2 + 4) = (2x + 3)(x^3) + (2x + 3)(-5x^2) + (2x + 3)(4)$
$= 2x^4 + 3x^3 - 10x^3 - 15x^2 + 8x + 12$
$= 2x^4 - 7x^3 - 15x^2 + 8x + 12$

21. $(2x + 1)(x^2 - 4x - 3) = (2x + 1)(x^2) + (2x + 1)(-4x) + (2x + 1)(-3)$
$= 2x^3 + x^2 - 8x^2 - 4x - 6x - 3$
$= 2x^3 - 7x^2 - 10x - 3$

22. $(x - 4)(2x^3 - 3x^2 + 2) = (x - 4)(2x^3) + (x - 4)(-3x^2) + (x - 4)(2)$
$= 2x^4 - 8x^3 - 3x^3 + 12x^2 + 2x - 8$
$= 2x^4 - 11x^3 + 12x^2 + 2x - 8$

23. $(x - 5)(-3x^3 - 4x - 1) = (x - 5)(-3x^3) + (x - 5)(-4x) + (x - 5)(-1)$
$= -3x^4 + 15x^3 - 4x^2 + 20x - x + 5$
$= -3x^4 + 15x^3 - 4x^2 + 19x + 5$

24. $(x - 3)(2 - x)(x - 1) = (x - 3)(2x - 2 - x^2 + x)$
$= (x - 3)(2x) + (x - 3)(-2) + (x - 3)(-x^2) + (x - 3)(x)$
$= 2x^2 - 6x - 2x + 6 - x^3 + 3x^2 + x^2 - 3x$
$= -x^3 + 6x^2 - 11x + 6$

25. $(x - 2)(2x + 3)(3 - x) = (x - 2)(6x - 2x^2 + 9 - 3x)$
$= (x - 2)(6x) + (x - 2)(-2x^2) + (x - 2)(9) + (x - 2)(-3x)$
$= 6x^2 - 12x - 2x^3 + 4x^2 + 9x - 18 - 3x^2 + 6x$
$= -2x^3 + 7x^2 + 3x - 18$

26. $(x + 1)^2(x - 2) = (x^2 + 2x + 1)(x - 2)$
$= x^2(x - 2) + 2x(x - 2) + 1(x - 2)$
$= x^3 - 2x^2 + 2x^2 - 4x + x - 2$
$= x^3 - 3x - 2$

27. $(2x - 4)(x + 1)^2 = (2x - 4)(x^2 + 2x + 1)$
$= (2x - 4)(x^2) + (2x - 4)(2x) + (2x - 4)(1)$
$= 2x^3 - 4x^2 + 4x^2 - 8x + 2x - 4$
$= 2x^3 - 6x - 4$

28. $(2x + 1)^3 = (2x + 1)(2x + 1)^2$
$= (2x + 1)(4x^2 + 4x + 1)$
$= (2x + 1)(4x^2) + (2x + 1)(4x) + (2x + 1)(1)$
$= 8x^3 + 4x^2 + 8x^2 + 4x + 2x + 1$
$= 8x^3 + 12x^2 + 6x + 1$

$= (3x + 2)(3x + 2)^2$
$= (3x + 2)(9x^2 + 12x + 4)$
$= (3x + 2)(9x^2) + (3x + 2)(12x) + (3x + 2)(4)$
$= 27x^3 + 18x^2 + 36x^2 + 24x + 12x + 8$
$= 27x^3 + 54x^2 + 36x + 8$

Optimized using
trial version
www.balesio.com

30. $(x-1)^2(x^2-3x+2) = (x-1)^2(x^2) + (x-1)^2(-3x) + (x-1)^2(2)$
$= (x^2-2x+1)(x^2) + (x^2-2x+1)(-3x) + (x^2-2x+1)(2)$
$= x^4 - 2x^3 + x^2 - 3x^3 + 6x^2 - 3x + 2x^2 - 4x + 2$
$= x^4 - 5x^3 + 9x^2 - 7x + 2$

31. $(-3x^2 - x + 2)(x+1)^2 = (-3x^2)(x+1)^2 + (-x)(x+1)^2 + (2)(x+1)^2$
$= (-3x^2)(x^2+2x+1) + (-x)(x^2+2x+1) + (2)(x^2+2x+1)$
$= -3x^4 - 6x^3 - 3x^2 - x^3 - 2x^2 - x + 2x^2 + 4x + 2$
$= -3x^4 - 7x^3 - 3x^2 + 3x + 2$

32. $\left(x-\frac{5}{7}\right)\left(\frac{2}{5}x^2 - \frac{1}{5}x + \frac{3}{5}\right) = \left(x-\frac{5}{7}\right)\left(\frac{2}{5}x^2\right) + \left(x-\frac{5}{7}\right)\left(-\frac{1}{5}x\right) + \left(x-\frac{5}{7}\right)\left(\frac{3}{5}\right)$
$= \frac{2}{5}x^3 - \frac{10}{35}x^2 - \frac{1}{5}x^2 + \frac{5}{35}x + \frac{3}{5}x - \frac{15}{35}$
$= \frac{2}{5}x^3 - \left(\frac{10}{35} + \frac{7}{35}\right)x^2 + \left(\frac{5}{35} + \frac{21}{35}\right)x - \frac{3}{7}$
$= \frac{2}{5}x^3 - \frac{17}{35}x^2 + \frac{26}{35}x - \frac{3}{7}$

33. $\left(x-\frac{1}{4}\right)\left(\frac{2}{3}x^2 + \frac{1}{3}x + \frac{2}{3}\right) = \left(x-\frac{1}{4}\right)\left(\frac{2}{3}x^2\right) + \left(x-\frac{1}{4}\right)\left(\frac{1}{3}x\right) + \left(x-\frac{1}{4}\right)\left(\frac{2}{3}\right)$
$= \frac{2}{3}x^3 - \frac{1}{6}x^2 + \frac{1}{3}x^2 - \frac{1}{12}x + \frac{2}{3}x - \frac{2}{12}$
$= \frac{2}{3}x^3 + \left(-\frac{1}{6} + \frac{2}{6}\right)x^2 + \left(-\frac{1}{12} + \frac{8}{12}\right)x - \frac{1}{6}$
$= \frac{2}{3}x^3 + \frac{1}{6}x^2 + \frac{7}{12}x - \frac{1}{6}$

34. $x^3 + 8x^2 + 15x = x(x^2 + 8x + 15)$
$= x(x+3)(x+5)$

35. $x^3 + 6x^2 + 8x = x(x^2 + 6x + 8)$
$= x(x+2)(x+4)$

36. $x^3 - 10x^2 + 24x = x(x^2 - 10x + 24)$
$= x(x-6)(x-4)$

37. $x^3 + 2x^2 - 3x = x(x^2 + 2x - 3)$
$= x(x+3)(x-1)$

38. $x^3 - x^2 - 30x = x(x^2 - x - 30)$
$= x(x-6)(x+5)$

39. $x^3 - 2x^2 - 3x = x(x^2 - 2x - 3)$
$= x(x-3)(x+1)$

40. $3x^3 - 300x = 3x(x^2 - 100)$
$= 3x(x+10)(x-10)$

41. $18x^3 - 60x^2 + 50x = 2x(9x^2 - 30x + 25)$
$= 2x(3x-5)^2$

42. $x^3 + 3x^2 - 2x - 6 = (x^3 + 3x^2) - (2x + 6)$
$= x^2(x+3) - 2(x+3)$
$= (x^2 - 2)(x+3)$

43. $x^3 - 3x^2 + 4x - 12 = (x^3 - 3x^2) + (4x - 12)$
$= x^2(x-3) + 4(x-3)$
$= (x^2 + 4)(x-3)$

44. $x^3 + 3x^2 - x - 3 = (x^3 + 3x^2) - (x + 3)$
$= x^2(x+3) - 1(x+3)$
$= (x^2 - 1)(x+3)$
$= (x-1)(x+1)(x+3)$

45. $x^3 - 2x^2 - 5x + 10 = (x^3 - 2x^2) - (5x - 10)$
$= x^2(x-2) - 5(x-2)$
$= (x^2 - 5)(x-2)$

46. $x^3 + x^2 + x + 1 = (x^3 + x^2) + (x + 1)$
$= x^2(x+1) + 1(x+1)$
$= (x^2 + 1)(x+1)$

47. $1 - x + x^2 - x^3 = (1 - x) + (x^2 - x^3)$
$= 1(1-x) + x^2(1-x)$
$= (1 + x^2)(1-x)$

48. $x^3 + 2x^2 + 14x + 7x^2 = (x^3 + 2x^2) + (7x^2 + 14x)$
$= x^2(x+2) + 7x(x+2)$
$= (x^2 + 7x)(x+2)$
$= x(x+7)(x+2)$

49. $x^3 + x^2 + 2 + 2x = (x^3 + x^2) + (2x + 2)$
$= x^2(x+1) + 2(x+1)$
$= (x^2 + 2)(x+1)$

PDF
Optimized using trial version www.balesio.com

$^2 + 4x + 16)$

5^3

$)(x^4 - 5x^2 + 25)$

$+ 2x + 4)$

^{3}

$4x^2 + 2x + 1)$

51. $x^3 + 1000 = x^3 + 10^3$
$= (x+10)(x^2 - 10x + 100)$

53. $x^6 + 27 = (x^2)^3 + 3^3$
$= (x^2 + 3)(x^4 - 3x^2 + 9)$

55. $x^3 - 216 = x^3 - 6^3$
$= (x-6)(x^2 + 6x + 36)$

57. $27x^3 - 125 = (3x)^3 - 5^3$
$= (3x-5)(9x^2 + 15x + 25)$

58. $x^6 - 1 = (x^3)^2 - 1^2$
$= (x^3 + 1)(x^3 - 1)$
$= (x + 1)(x^2 - x + 1)(x - 1)(x^2 + x + 1)$

59. $64 - x^3 = 4^3 - x^3$
$= (4 - x)(16 + 4x + x^2)$

60. $27 + 8x^3 = 3^3 + (2x)^3$
$= (3 + 2x)(9 - 6x + 4x^2)$

61. $f(x) = x^2 + x + 1$
$f(1) = 1^2 + 1 + 1$
$= 3$
No, $x - 1$ is not a factor of $x^2 + x + 1$.

62. $f(x) = x^2 + 2x + 1$
$x + 2 = x - (-2)$
$f(-2) = (-2)^2 + 2(-2) + 1$
$= 4 - 4 + 1$
$= 1$
No, $x + 2$ is not a factor of $x^2 + 2x + 1$.

63. $f(x) = x^3 + 3x^2 - 33x - 35$
$x + 1 = x - (-1)$
$f(-1) = (-1)^3 + 3(-1)^2 - 33(-1) - 35$
$= -1 + 3 + 33 - 35$
$= 0$
Yes, $x + 1$ is a factor of $x^3 + 3x^2 - 33x - 35$.

64. $f(x) = x^3 + 5x^2 - 18x - 48$
$x + 6 = x - (-6)$
$f(-6) = (-6)^3 + 5(-6)^2 - 18(-6) - 48$
$= -216 + 180 + 108 - 48$
$= 24$
No, $x + 6$ is not a factor of $x^3 + 5x^2 - 18x - 48$.

65. $f(x) = x^3 + 3x^2 - 18x - 40$
$f(4) = (4)^3 + 3(4)^2 - 18(4) - 40$
$= 64 + 48 - 72 - 40$
$= 0$
Yes, $x - 4$ is a factor of $x^3 + 3x^2 - 18x - 40$.

66. $f(x) = x^3 - 8x^2 + 9x + 18$
$f(6) = (6)^3 - 8(6)^2 + 9(6) + 18$
$= 216 - 288 + 54 + 18$
$= 0$
Yes, $x - 6$ is a factor of $x^3 - 8x^2 + 9x + 18$.

67. $f(x) = x^3 + 6x^2 - x - 30$
$f(2) = (2)^3 + 6(2)^2 - 2 - 30$
$= 8 + 24 - 2 - 30$
$= 0$
Yes, $x - 2$ is a factor of $x^3 + 6x^2 - x - 30$.

68. $f(x) = x^3 - x^2 - 17x - 15$
$x + 3 = x - (-3)$
$f(-3) = (-3)^3 - (-3)^2 - 17(-3) - 15$
$= -27 - 9 + 51 - 15$
$= 0$
Yes, $x + 3$ is a factor of $x^3 - x^2 - 17x - 15$.

69. $f(x) = 2x^3 + 9x^2 + 6x + 8$
$x + 4 = x - (-4)$
$f(-4) = 2(-4)^3 + 9(-4)^2 + 6(-4) + 8$
$= -128 + 144 - 24 + 8$
$= 0$
Yes, $x + 4$ is a factor of $2x^3 + 9x^2 + 6x + 8$.

70. $f(x) = 2x^3 - x^2 - 12x - 9$
$f(3) = 2(3)^3 - (3)^2 - 12(3) - 9$
$= 54 - 9 - 36 - 9$
$= 0$
Yes, $x - 3$ is a factor of $2x^3 - x^2 - 12x - 9$.

71.
$$\begin{array}{r} x + 2 \\ x + 2\overline{)x^2 + 4x + 4} \\ -(x^2 + 2x) \\ \hline 2x + 4 \\ -(2x + 4) \\ \hline 0 \end{array}$$

72.
$$\begin{array}{r} x - 2 \\ x - 1\overline{)x^2 - 3x + 2} \\ -(x^2 - x) \\ \hline -2x + 2 \\ -(-2x + 2) \\ \hline 0 \end{array}$$

73.
$$\begin{array}{r} x^2 - x - 6 \\ x + 1\overline{)x^3 + 0x^2 - 7x - 6} \\ -(x^3 + x^2) \\ \hline -x^2 - 7x \\ -(-x^2 - x) \\ \hline -6x - 6 \\ -(-6x - 6) \\ \hline 0 \end{array}$$

74.
$$\begin{array}{r} x^2 + 6x + 9 \\ x + 5\overline{)x^3 + 11x^2 + 39x + 45} \\ -(x^3 + 5x^2) \\ \hline 6x^2 + 39x \\ -(6x^2 + 30x) \\ \hline 9x + 45 \\ -(9x + 45) \\ \hline 0 \end{array}$$

Optimized using
trial version
www.balesio.com

75.
$$\begin{array}{r} x - 1 \\ x^3 + 3x^2 - x - 3\overline{)} \\ x^3 + 4x^2 + 3x) \\ \hline -x^2 - 4x - 3 \\ -(-x^2 - 4x - 3) \\ \hline 0 \end{array}$$

76.
$$\begin{array}{r} x - 2 \\ x^2 + 8x + 15\overline{)x^3 + 6x^2 - x - 30} \\ -(x^3 + 8x^2 + 15x) \\ \hline -2x^2 - 16x - 30 \\ -(-2x^2 - 16x - 30) \\ \hline 0 \end{array}$$

77. $x^2 + 6x - 7 \overline{)x^3 + 0x^2 - 43x + 42}$

$x - 6$

$-(x^3 + 6x^2 - 7x)$

$-6x^2 - 36x + 42$

$-(-6x^2 - 36x + 42)$

0

78. $x^2 - x + 6 \overline{)x^3 - 5x^2 + 10x - 24}$

$x - 4$

$-(x^3 - x^2 + 6x)$

$-4x^2 + 4x - 24$

$-(-4x^2 + 4x - 24)$

0

79. $x - \frac{1}{2} \overline{)x^2 - \frac{1}{6}x - \frac{1}{6}}$

$x + \frac{1}{3}$

$-\left(x^2 - \frac{1}{2}x\right)$

$\frac{1}{3}x - \frac{1}{6}$

$-\left(\frac{1}{3}x - \frac{1}{6}\right)$

0

80. $x + \frac{3}{4} \overline{)x^2 + \frac{1}{2}x - \frac{3}{16}}$

$x - \frac{1}{4}$

$-\left(x^2 + \frac{3}{4}x\right)$

$-\frac{1}{4}x - \frac{3}{16}$

$-\left(-\frac{1}{4}x - \frac{3}{16}\right)$

0

81.

4	1	−4	−12
		4	0
	1	0	−12

Quotient: $x^2 - \frac{12}{x-4}$

82.

1	1	−3	2
		1	−2
	1	−2	0

Quotient: $x - 2$

83.

−1	1	1	−9	−9
		−1	0	9
	1	0	−9	0

Quotient: $x^2 - 9$

84.

4	1	−2	−22	40
		4	8	−56
	1	2	−14	−16

Quotient: $x^2 + 2x - 14 - \frac{16}{x-4}$

85.

−3	1	5	0	−18
		−3	−6	18
	1	2	−6	0

Quotient: $x^2 + 2x - 6$

86.

3	1	0	0	−27
		3	9	27
	1	3	9	0

Quotient: $x^2 + 3x + 9$

87.

1	1	0	0	3
		1	1	1
	1	1	1	4

Quotient: $x^2 + x + 1 + \frac{4}{x-1}$

88.

−4	1	0	−6
		−4	16
	1	−4	10

Quotient: $x - 4 + \frac{10}{x+4}$

89.

2	1	2	0	−3	−6
		2	8	16	26
	1	4	8	13	20

Quotient: $x^3 + 4x^2 + 8x + 13 + \frac{20}{x-2}$

90.

3	1	−5	6	0	5	−15
		3	−6	0	0	15
	1	−2	0	0	5	0

Quotient: $x^4 - 2x^3 + 5$

Optimized using
trial version
www.balesio.com

91. $P(x) = x^2 + 1$; $P(1)$

Method 1

1	1	0	1
		1	1
	1	1	2

$P(1) = 2$

Method 2

$P(1) = (1)^2 + 1$
$= 2$

92. $P(x) = x^2 + 1$; $P(2)$

Method 1

2	1	0	1
		2	4
	1	2	5

$P(2) = 5$

Method 2

$P(2) = (2)^2 + 1$
$= 4 + 1$
$= 5$

93. $P(x) = x^2 + x$; $P(2)$

Method 1

2	1	1	0
		2	6
	1	3	6

$P(2) = 6$

Method 2

$P(2) = (2)^2 + 2$
$= 4 + 2$
$= 6$

94. $P(x) = x^2 + x$; $P(1)$

Method 1

1	1	1	0
		1	2
	1	2	2

$P(1) = 2$

Method 2

$P(1) = (1)^2 + 1$
$= 1 + 1$
$= 2$

95. $P(x) = 4x^2 - 2x + 3$; $P(3)$

Method 1

3	4	−2	3
		12	30
	4	10	33

$P(3) = 33$

Method 2

$P(3) = 4(3)^2 - 2(3) + 3$
$= 36 - 6 + 3$
$= 33$

96. $P(x) = 3x^3 + 2x^2 + 3x + 1$; $P(-2)$

Method 1

−2	3	2	3	1
		−6	8	−22
	3	−4	11	−21

$P(-2) = -21$

Method 2

$P(-2) = 3(-2)^3 + 2(-2)^2 + 3(-2) + 1$
$= -24 + 8 - 6 + 1$
$= -21$

97. $P(x) = 2x^4 + x^3 - 3x^2 + 2x$; $P(-4)$

Method 1

−4	2	1	−3	2	0
		−8	28	−100	392
	2	−7	25	−98	392

$P(-4) = 392$

Method 2

$P(-4) = 2(-4)^4 + (-4)^3 - 3(-4)^2 + 2(-4)$
$= 512 - 64 - 48 - 8$
$= 392$

98. $P(x) = 2x^3 - 3x^2 + 2x - 2$; $P(3)$

Method 1

3	2	−3	2	−2
		6	9	33
	2	3	11	31

$3)^3 - 3(3)^2 + 2(3) - 2$
$- 27 + 6 - 2$

Optimized using
trial version
www.balesio.com

99. Let $f(x) = x^3 + 3x^2 - x + k$.
Solve $f(2) = 0$ for k.

$$\begin{aligned} f(2) &= (2)^3 + 3(2)^2 - 2 + k \\ &= 8 + 12 - 2 + k \\ &= 18 + k \\ 18 + k &= 0 \\ k &= -18 \end{aligned}$$

100. Let $f(x) = kx^3 - 2x^2 + x - 6$.
Solve $f(-3) = 0$ for k.

$$\begin{aligned} f(-3) &= k(-3)^3 - 2(-3)^2 + (-3) - 6 \\ &= -27k - 18 - 3 - 6 \\ &= -27k - 27 \\ -27k - 27 &= 0 \\ -27k &= 27 \\ k &= -1 \end{aligned}$$

101. a.
$$\begin{aligned} V(x) &= x(14 - 2x)(32 - 2x) \\ &= (14x - 2x^2)(32 - 2x) \\ &= 448x - 28x^2 - 64x^2 + 4x^3 \\ &= 4x^3 - 92x^2 + 448x \end{aligned}$$

b.
$$\begin{aligned} V(2) &= 4(2)^3 - 92(2)^2 + 448(2) \\ &= 32 - 368 + 896 \\ &= 560 \text{ cubic inches} \end{aligned}$$

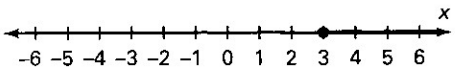
102. a.
$$\begin{aligned} V(x) &= \tfrac{1}{2}x(19 - 2x)(40 - 5x) \\ &= \tfrac{1}{2}(19x - 2x^2)(40 - 5x) \\ &= \tfrac{1}{2}[19x(40 - 5x) - 2x^2(40 - 5x)] \\ &= \tfrac{1}{2}(760x - 95x^2 - 80x^2 + 10x^3) \\ &= 5x^3 - \tfrac{175}{2}x^2 + 380x \end{aligned}$$

b.
$$\begin{aligned} V(2) &= 5(2)^3 - \tfrac{175}{2}(2)^2 + 380(2) \\ &= 40 - 350 + 760 \\ &= 450 \text{ cubic inches} \end{aligned}$$

c.
$$\begin{aligned} V(2.5) &= 5(2.5)^3 - \tfrac{175}{2}(2.5)^2 + 380(2.5) \\ &= 78.125 - 546.875 + 950 \\ &= 481.25 \text{ cubic inches} \end{aligned}$$

PAGE 447, LOOK BACK

103.
$$\begin{aligned} x + 3 &\leq 3(x - 1) \\ x + 3 &\leq 3x - 3 \\ 6 &\leq 2x \\ 3 &\leq x \end{aligned}$$



104. Yes, the set represents a function because each element of the domain is paired with exactly one element of the range.

105. $5a^2 - 5b^2 = 5(a^2 - b^2) = 5(a + b)(a - b)$

106. $2x^2 - 32y^2 = 2(x^2 - 16y^2) = 2(x + 4y)(x - 4y)$

107. $n^2 + n - 12 = (n + 4)(n - 3)$

108. $5 - 6s + s^2 = (5 - s)(1 - s)$

109.
$$\begin{aligned} 4x^2 + 4x + 1 &= (2x + 1)(2x + 1) \\ &= (2x + 1)^2 \end{aligned}$$

110. $2x^2 + 11x + 15 = (x + 3)(2x + 5)$

111.
$$\begin{aligned} 10^x &= 32 \\ \log(10^x) &= \log(32) \\ x &\approx 1.51 \end{aligned}$$

112.
$$\begin{aligned} 3^x &= 7 \\ x \log 3 &= \log 7 \\ x &= \frac{\log 7}{\log 3} \\ x &\approx 1.77 \end{aligned}$$

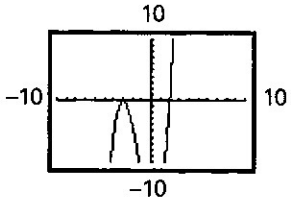
113.
$$\begin{aligned} e^x &= 5 \\ \ln(e^x) &= \ln 5 \\ x &\approx 1.61 \end{aligned}$$

114.
$$\begin{aligned} e^{2x} &= 7 \\ \ln(e^{2x}) &= \ln 7 \\ 2x &= \ln 7 \\ x &\approx 0.97 \end{aligned}$$

Optimized using
trial version
www.balesio.com

PAGE 447, LOOK BEYOND

115. There are at most 2 other values of x that are zeros of $f(x)$. The graph of $f(x)$ crosses the x-axis once (at $x = 2$) and *touches* the x-axis at $x = -3$.

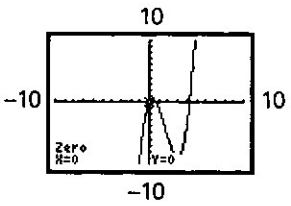


7.4 PAGES 452–453, GUIDED SKILLS PRACTICE

4. $x^3 - x^2 - 12x = 0$
$x(x^2 - x - 12) = 0$
$x(x - 4)(x + 3) = 0$
$x = 0$, $x = 4$, or $x = -3$

5. $y^3 + 15y^2 + 54y = 0$
$y(y^2 + 15y + 54) = 0$
$y(y + 6)(y + 9) = 0$
$y = 0$, $y = -6$, or $y = -9$

6. $x^3 - 5x^2 + 3x = 0$



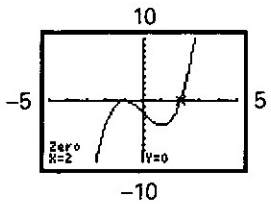
0	1	−5	3	0
	0	0	0	
	1	−5	3	0

The quotient is $x^2 - 5x + 3$.
$x^3 - 5x^2 + 3x = 0$
$x(x^2 - 5x + 3) = 0$
$a = 1$, $b = -5$, $c = 3$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$$
$$= \frac{5 \pm \sqrt{13}}{2}$$

$x = 0$, $x = \frac{5 + \sqrt{13}}{2}$, and $x = \frac{5 - \sqrt{13}}{2}$

7. $x^3 - 3x - 2 = 0$



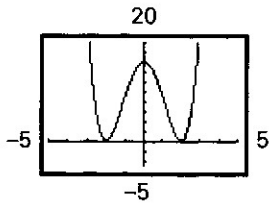
2	1	0	−3	−2
		2	4	2
	1	2	1	0

The quotient is $x^2 + 2x + 1$.
$x^3 - 3x - 2 = 0$
$(x - 2)(x^2 + 2x + 1) = 0$
$(x - 2)(x + 1)^2 = 0$

The roots are $x = 2$ and $x = -1$, with the root -1 occurring twice.

8. Substitute u for x^2 and solve for u.
$x^4 - 8x^2 + 16 = 0$
$(x^2)^2 - 8x^2 + 16 = 0$
$u^2 - 8u + 16 = 0$
$(u - 4)(u - 4) = 0$
$(u - 4)^2 = 0$

Replace u with x^2 and solve for x.
$(x^2 - 4)^2 = 0$
$x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$



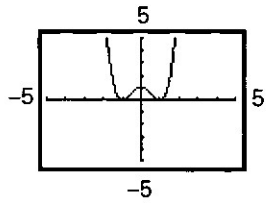
Optimized using
trial version
www.balesio.com

9. Substitute u for x^2 and solve for u.

$$x^4 - 2x^2 + 1 = 0$$
$$(x^2)^2 - 2x^2 + 1 = 0$$
$$u^2 - 2u + 1 = 0$$
$$(u - 1)(u - 1) = 0$$
$$(u - 1)^2 = 0$$

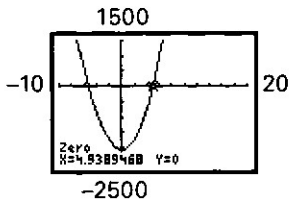
Replace u with x^2 and solve for x.

$$(x^2 - 1)^2 = 0$$
$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$x = \pm 1$$



10. $\frac{1}{3}\pi r^3 + 25\,\pi r^2 = 2042$

Solve for r by graphing $y = \frac{1}{3}\pi r^3 + 25\pi r^2 - 2042$.



A radius of about 4.9 feet will give a volume of approximately 2042 cubic feet.

PAGES 453-454, PRACTICE AND APPLY

11. $x^3 + 2x^2 - 35x = 0$

$x(x^2 + 2x - 35) = 0$

$x(x - 5)(x + 7) = 0$

$x = 0$, $x = 5$, and $x = -7$

12. $x^3 + 2x^2 - 48x = 0$

$x(x^2 + 2x - 48) = 0$

$x(x - 6)(x + 8) = 0$

$x = 0$, $x = 6$, and $x = -8$

13. $y^3 - 6y^2 - 27y = 0$

$y(y^2 - 6y - 27) = 0$

$y(y + 3)(y - 9) = 0$

$y = 0$, $y = -3$, and $y = 9$

14. $a^3 - 8a^2 - 48a = 0$

$a(a^2 - 8a - 48) = 0$

$a(a + 4)(a - 12) = 0$

$a = 0$, $a = -4$, and $a = 12$

15. $x^3 - 13x^2 + 40x = 0$

$x(x^2 - 13x + 40) = 0$

$x(x - 5)(x - 8) = 0$

$x = 0$, $x = 5$, and $x = 8$

16. $x^3 - 7x^2 + 10x = 0$

$x(x^2 - 7x + 10) = 0$

$x(x - 5)(x - 2) = 0$

$x = 0$, $x = 5$, and $x = 2$

17. $x^3 = 25x$

$x^3 - 25x = 0$

$x(x^2 - 25) = 0$

$x(x - 5)(x + 5) = 0$

$x = 0$, $x = 5$, and $x = -5$

18. $y^3 = 49y$

$y^3 - 49y = 0$

$y(y^2 - 49) = 0$

$y(y + 7)(y - 7) = 0$

$y = 0$, $y = -7$, and $y = 7$

19. $2x^3 - 10x^2 - 100x = 0$

$2x(x^2 - 5x - 50) = 0$

$2x(x + 5)(x - 10) = 0$

$x = 0$, $x = -5$, and $x = 10$

20. $16x - 6x^2 - x^3 = 0$

$x(16 - 6x - x^2) = 0$

$x(8 + x)(2 - x) = 0$

$x = 0$, $x = -8$, and $x = 2$

21. $3y^3 + 9y^2 - 162y = 0$

$3y(y^2 + 3y - 54) = 0$

$3y(y + 9)(y - 6) = 0$

$y = 0$, $y = -9$, and $y = 6$

22. $20a^2 + 5a^3 - 60a = 0$

$5a(4a + a^2 - 12) = 0$

$5a(a + 6)(a - 2) = 0$

$a = 0$, $a = -6$, and $a = 2$

23. $110x - 2x^3 = 12x^2$

$2x^3 + 12x^2 - 110x = 0$

$2x(x^2 + 6x - 55) = 0$

$2x(x + 11)(x - 5) = 0$

$x = 0$, $x = -11$, and $x = 5$

24. $3y^3 + 36y^2 = 3y^4$

$3y^4 - 3y^3 - 36y^2 = 0$

$3y^2(y^2 - y - 12) = 0$

$3y^2(y - 4)(y + 3) = 0$

$y = 0$, $y = 4$, and $y = -3$

25. $28a - 5a^2 - 3a^3 = 0$

-4

26. $15y - 4y^2 - 3y^3 = 0$

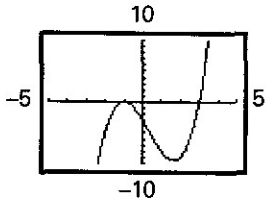
$y(-3y^2 - 4y + 15) = 0$

$y(-3y + 5)(y + 3) = 0$

$y = 0$, $y = \frac{5}{3}$, and $y = -3$

Optimized using
trial version
www.balesio.com

27. $a^3 - a^2 - 5a - 3 = 0$



3	1	−1	−5	−3
		3	6	3
	1	2	1	0

The quotient is $a^2 + 2a + 1$.

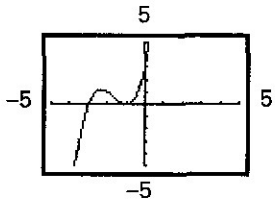
$a^3 - a^2 - 5a - 3 = 0$

$(a - 3)(a^2 + 2a + 1) = 0$

$(a - 3)(a + 1)^2 = 0$

The roots are $a = 3$ and $a = -1$, with the root $a = -1$ occurring twice.

28. $x^3 + 5x^2 + 7x + 3 = 0$



−1	1	5	7	3
		−1	−4	−3
	1	4	3	0

The quotient is $x^2 + 4x + 3$.

$x^3 + 5x^2 + 7x + 3 = 0$

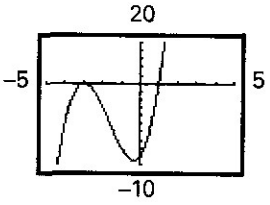
$(x + 1)(x^2 + 4x + 3) = 0$

$(x + 1)(x + 1)(x + 3) = 0$

$(x + 1)^2(x + 3) = 0$

The roots are $x = -1$ and $x = -3$, with the root $x = -1$ occurring twice.

29. $x^3 + 5x^2 + 3x - 9 = 0$



1	1	5	3	−9
		1	6	9
	1	6	9	0

The quotient is $x^2 + 6x + 9$.

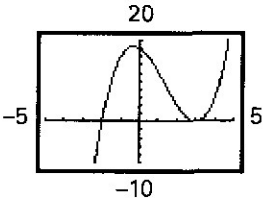
$x^3 + 5x^2 + 3x - 9 = 0$

$(x - 1)(x^2 + 6x + 9) = 0$

$(x - 1)(x + 3)^2 = 0$

The roots are $x = 1$ and $x = -3$, with the root $x = -3$ occurring twice.

30. $x^3 - 4x^2 - 3x + 18 = 0$



3	1	−4	−3	18
		3	−3	−18
	1	−1	−6	0

The quotient is $x^2 - x - 6$.

$x^3 - 4x^2 - 3x + 18 = 0$

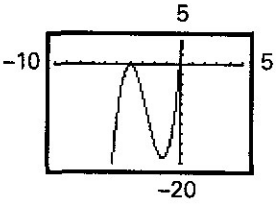
$(x - 3)(x^2 - x - 6) = 0$

$(x - 3)(x - 3)(x + 2) = 0$

$(x - 3)^2(x + 2) = 0$

The roots are $x = 3$ and $x = -2$, with the root $x = 3$ occurring twice.

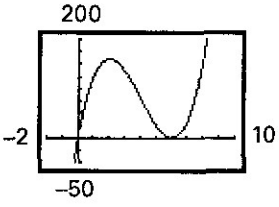
31. $2b^3 + 16b^2 + 32b = 0$



−4	2	16	32	0
		−8	−32	0
	2	8	0	0

Optimized using
trial version
www.balesio.com

32. $5h^3 - 60h^2 + 180h = 0$



6	5	−60	180	0
		30	−180	0
	5	−30	0	0

The quotient is $5h^2 - 30h$.

$5h^3 - 60h^2 + 180h = 0$

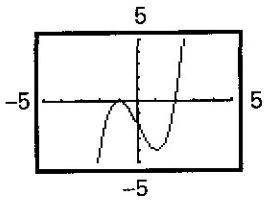
$(h - 6)(5h^2 - 30h) = 0$

$(h - 6)(h - 6)(5h) = 0$

$(h - 6)^2(5h) = 0$

The roots are $h = 6$ and $h = 0$ with the root $h = 6$ occurring twice.

33. $x^3 - 3x - 2 = 0$



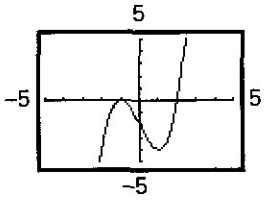
2	1	0	−3	−2
		2	4	2
	1	2	1	0

The quotient is $x^2 + 2x + 1$.

$$x^3 - 3x - 2 = 0$$
$$(x - 2)(x^2 + 2x + 1) = 0$$
$$(x - 2)(x + 1)(x + 1) = 0$$
$$(x - 2)(x + 1)^2 = 0$$

The roots are $x = 2$ and $x = -1$, with the root $x = -1$ occurring twice.

34. $x^3 - 3x + 2 = 0$



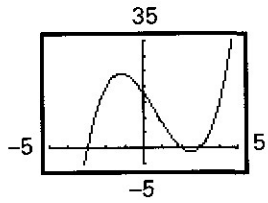
−2	1	0	−3	2
		−2	4	−2
	1	−2	1	0

The quotient is $x^2 - 2x + 1$.

$$x^3 - 3x + 2 = 0$$
$$(x + 2)(x^2 - 2x + 1) = 0$$
$$(x + 2)(x - 1)(x - 1) = 0$$
$$(x + 2)(x - 1)^2 = 0$$

The roots are $x = -2$ and $x = 1$, with the root $x = 1$ occurring twice.

35. $x^3 - 2x^2 - 9x + 18 = 0$



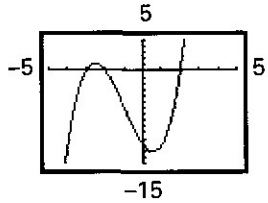
−3	1	−2	−9	18
		−3	15	−18
	1	−5	6	0

The quotient is $x^2 - 5x + 6$.

$$x^3 - 2x^2 - 9x + 18 = 0$$
$$(x + 3)(x^2 - 5x + 6) = 0$$
$$(x + 3)(x - 2)(x - 3) = 0$$

The roots are $x = -3$, $x = 2$, and $x = 3$.

36. $x^3 + 3x^2 - 4x - 12 = 0$



2	1	3	−4	−12
		2	10	12
	1	5	6	0

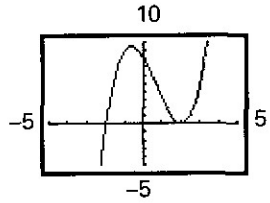
The quotient is $x^2 + 5x + 6$.

$$x^3 + 3x^2 - 4x - 12 = 0$$
$$(x - 2)(x^2 + 5x + 6) = 0$$
$$(x - 2)(x + 3)(x + 2) = 0$$

The roots are $x = 2$, $x = -3$, and $x = -2$.

37. $n^3 + 8 = 2n^2 + 4n$

$n^3 - 2n^2 - 4n + 8 = 0$



2	1	−2	−4	8
		2	0	−8
	1	0	−4	0

$- 4.$

$+ 8 = 0$

$4) = 0$

$2) = 0$

$2) = 0$

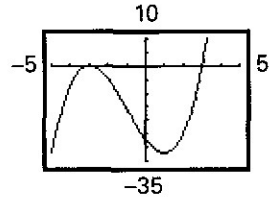
and $n = -2$, with the root

ce.

Optimized using
trial version
www.balesio.com

38. $x^3 + 3x^2 = 27 + 9x$

$x^3 + 3x^2 - 9x - 27 = 0$



3	1	3	−9	−27
		3	18	27
	1	6	9	0

The quotient is $x^2 + 6x + 9$.

$$x^3 + 3x^2 - 9x - 27 = 0$$
$$(x - 3)(x^2 + 6x + 9) = 0$$
$$(x - 3)(x + 3)(x + 3) = 0$$
$$(x - 3)(x + 3)^2 = 0$$

The roots are $x = 3$ and $x = -3$, with the root $x = -3$ occurring twice.

39. Substitute u for x^2 and solve for u.

$$x^4 - 4x^2 + 4 = 0$$
$$(x^2)^2 - 4x^2 + 4 = 0$$
$$u^2 - 4u + 4 = 0$$
$$(u - 2)^2 = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 2)^2 = 0$$
$$x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

40. Substitute u for x^2 and solve for u.

$$x^4 - 6x^2 + 9 = 0$$
$$(x^2)^2 - 6x^2 + 9 = 0$$
$$u^2 - 6u + 9 = 0$$
$$(u - 3)^2 = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 3)^2 = 0$$
$$x^2 - 3 = 0$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

41. Substitute u for y^2 and solve for u.

$$y^4 - 18y^2 + 81 = 0$$
$$(y^2)^2 - 18y^2 + 81 = 0$$
$$u^2 - 18u + 81 = 0$$
$$(u - 9)^2 = 0$$

Replace u with y^2 and solve for y.

$$(y^2 - 9)^2 = 0$$
$$y^2 - 9 = 0$$
$$y^2 = 9$$
$$y = \pm 3$$

42. Substitute u for a^2 and solve for u.

$$a^4 - 24a^2 + 144 = 0$$
$$(a^2)^2 - 24a^2 + 144 = 0$$
$$u^2 - 24u + 144 = 0$$
$$(u - 12)^2 = 0$$

Replace u with a^2 and solve for a.

$$(a^2 - 12)^2 = 0$$
$$a^2 - 12 = 0$$
$$a^2 = 12$$
$$a = \pm\sqrt{12}$$
$$a = \pm 2\sqrt{3}$$

43. Substitute u for x^2 and solve for u.

$$x^4 - 13x^2 + 36 = 0$$
$$(x^2)^2 - 13x^2 + 36 = 0$$
$$u^2 - 13u + 36 = 0$$
$$(u - 9)(u - 4) = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 9)(x^2 - 4) = 0$$
$$x^2 = 9 \quad \text{or} \quad x^2 = 4$$
$$x = \pm 3 \text{ or } x = \pm 2$$

44. Substitute u for x^2 and solve for u.

$$x^4 - 9x^2 + 18 = 0$$
$$(x^2)^2 - 9x^2 + 18 = 0$$
$$u^2 - 9u + 18 = 0$$
$$(u - 6)(u - 3) = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 6)(x^2 - 3) = 0$$
$$x^2 = 6 \quad \text{or} \quad x^2 = 3$$
$$x = \pm\sqrt{6} \text{ or } x = \pm\sqrt{3}$$

45.

$$x^5 - 9x^3 + 8x = 0$$
$$x(x^4 - 9x^2 + 8) = 0$$
$$x = 0 \text{ or } x^4 - 9x^2 + 8 = 0$$

Substitute u for x^2 and solve for u.

$$x^4 - 9x^2 + 8 = 0$$
$$(x^2)^2 - 9x^2 + 8 = 0$$
$$u^2 - 9u + 8 = 0$$
$$(u - 8)(u - 1) = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 8)(x^2 - 1) = 0$$
$$x^2 = 8 \quad \text{or} \quad x^2 = 1$$
$$x = \pm\sqrt{8} \quad \text{or} \quad x = \pm 1$$
$$x = 0 \text{ or } x = \pm 2\sqrt{2} \text{ or } x = \pm 1$$

46.

$$z^5 - 28z^3 + 27z = 0$$
$$z(z^4 - 28z^2 + 27) = 0$$
$$z = 0 \text{ or } z^4 - 28z^2 + 27 = 0$$

Substitute u for z^2 and solve for u.

$$z^4 - 28z^2 + 27 = 0$$
$$(z^2)^2 - 28z^2 + 27 = 0$$
$$u^2 - 28u + 27 = 0$$
$$(u - 27)(u - 1) = 0$$

Replace u with z^2 and solve for z.

$$(z^2 - 27)(z^2 - 1) = 0$$
$$z^2 = 27 \quad \text{or} \quad z^2 = 1$$
$$z = \pm\sqrt{27} \text{ or } z = \pm 1$$
$$z = 0 \text{ or } z = \pm 3\sqrt{3} \text{ or } z = \pm 1$$

47. Substitute u for x^2 and solve for u.

$$x^4 - 12x^2 = -36$$
$$(x^2)^2 - 12x^2 + 36 = 0$$
$$u^2 - 12u + 36 = 0$$
$$(u - 6)^2 = 0$$

with x^2 and solve for x.

$$= 0$$
$$= 0$$
$$= 6$$
$$= \pm\sqrt{6}$$

Optimized using
trial version
www.balesio.com

48. Substitute u for x^2 and solve for u.

$$x^4 - 14x^2 = -49$$
$$(x^2)^2 - 14x^2 + 49 = 0$$
$$u^2 - 14u + 49 = 0$$
$$(u - 7)^2 = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 7)^2 = 0$$
$$x^2 - 7 = 0$$
$$x^2 = 7$$
$$x = \pm\sqrt{7}$$

49. Substitute u for h^2 and solve for u.

$$h^4 + 12 = 7h^2$$
$$(h^2)^2 - 7h^2 + 12 = 0$$
$$u^2 - 7u + 12 = 0$$
$$(u - 3)(u - 4) = 0$$

Replace u with h^2 and solve for h.

$$(h^2 - 3)(h^2 - 4) = 0$$
$h^2 = 3$ or $h^2 = 4$
$h = \pm\sqrt{3}$ or $h = \pm 2$

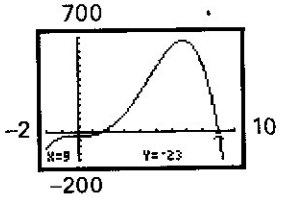
50. Substitute u for t^2 and solve for u.

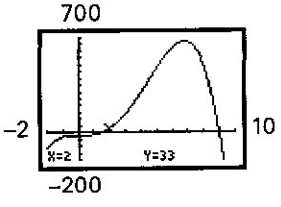
$$t^4 + 14 = 9t^2$$
$$(t^2)^2 - 9t^2 + 14 = 0$$
$$u^2 - 9u + 14 = 0$$
$$(u - 7)(u - 2) = 0$$

Replace u with t^2 and solve for t.

$$(t^2 - 7)(t^2 - 2) = 0$$
$t^2 = 7$ or $t^2 = 2$
$t = \pm\sqrt{7}$ or $t = \pm\sqrt{2}$

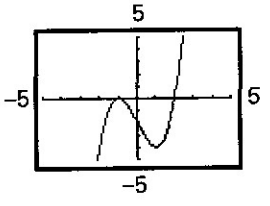
51. $f(x) = 9x^3 - x^4 - 23$





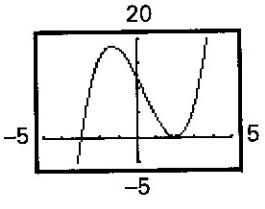
There are zeros of $f(x)$ between 8 and 9 (about 8.97) and between 1 and 2 (about 1.45).

52. $g(x) = x^3 - 3x - 2$



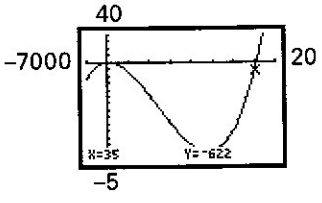
There are zeros of $g(x)$ at $x = -1$ and $x = 2$.

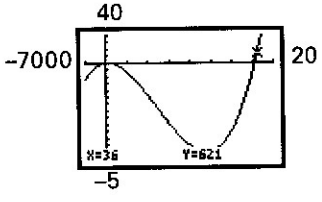
53. $f(a) = a^3 - a^2 - 8a + 12$

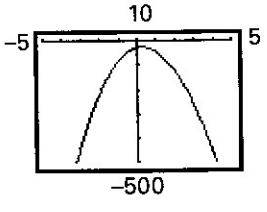


There are zeros of $f(a)$ at $a = -3$ and $a = 2$.

54. $h(x) = x^3 - 36x^2 + 18x - 27$

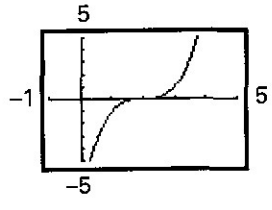






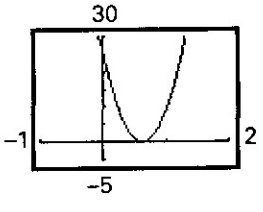
There is a zero of $h(x)$ between 35 and 36 (about 35.51).

55. $m(n) = n^3 - 6n^2 + 12n - 8$



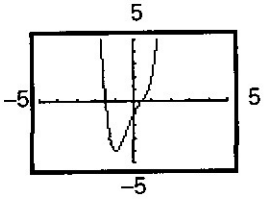
There is a zero of $m(n)$ at $n = 2$.

56. $f(t) = 64t^2 - 80t + 25$



There is a zero of $f(t)$ at $t = 0.625$.

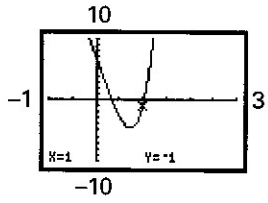
57. $g(x) = 2x^4 - 2x^2 + 3x - 1$



There are zeros of $g(x)$ between -2 and -1 (about -1.49) and between 0 and 1 (about 0.44).

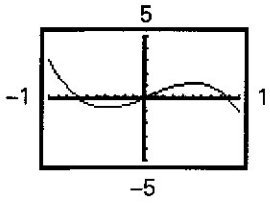
Optimized using
trial version
www.balesio.com

58. $h(x) = 13x^4 - 21x + 7$



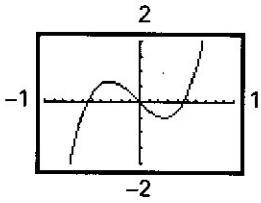
There are zeros of $h(x)$ between 0 and 1 (about 0.34) and between 1 and 2 (about 1.03).

59. $f(x) = x^2 - 5x^3 + 3x$



There are zeros of $f(x)$ at $x = 0$ and between -0.7 and -0.6 (about -0.68) and between 0.8 0.9 (about 0.88).

60. $a(x) = x^2 + 12x^3 - 3x$

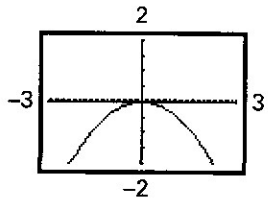


There are zeros of $a(x)$ at $x = 0$ and between -0.6 and -0.5 (about -0.54) and between 0.4 and 0.5 (about 0.46).

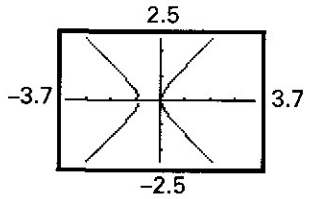
61. The graph of $2x^3 - x^2 - 6x$ passes through 0, and is less than 0 between $x = 0$ and $x = 1$. $f(0) = 0$ and $f(1) = -5$. So $f(x) + c$ has a zero between $x = 0$ and $x = 1$ for the values of $0 < c < 5$.

62. a. $f(x) = x^3 - 7x - 6$
$a = 6, b = 7$

b.

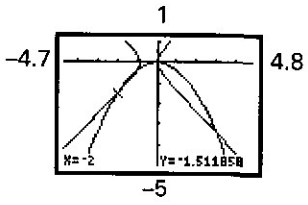


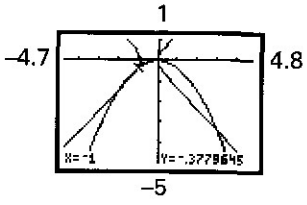
$y = -\frac{1}{\sqrt{7}}x^2$

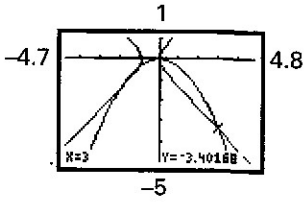


$y = \pm\sqrt{x^2 + \frac{6}{7}x}$

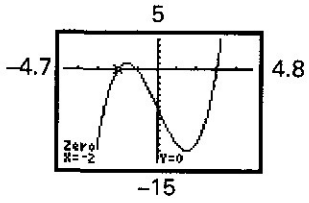
c.

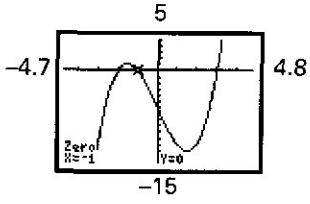


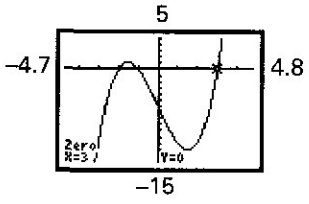




d.





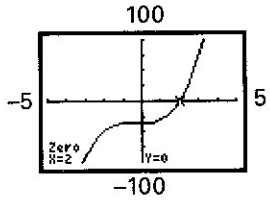


phs intersect at $x = -2, -1$, and 3.

Optimized using
trial version
www.balesio.com

63. $3x^3 + 3x^2 = 36$

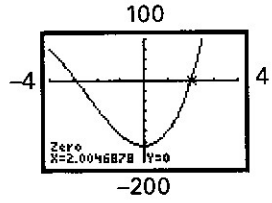
$3x^3 + 3x^2 - 36 = 0$



The width is 2, so the length is $3(2) = 6$ and the height is $1 + 2 = 3$. The dimensions are 2 feet by 6 feet by 3 feet.

64. $10\pi r^2 + \frac{4}{3}\pi r^3 = 160$

$10\pi r^2 + \frac{4}{3}\pi r^3 - 160 = 0$



A radius of about 2 millimeters will give a volume of 160 cubic millimeters.

PAGE 455, LOOK BACK

65. $x^2 - 2x = 63$

$x^2 - 2x - 63 = 0$

$(x - 9)(x + 7) = 0$

$x = 9$ or $x = -7$

66. $2x^2 - 5 = -9x$

$2x^2 + 9x - 5 = 0$

$(2x - 1)(x + 5) = 0$

$x = \frac{1}{2}$ or $x = -5$

67. $3x^2 + 60 = 27x$

$3x^2 - 27x + 60 = 0$

$3(x^2 - 9x + 20) = 0$

$3(x - 4)(x - 5) = 0$

$x = 4$ or $x = 5$

68. $\sqrt{-49} = 7i$

69. $\sqrt{-81} = 9i$

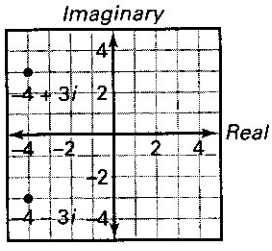
70. $\sqrt{-11} = i\sqrt{11}$

71. The complex conjugate of $6 - 2i$ is $6 + 2i$.

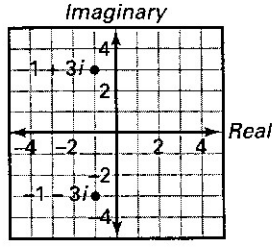
72. The complex conjugate of $4i - 2$ is $-4i - 2$.

73. The complex conjugate of 5 is 5.

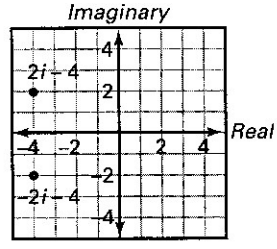
74.



75.



76.



77. $x^2 + 2x + 7 = 0$

$a = 1, b = 2, c = 7$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(7)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{-24}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{6}}{2}$$

$$= -1 \pm i\sqrt{6}$$

78. $4x^2 - 3x + 5 = 0$

$a = 4, b = -3, c = 5$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(5)}}{2(4)}$$

$$= \frac{3 \pm \sqrt{-71}}{8}$$

$$= \frac{3}{8} \pm \frac{\sqrt{71}}{8}i$$

79. $x^2 - 3x = -9$

$x^2 - 3x + 9 = 0$

$a = 1, b = -3, c = 9$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{-27}}{2}$$

$$= \frac{3 \pm 3i\sqrt{3}}{2}$$

$$= \frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$$

ND

$3i)^3 - 2(2 + 3i)^2 + 5(2 + 3i) + 26$

$3i)(2 + 3i)^2 - 2(4 + 12i - 9) + 10 + 15i + 26$

$3i)(-5 + 12i) - 8 - 24i + 18 + 10 + 15i + 26$

$+ 24i - 15i - 36 - 8 - 24i + 18 + 10 + 15i + 26$

Optimized using trial version www.balesio.com

$$P(2-3i) = (2-3i)^3 - 2(2-3i)^2 + 5(2-3i) + 26$$
$$= (2-3i)(2-3i)^2 - 2(4-12i-9) + 10 - 15i + 26$$
$$= (2-3i)(-5-12i) - 8 + 24i + 18 + 10 - 15i + 26$$
$$= -10 - 24i + 15i - 36 - 8 + 24i + 18 + 10 - 15i + 26$$
$$= 0$$

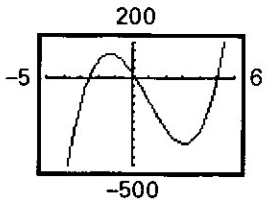
b. $(x-(2+3i))(x-(2-3i)) = x^2 - 4x + 13$, so using long division we find that $P(x) = (2+x)(13-4x+x^2)$
$= (x+2)[x-(2+3i)][x-(2-3i)]$

7.5 PAGE 463, GUIDED SKILLS PRACTICE

5. $P(x) = 12x^3 - 32x^2 - 145x + 25$.

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm1, \pm5, \pm25$ and $q = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$. List the possibilities:
$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}, \pm\frac{5}{1}, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{4}, \pm\frac{5}{6}, \pm\frac{5}{12}, \pm\frac{25}{1}, \pm\frac{25}{2}, \pm\frac{25}{3}, \pm\frac{25}{4}, \pm\frac{25}{6}, \pm\frac{25}{12}$

Use a graph to determine the roots from these possibilities.



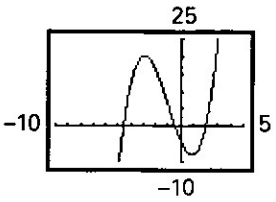
From the graph, there is a zero at 5. There is a zero between -3 and -2, so $\frac{5}{2}$ and $\frac{25}{12}$ are possibilities. Finally, there is a zero between 0 and 1, close to 0 so $-\frac{1}{3}$, $-\frac{1}{4}$, $-\frac{1}{6}$, and $-\frac{1}{12}$ would be possibilities. Use synthetic division to test the possible zeros that are between these values.

$-\frac{5}{2}$	12	−32	−145	25
		−30	155	−25
	12	−62	10	0

$\frac{1}{6}$	12	−32	−145	25
		2	−5	−25
	12	−30	−150	0

The rational roots are 5, $-\frac{5}{2}$, and $\frac{1}{6}$.

6. $Q(x) = x^3 + 3x^2 - 8x - 4$



From the graph, there is a zero at 2. Then $x-2$ is a factor of $x^3 + 3x^2 - 8x - 4$.

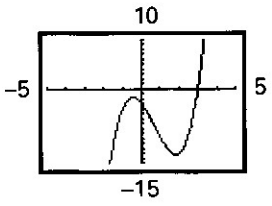
2	1	3	−8	−4
		2	10	4
	1	5	2	0

$$x^3 + 3x^2 - 8x - 4 = 0$$
$$(x-2)(x^2+5x+2) = 0$$
$$x - 2 = 0 \text{ or } x^2 + 5x + 2 = 0$$
$$x = 2 \text{ or } \quad x = \frac{-5 \pm \sqrt{25-8}}{2}$$
$$x = -\frac{5}{2} \pm \frac{\sqrt{17}}{2}$$

$Q(x)$ are 2, $-\frac{5}{2} + \frac{\sqrt{17}}{2}$ and $-\frac{5}{2} - \frac{\sqrt{17}}{2}$.

Optimized using
trial version
www.balesio.com

7. $Q(x) = 2x^3 - 4x^2 - 5x - 3$



From the graph, there is a zero at 3. Then $x - 3$ is a factor of $2x^3 - 4x^2 - 5x - 3$.

3	2	−4	−5	−3
		6	6	3
	2	2	1	0

$2x^3 - 4x^2 - 5x - 3 = 0$
$(x - 3)(2x^2 + 2x + 1) = 0$
$x - 3 = 0$ or $2x^2 + 2x + 1 = 0$

$x = 3$

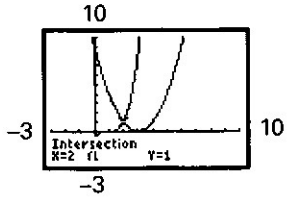
$$x = \frac{-2 \pm \sqrt{4 - 8}}{4}$$

$$x = \frac{-2 \pm 2i}{4}$$

$$x = -\frac{1}{2} \pm \frac{1}{2}i$$

The zeros of $Q(x)$ are 3, $-\frac{1}{2} + \frac{1}{2}i$, and $-\frac{1}{2} - \frac{1}{2}i$.

8. Graph $P(x) = (x - 1)^3$ and $Q(x) = (x - 3)^2$ in the same window.

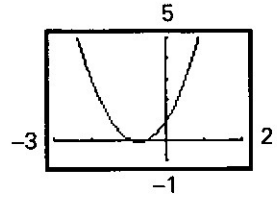


The graphs intersect only once, at $x = 2$.

9. $P(x) = a(x - 3)^2(x - (3 + 4i))(x - (3 - 4i))$
$P(x) = a(x - 3)^2(x^2 - 6x + 25)$
$-6 = a(0 - 3)^2(0^2 - 6(0) + 25)$
$-6 = a(-3)^2(25)$
$-6 = 225a$
$a = -\frac{2}{75}$
$P(x) = -\frac{2}{75}(x - 3)^2(x^2 - 6x + 25)$
$= -\frac{2}{75}x^4 + \frac{8}{25}x^3 - \frac{28}{15}x^2 + \frac{136}{25}x - 6$

PAGES 463-465, PRACTICE AND APPLY

10. $2x^2 + 3x + 1 = 0$
Possible rational zeros are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1$ and $q = \pm 1, \pm 2$. Use a graph to narrow the possibilities.



From the graph, there appear to be zeros at -1 and $-\frac{1}{2}$.

−1	2	3	1
		−2	−1
	2	1	0

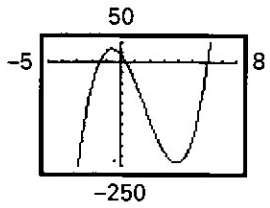
$-\frac{1}{2}$	2	3	1
		−1	−1
	2	2	0

-1 and $-\frac{1}{2}$.

Optimized using
trial version
www.balesio.com

11. $6x^3 - 29x^2 - 45x + 18 = 0$

Possible rational zeros are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm1, \pm2, \pm3, \pm6, \pm9, \pm18$ and $q = \pm1, \pm2, \pm3, \pm6$. Use a graph to narrow the possibilities.



From the graph, there is a zero at 6. There are also zeros between 0 and 1, and between -2 and -1. Use synthetic division to test the possible zeros that are between these values.

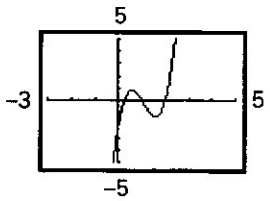
$-\frac{3}{2}$	6	-29	-45	18
		-9	57	-18
	6	-38	12	0

$\frac{1}{3}$	6	-29	-45	18
		2	-9	-18
	6	-27	-54	0

The rational roots are 6, $-\frac{3}{2}$, and $\frac{1}{3}$.

12. $4x^3 - 13x^2 + 11x - 2 = 0$

Possible rational zeros are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm1, \pm2$, and $q = \pm1, \pm2, \pm4$. Use a graph to narrow the possibilities.



From the graph, there are zeros at 1 and 2. There is also a zero between 0 and 1. Use synthetic division to test the possible zeros that are between these values.

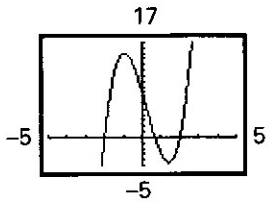
$\frac{1}{4}$	4	-13	11	-2
		1	-3	2
	4	-12	8	0

il roots are $\frac{1}{4}$, 1, and 2.

Optimized using
trial version
www.balesio.com

13. $3x^3 - 2x^2 - 12x + 8 = 0$

Possible rational zeros are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2, \pm 4, \pm 8$ and $q = \pm 1, \pm 3$. Use a graph to narrow the possibilities.



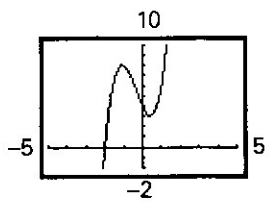
From the graph, there are zeros at -2 and 2. There is also a zero between 0 and 1. Use synthetic division to test possible zeros that are between these values.

$\frac{2}{3}$	3	-2	-12	8
		2	0	-8
	3	0	-12	0

The rational roots are -2, 2, and $\frac{2}{3}$.

14. $3x^3 + 3x^2 - 4x + 4 = 0$

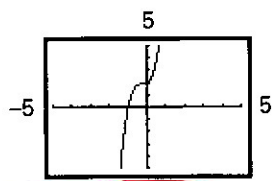
Possible rational zeros are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2, \pm 4$ and $q = \pm 1, \pm 3$. Use a graph to narrow the possibilities.



From the graph, there is only one zero and it is at $x = -2$.

15. $4x^3 + 3x^2 + x + 2 = 0$

Possible rational zeros are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2$ and $q = \pm 1, \pm 2, \pm 4$. Use a graph to narrow the possibilities.

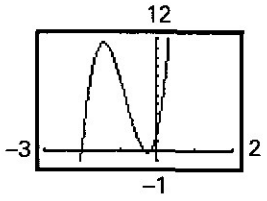


only one zero and it is at $x = -1$.

Optimized using
trial version
www.balesio.com

16. $15a^3 + 38a^2 + 17a + 2 = 0$

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2$ and $q = \pm 1, \pm 3, \pm 5, \pm 15$. Use a graph to narrow the possibilities.



From the graph, there is a zero at -2. There are also two zeros between -1 and 0. Use synthetic division to test the possible zeros that are between these values.

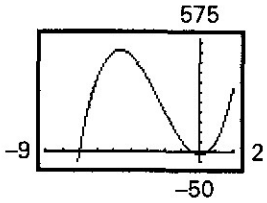
$-\frac{1}{3}$	15	38	17	2
		-5	-11	-2
	15	33	6	0

$-\frac{1}{5}$	15	38	17	2
		-3	-7	-2
	15	35	10	0

The rational roots are -2, $-\frac{1}{3}$, and $-\frac{1}{5}$.

17. $10x^3 + 69x^2 - 9x - 14 = 0$

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2, \pm 7, \pm 14$ and $q = \pm 1, \pm 2, \pm 5, \pm 10$. Use a graph to narrow the possibilities.



From the graph, there is a zero at -7. There are also zeros between -1 and 0, and between 0 and 1. Use synthetic division to test the possible zeros that are between these values.

$-\frac{2}{5}$	10	69	-9	-14
		-4	-26	14
	10	65	-35	0

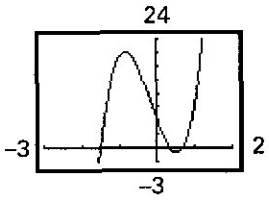
$\frac{1}{2}$	10	69	-9	-14
		5	37	14
	10	74	28	0

The rational roots are -7, $-\frac{2}{5}$, and $\frac{1}{2}$.

Optimized using
trial version
www.balesio.com

18. $18c^3 - 23c + 6 + 9c^2 = 0$

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2, \pm 3, \pm 6$ and $q = \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$. Use a graph to narrow the possibilities.



From the graph, there is one zero between -2 and -1, and two zeros between 0 and 1. Use synthetic division to test the possible zeros that are between these values.

$-\frac{3}{2}$	18	9	-23	6
		-27	27	-6
	18	-18	4	0

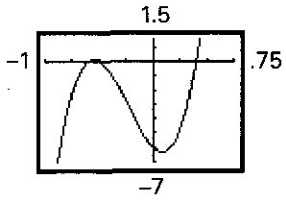
$\frac{1}{3}$	18	9	-23	6
		6	5	-6
	18	15	-18	0

$\frac{2}{3}$	18	9	-23	6
		12	14	-6
	18	21	-9	0

The rational roots are $-\frac{3}{2}, \frac{1}{3}$, and $\frac{2}{3}$.

19. $50x^3 - 7x + 35x^2 - 6 = 0$

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2, \pm 3, \pm 6$ and $q = \pm 1, \pm 2, \pm 5, \pm 10, \pm 25, \pm 50$. Use a graph to narrow the possibilities.



From the graph, there are two zeros between $-\frac{1}{4}$ and $\frac{3}{4}$, and one zero between $\frac{1}{4}$ and $\frac{1}{2}$. Use synthetic division to test the possible zeros that are between these values.

$-\frac{3}{5}$	50	35	-7	-6
		-30	-3	6
	50	5	-10	0

$-\frac{1}{2}$	50	35	-7	-6
		-25	-5	6
	50	10	-12	0

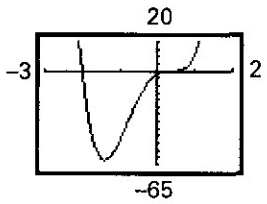
		35	-7	-6
		~~-20~~	22	6
	55	15		0

Optimized using trial version www.balesio.com

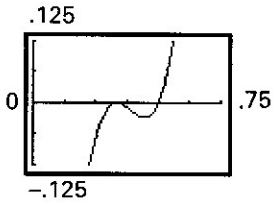
$-\frac{3}{5}, -\frac{1}{2}$, and $\frac{2}{5}$.

20. $18x^4 + 15x + 15x^3 - 34x^2 - 2 = 0$

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2,$ and $q = \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$. Use a graph to narrow the possibilities.



From the graph, there is a zero at -2. Zoom in closer on the graph between 0 and 1.



There is a zero at $\frac{1}{2}$, and it looks like there is a root with a multiplicity of 2 between $\frac{1}{4}$ and $\frac{1}{2}$: Use synthetic division to test the possible zeros that are between these values.

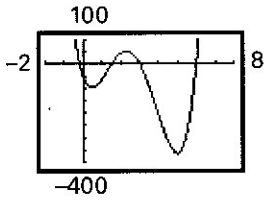
$\frac{1}{3}$	18	15	-34	15	-2
		6	-9	7	-9	2
	18	21	-27	6	0

The rational roots are -2, $\frac{1}{2}$, and $\frac{1}{3}$.

Optimized using
trial version
www.balesio.com

21. $10x^4 - 103x^3 - 207x + 294x^2 - 54 = 0$

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm 27, \pm 54$, and $q = \pm 1, \pm 2, \pm 5, \pm 10$. Use a graph to narrow the possibilities.

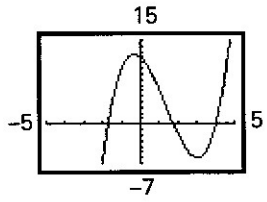


From the graph, there are zeros at 3 and 6. There are also zeros between -1 and 0, and between 1 and 2. Use synthetic division to test the possible zeros that are between these values.

$-\frac{1}{5}$	10	-103	294	-207	-54
		-2	21	-63	54
	10	-105	315	-270	0
$\frac{3}{2}$	10	-103	294	-207	-54
		15	-132	243	54
	10	-88	162	36	0

The rational roots are 3, 6, $-\frac{1}{5}$, and $\frac{3}{2}$.

22. $B(x) = x^3 - 4x^2 - 3x + 12$



From the graph, there is a zero at 4.
Then $x - 4$ is a factor of $B(x)$.

$x^3 - 4x^2 - 3x + 12 = 0$

$(x - 4)(x^2 - 3) = 0$

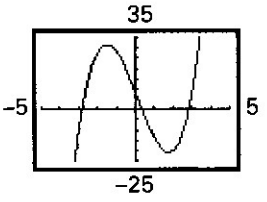
$x - 4 = 0$ or $x^2 - 3 = 0$

$x = 4$ $\qquad x^2 = 3$

$x = \pm\sqrt{3}$

The zeros of $B(x)$ are 4, $\sqrt{3}$, and $-\sqrt{3}$.

23. $P(x) = 3x^3 - x^2 - 24x + 8$



From the graph, there is a zero at $\frac{1}{3}$. Then $(3x - 1)$ is a factor of $P(x)$.

$3x^3 - x^2 - 24x + 8 = 0$

$(3x - 1)(x^2 - 8) = 0$

$3x - 1 = 0$ or $x^2 - 8 = 0$

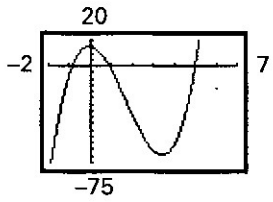
$x = \frac{1}{3}$ $\qquad x^2 = 8$

$x = \pm 2\sqrt{2}$

The zeros of $P(x)$ are $\frac{1}{3}$, $-2\sqrt{2}$, and $2\sqrt{2}$.

Optimized using
trial version
www.balesio.com

24. $f(x) = 4x^3 - 20x^2 - 3x + 15$



From the graph, there is a zero at 5.
Then $x - 5$ is a factor of $f(x)$.

$4x^3 - 20x^2 - 3x + 15 = 0$

$(x - 5)(4x^2 - 3) = 0$

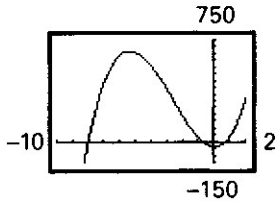
$x - 5 = 0$ or $4x^2 - 3 = 0$

$x = 5$ $\qquad x^2 = \frac{3}{4}$

$x = \pm\frac{\sqrt{3}}{2}$

The zeros of $f(x)$ are 5, $\frac{\sqrt{3}}{2}$, and $-\frac{\sqrt{3}}{2}$.

25. $f(x) = 9x^3 + 72x^2 - 5x - 40$



From the graph, there is a zero at -8.
Then $x + 8$ is a factor of $f(x)$.

$9x^3 + 72x^2 - 5x - 40 = 0$

$(x + 8)(9x^2 - 5) = 0$

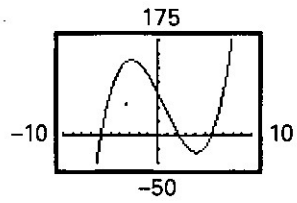
$x + 8 = 0$ or $9x^2 - 5 = 0$

$x = -8$ $\qquad x^2 = \frac{5}{9}$

$x = \pm\frac{\sqrt{5}}{3}$

The zeros of $f(x)$ are -8, $\frac{\sqrt{5}}{3}$, and $-\frac{\sqrt{5}}{3}$.

26. $t(x) = x^3 - 2x^2 + 78 - 35x$



From the graph, there is a zero at -6.
Then $x + 6$ is a factor of $t(x)$.

$x^3 - 2x^2 + 78 - 35x = 0$

$(x + 6)(x^2 - 8x + 13) = 0$

$x + 6 = 0$ or $x^2 - 8x + 13 = 0$

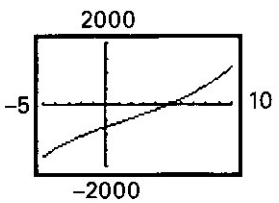
$x = -6$ $\qquad x = \frac{8 \pm \sqrt{64 - 52}}{2}$

$x = \frac{8 \pm 2\sqrt{3}}{2}$

$x = 4 \pm \sqrt{3}$

The zeros of $t(x)$ are -6, $4 + \sqrt{3}$, and $4 - \sqrt{3}$.

27. $g(x) = x^3 - 5x^2 + 144x - 720$



From the graph, there is a zero at 5.
Then $x - 5$ is a factor of $g(x)$.

$x^3 - 5x^2 + 144x - 720 = 0$

$(x - 5)(x^2 + 144) = 0$

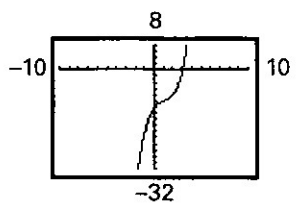
$x - 5 = 0$ or $x^2 + 144 = 0$

$x = 5$ $\qquad x^2 = -144$

$x = \pm 12i$

The zeros of $g(x)$ are 5, $12i$, and $-12i$.

28. $N(x) = x^3 - 3x^2 + 4x - 12$



From the graph, there is a zero at 3.
Then $x - 3$ is a factor of $N(x)$.

$= 0$

$= 0$

$= 0$

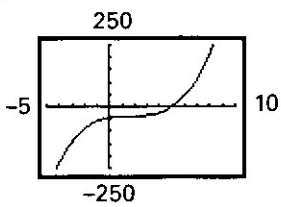
$= -4$

$= \pm 2i$

3, $2i$, and $-2i$.

Optimized using
trial version
www.balesio.com

29. $f(b) = b^3 - 5b^2 + 9b - 45$



From the graph, there is a zero at 5.
Then $b - 5$ is a factor of $f(b)$.

$b^3 - 5b^2 + 9b - 45 = 0$

$(b - 5)(b^2 + 9) = 0$

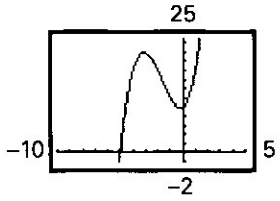
$b - 5 = 0$ or $b^2 + 9 = 0$

$b = 5$ $\qquad b^2 = -9$

$b = \pm 3i$

The zeros of $f(b)$ are 5, $3i$, and $-3i$.

30. $G(x) = x^3 + 5x^2 + 10 + 2x$



From the graph, there is a zero at -5.
Then $x + 5$ is a factor of $G(x)$.

$x^3 + 5x^2 + 10 + 2x = 0$

$(x + 5)(x^2 + 2) = 0$

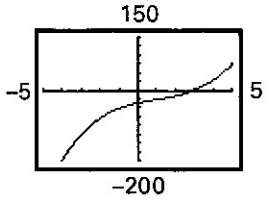
$x + 5 = 0$ or $x^2 + 2 = 0$

$x = -5 \qquad x^2 = -2$

$x = \pm i\sqrt{2}$

The zeros of $G(x)$ are -5, $i\sqrt{2}$, and $-i\sqrt{2}$.

31. $a(x) = x^3 - 3x^2 + 12x - 36$



From the graph, there is a zero at 3.
Then $x - 3$ is a factor of $a(x)$.

$x^3 - 3x^2 + 12x - 36 = 0$

$(x - 3)(x^2 + 12) = 0$

$x - 3 = 0$ or $x^2 + 12 = 0$

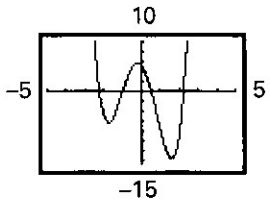
$x = 3 \qquad x^2 = -12$

$x = \pm i\sqrt{12}$

$x = \pm 2i\sqrt{3}$

The zeros of $a(x)$ are 3, $2i\sqrt{3}$, and $-2i\sqrt{3}$.

32. $W(x) = x^3 + 2x^4 - 5x + 5 - 11x^2$



From the graph, there are zeros at -1 and $\frac{1}{2}$.
Then $x + 1$ and $2x - 1$ are factors of $W(x)$.

$x^3 + 2x^4 - 5x + 5 - 11x^2 = 0$

$(x + 1)(2x - 1)(x^2 - 5) = 0$

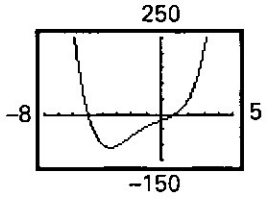
$x + 1 = 0$ or $2x - 1 = 0$ or $x^2 - 5 = 0$

$x = -1 \qquad x = \frac{1}{2} \qquad x^2 = 5$

$x = \pm\sqrt{5}$

The zeros of $W(x)$ are -1, $\frac{1}{2}$, $\sqrt{5}$, and $-\sqrt{5}$.

33. $H(x) = x^4 + 16x - x^2 + 4x^3 - 20$



From the graph, there are zeros at -5 and 1.
Then $x + 5$ and $x - 1$ are factors of $H(x)$.

$x^4 + 16x - x^2 + 4x^3 - 20 = 0$

$(x + 5)(x - 1)(x^2 + 4) = 0$

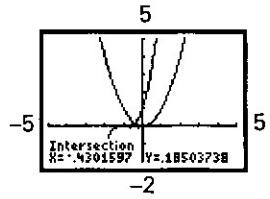
$x + 5 = 0$ or $x - 1 = 0$ or $x^2 + 4 = 0$

$x = -5 \qquad x = 1 \qquad x^2 = -4$

$x = \pm 2i$

The zeros of $H(x)$ are -5, 1, $2i$, and $-2i$.

34. Graph $P(x) = x^2$ and $Q(x) = x^3 + 3x^2 + 3x + 1$ in the same window.



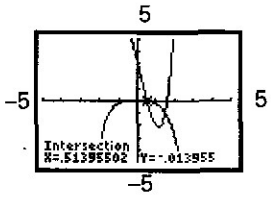
The graphs intersect only once, at $x \approx -0.43$.

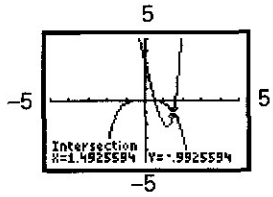
35. Graph $P(x) = x^2 - 2x + 1$ and $Q(x) = -x^3 + 3x^2 - 3x + 3$ in the same window.

sect only once, at $x = 2$.

Optimized using
trial version
www.balesio.com

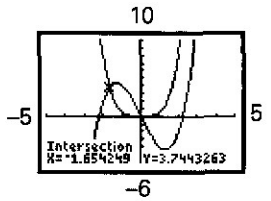
36. Graph $P(x) = x^4 - 6x + 3$ and $Q(x) = -0.2x^4$ in the same window.

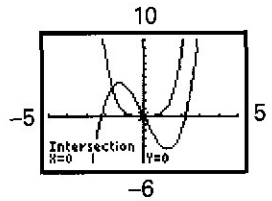




The graphs intersect twice, at $x \approx 0.51$ and $x \approx 1.49$.

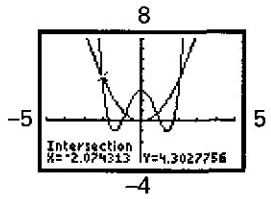
37. Graph $P(x) = x^3 - 5x$ and $Q(x) = 0.5x^4$ in the same window.

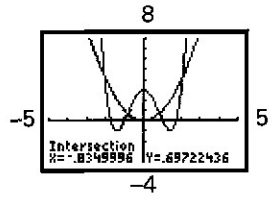


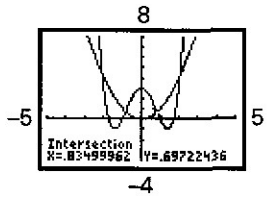


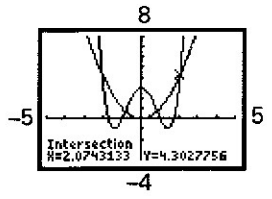
The graphs intersect twice, at $x = 0$ and $x \approx -1.65$.

38. Graph $P(x) = x^4 - 4x^2 + 3$ and $Q(x) = x^2$ in the same window.



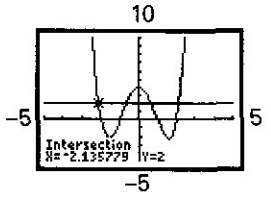


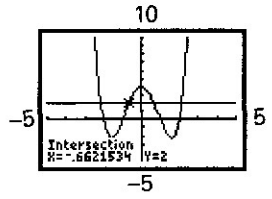


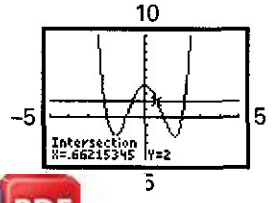


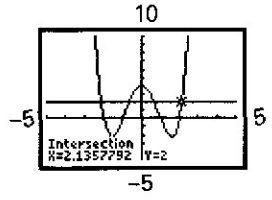
The graphs intersect four times, at $x \approx -2.07$, $x \approx -0.83$, $x \approx 0.83$, and $x \approx 2.07$.

39. Graph $P(x) = x^4 - 5x^2 + 4$ and $Q(x) = 2$ in the same window.





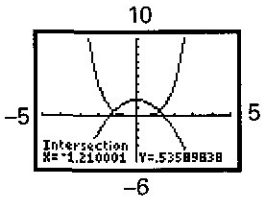


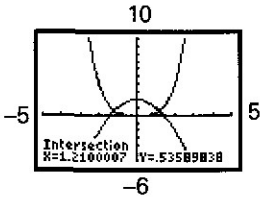


ntersect four times, at $x \approx -2.14$, $x \approx -0.66$, $x \approx 0.66$,

Optimized using
trial version
www.balesio.com

40. Graph $P(x) = 0.25x^4$ and $Q(x) = -x^2 + 2$ in the same window.





The graphs intersect twice, at $x \approx -1.21$ and $x \approx 1.21$.

41. $P(x) = a(x-2)(x-3)$
$12 = a(0-2)(0-3)$
$12 = a(-2)(-3)$
$12 = 6a$
$a = 2$
$P(x) = 2(x-2)(x-3) = 2x^2 - 10x + 12$

42. $P(x) = a(x+1)(x-4)$
$4 = a(0+1)(0-4)$
$4 = -4a$
$a = -1$
$P(x) = -(x+1)(x-4) = -x^2 + 3x + 4$

43. $P(x) = a(x+2)(x-1)(x-2)$
$20 = a(0+2)(0-1)(0-2)$
$20 = a(2)(-1)(-2)$
$20 = 4a$
$a = 5$
$P(x) = 5(x+2)(x-1)(x-2)$
$= 5x^3 - 5x^2 - 20x + 20$

44. $P(x) = a(x+1)(x-2)(x-4)$
$24 = a(0+1)(0-2)(0-4)$
$24 = a(1)(-2)(-4)$
$24 = 8a$
$a = 3$
$P(x) = 3(x+1)(x-2)(x-4)$
$= 3x^3 - 15x^2 + 6x + 24$

45. $P(x) = a(x-1)^2(x-2)^2$
$1 = a(0-1)^2(0-2)^2$
$1 = a(-1)^2(-2)^2$
$1 = 4a$
$a = \frac{1}{4}$
$P(x) = \frac{1}{4}(x-1)^2(x-2)^2$
$= \frac{1}{4}x^4 - \frac{3}{2}x^3 + \frac{13}{4}x^2 - 3x + 1$

46. $P(x) = a(x-1)^3(x-2)^2$
$2 = a(0-1)^3(0-2)^2$
$2 = a(-1)^3(-2)^2$
$2 = -4a$
$a = -\frac{1}{2}$
$P(x) = -\frac{1}{2}(x-1)^3(x-2)^2$
$= -\frac{1}{2}x^5 - \frac{7}{2}x^4 + \frac{19}{2}x^3 - \frac{25}{2}x^2 + 8x - 2$

47. $P(x) = a(x-1)(x-i)(x-(-i))$
$-1 = a(0-1)(0-i)(0+i)$
$-1 = a(-1)(-i)(i)$
$-1 = -a$
$a = 1$
$P(x) = (x-1)(x-i)(x+i)$
$= x^3 - x^2 + x - 1$

48. $P(x) = a(x-(-1))(x-2i)(x-(-2i))$
$4 = a(0+1)(0-2i)(0+2i)$
$4 = a(1)(-2i)(2i)$
$4 = 4a$
$a = 1$
$P(x) = (x+1)(x-2i)(x+2i)$
$= x^3 + x^2 + 4x + 4$

49. $P(x) = a(x-1)(x-2)(x-5i)(x-(-5i))$
$3 = a(0-1)(0-2)(0-5i)(0+5i)$
$3 = a(-1)(-2)(-5i)(5i)$
$3 = 50a$
$a = \frac{3}{50}$
$P(x) = -\frac{3}{50}(x-1)(x-2)(x-5i)(x+5i)$
$= \frac{3}{50}x^4 - \frac{9}{50}x^3 + \frac{81}{50}x^2 - \frac{9}{2}x + 3$

50. $P(x) = a(x-2)(x-5)(x-3i)(x-(-3i))$
$-3 = a(0-2)(0-5)(0-3i)(0+3i)$
$-3 = a(-2)(-5)(-3i)(3i)$
$-3 = 90a$
$a = -\frac{1}{30}$
$P(x) = -\frac{1}{30}(x-2)(x-5)(x-3i)(x+3i)$
$= -\frac{1}{30}x^4 + \frac{7}{30}x^3 - \frac{19}{30}x^2 + \frac{21}{10}x - 3$

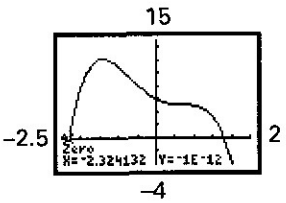
51. $P(x) = x^2 - 3$

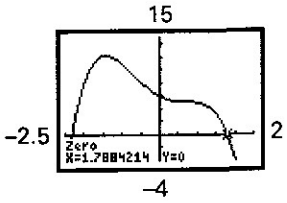
$\sqrt{3})^2 - 3 = 3 - 3 = 0$, we know that $\sqrt{3}$ is a root of $P(x)$.

ɔt Theorem, if $\sqrt{3} = \frac{p}{q}$ for some integers p and q, then p d q is a factor of 1. However, the only possible values of Therefore $\sqrt{3}$ is irrational.

Optimized using
trial version
www.balesio.com

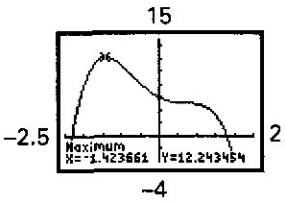
52. $f(x) = -x^4 + 3x^2 - 3x + 6$





a. The approximate zeros are $x = -2.32$ and $x = 1.79$.

b.



The approximate maximum value is 12.24 at $x = -1.42$.

c. $y = kx$

$3200 = k(12.24)$

$k = \frac{3200}{12.24}$

$k \approx 261.4$

The scale factor is about 261.4.

53. $f(x) = 0.025t^2 + 2t$

To determine the time it takes the balloon to rise to 800 feet:

$$800 = 0.025t^2 + 2t$$

$$0.025t^2 + 2t - 800 = 0$$

$$t^2 + 80t - 32{,}000 = 0$$

$$t = \frac{-80 \pm \sqrt{6400 - 4(1)(-32{,}000)}}{2}$$

$$= \frac{-80 \pm \sqrt{134{,}400}}{2}$$

$$= \frac{-80 \pm 80\sqrt{21}}{2}$$

$$= -40 \pm 40\sqrt{21}$$

$$\approx 14.3.$$

The balloon reaches the altitude of 800 feet after about 143 seconds.

PAGE 465, LOOK BACK

54. $f \circ g = f(2 - x)$
$= 5(2 - x) + 4$
$= 10 - 5x + 4$
$= 14 - 5x$

55. $g \circ f = g(5x + 4)$
$= 2 - (5x + 4)$
$= 2 - 5x - 4$
$= -2 - 5x$

56. $f \circ f = f(5x + 4)$
$= 5(5x + 4) + 4$
$= 25x + 20 + 4$
$= 25x + 24$

57. $(f \circ g)(x) = 14 - 5x$
$(f \circ g)(-2) = 14 - 5(-2)$
$= 14 + 10$
$= 24$

58. $(g \circ f)(x) = -2 - 5x$
$(g \circ f)(5) = -2 - 5(5)$
$= -2 - 25$
$= -27$

59. $(f \circ f)(x) = 25x + 24$
$(f \circ f)(0.2) = 25(0.2) + 24$
$= 5 + 24$
$= 29$

Optimized using
trial version
www.balesio.com

60. $y = 6x^2 - x - 12$

$x = -\frac{b}{2a} = -\frac{-1}{2(6)} = \frac{1}{12}$

$f\left(\frac{1}{12}\right) = 6\left(\frac{1}{12}\right)^2 - \left(\frac{1}{12}\right) - 12$

$= \frac{6}{144} - \frac{12}{144} - \frac{1728}{144}$

$= \frac{-1734}{144}$

$= -\frac{289}{24}$

The equation for the axis of symmetry is $x = \frac{1}{12}$, and the coordinates of the vertex are $\left(\frac{1}{12}, -\frac{289}{24}\right)$.

61. $y = 2x^2 + 5x + 2$

$x = -\frac{b}{2a} = -\frac{5}{2(2)} = -\frac{5}{4}$

$f\left(-\frac{5}{4}\right) = 2\left(-\frac{5}{4}\right)^2 + 5\left(-\frac{5}{4}\right) + 2$

$= 2\left(\frac{25}{16}\right) - \frac{25}{4} + 2$

$= \frac{25}{8} - \frac{50}{8} + \frac{16}{8}$

$= -\frac{9}{8}$

The equation for the axis of symmetry is $x = -\frac{5}{4}$, and the coordinates of the vertex are $\left(-\frac{5}{4}, -\frac{9}{8}\right)$.

62. $y = x^2 + 3x - 2$

$x = -\frac{b}{2a} = -\frac{3}{2(1)} = -\frac{3}{2}$

$f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^2 + 3\left(\frac{3}{2}\right) - 2$

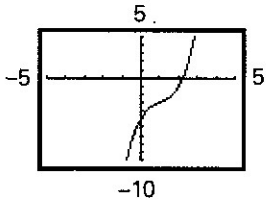
$= \frac{9}{4} - \frac{9}{2} - 2$

$= \frac{9}{4} - \frac{18}{4} - \frac{8}{4}$

$= -\frac{17}{4}$

The equation for the axis of symmetry is $x = -\frac{3}{2}$, and the coordinates of the vertex are $\left(-\frac{3}{2}, -\frac{17}{4}\right)$.

63. $f(x) = x^3 - 3x^2 + 4x - 5$



$f(x)$ is falling on the left and rising on the right.

64. $f(x) = -3(x + 1)^2(x - 1)^3(x - 2)^4$

$f(x)$ has degree 9, which is odd, so the graph will rise at one end and fall at the other end. Because the leading coefficient is negative, it rises on the left and falls on the right.

65.

1	3	0	−4	2	−1
		3	3	−1	1
	3	3	−1	1	0

The quotient is $3x^3 + 3x^2 - x + 1$.

66.

−2	1	4	5	−5	−14
		−2	−4	−2	14
	1	2	1	−7	0

The quotient is $x^3 + 2x^2 + x - 7$.

Optimized using
trial version
www.balesio.com

DND

$= \frac{(x + 2)(x + 3)}{(x + 7)x} \cdot \frac{(x - 2)x}{(x - 2)(x + 2)}$

$= \frac{x + 3}{x + 7}$

68. $\sqrt{x+1} = \sqrt{2x}$

$(\sqrt{x+1})^2 = (\sqrt{2x})^2$

$x + 1 = 2x$

$x = 1$

Check: $\sqrt{1+1} \stackrel{?}{=} \sqrt{2(1)}$

$\sqrt{2} = \sqrt{2}$ True

CHAPTER 7 REVIEW AND ASSESSMENT PAGES 468–470

1. $3a^3 + 11a^2 - 2a + 1$

The polynomial is a cubic polynomial.

2. $8x^5 - 6x^2 + 10x^3$

The polynomial is a quintic trinomial.

3. $-b^2 + 8b - 5b^4 - 3$

The polynomial is a quartic polynomial.

4. $-2x^2 - x^3 + 7x^4$

The polynomial is a quartic trinomial.

5. $-x^3 + 4x^2 - 2$

$= -(2)^3 + 4(2)^2 - 2$

$= -8 + 4(4) - 2$

$= -8 + 16 - 2$

$= 6$

$-x^3 + 4x^2 - 2$

$= -(-1)^3 + 4(-1)^2 - 2$

$= -(-1) + 4 - 2$

$= 1 + 4 - 2$

$= 3$

6. $x^3 + 2x^2 - 1$

$= (2)^3 + 2(2)^2 - 1$

$= 8 + 2(4) - 1$

$= 8 + 8 - 1$

$= 15$

$x^3 + 2x^2 - 1$

$= (-1)^3 + 2(-1)^2 - 1$

$= -1 + 2 - 1$

$= 0$

7. $x^4 - 22$

$= (2)^4 - 22$

$= 16 - 22$

$= -6$

$x^4 - 22$

$= (-1)^4 - 22$

$= 1 - 22$

$= -21$

8. $19 - x^2 - x^3$

$= 19 - (2)^2 - (2)^3$

$= 19 - 4 - 8$

$= 7$

$19 - x^2 - x^3$

$= 19 - (-1)^2 - (-1)^3$

$= 19 - 1 - (-1)$

$= 19 - 1 + 1$

$= 19$

9. $(3x^3 - 5x^2 + 8x + 1) + (11x^3 - x^2 + 2x - 3)$

$= 3x^3 + 11x^3 - 5x^2 - x^2 + 8x + 2x + 1 - 3$

$= 14x^3 - 6x^2 + 10x - 2$

The polynomial is a cubic polynomial.

10. $(7x^3 - 8x^2 + 2x - 3) - (2x^3 + x^2 - 6)$

$= 7x^3 - 2x^3 - 8x^2 - x^2 + 2x - 3 + 6$

$= 5x^3 - 9x^2 + 2x + 3$

The polynomial is a cubic polynomial.

11. $c(a) = -a^2 - 2a + 22$

The leading coefficient is negative and the degree is even, so the graph falls on both ends.

12. $d(x) = -2x^3 + 3x^2 - 7$

The leading coefficient is negative and the degree is odd, so the graph rises on the left and falls on the right.

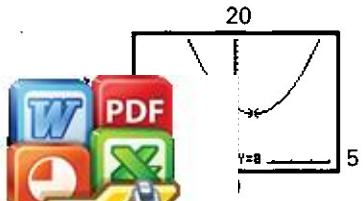
13. $f(x) = x^4 + 7x + 1$

The leading coefficient is positive and the degree is even, so the graph rises on both ends.

14. $f(x) = 4x + x^3 - 6$

The leading coefficient is positive and the degree is odd, so the graph falls on the left and rises on the right.

15. $f(x) = x^2 - 2x + 9$

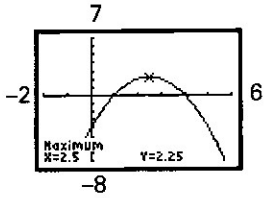


$f(x)$ has a local minimum of

decreasing when $x < 1$ and

hen $x > 1$.

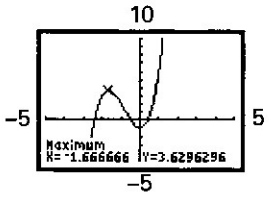
16. $g(x) = -x^2 + 5x - 4$



The graph of $g(x)$ has a local maximum of 2.25 at $x = 2.5$, is increasing when $x < 2.5$ and decreasing when $x > 2.5$.

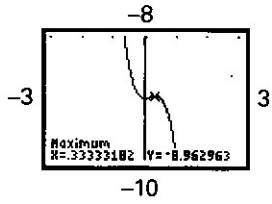
Optimized using
trial version
www.balesio.com

17. $f(a) = 2a^3 + 5a^2 - 1$



The graph of $f(a)$ has a local maximum of about 3.6 at $a = -1\frac{2}{3}$ and a local minimum of -1 at $a = 0$. The graph is increasing when $a < -1\frac{2}{3}$ and when $a > 0$. The graph is decreasing when $-1\frac{2}{3} < a < 0$.

18. $m(x) = -2x^3 + x^2 - 9$



The graph of $m(x)$ has a local minimum of -9 at $x = 0$ and a local maximum of about -8.96 at $x = \frac{1}{3}$. The graph is decreasing when $x < 0$ and $x > \frac{1}{3}$. The graph is increasing when $0 < x < \frac{1}{3}$.

19. $-2x^3(5x^4 - 3x + x^2 - 6 - x^3)$
$= -2x^3(5x^4) - (-2x^3)(3x) + (-2x^3)(x^2) - (-2x^3)(6) - (-2x^3)(x^3)$
$= -10x^7 + 6x^4 - 2x^5 + 12x^3 + 2x^6$
$= -10x^7 + 2x^6 - 2x^5 + 6x^4 + 12x^3$

20. $(x + 4)(x^3 - 7)(x + 1)$
$= (x + 4)(x^4 + x^3 - 7x - 7)$
$= (x + 4)(x^4) + (x + 4)(x^3) + (x + 4)(-7x) + (x + 4)(-7)$
$= x^5 + 4x^4 + x^4 + 4x^3 - 7x^2 - 28x - 7x - 28$
$= x^5 + 5x^4 + 4x^3 - 7x^2 - 35x - 28$

21. $x^3 + 4x^2 - 5x = x(x^2 + 4x - 5)$
$= x(x + 5)(x - 1)$

22. $x^3 - 3x^2 - 10x = x(x^2 - 3x - 10)$
$= x(x - 5)(x + 2)$

23. $x^3 - 125 = (x - 5)(x^2 + 5x + 25)$

24. $27x^3 + 1 = (3x)^3 + (1)^3$
$= (3x + 1)(9x^2 - 3x + 1)$

25. Let $P(x) = x^3 - 7x^2 + 4x + 12$
$P(2) = (2)^3 - 7(2)^2 + 4(2) + 12$
$= 8 - 28 + 8 + 12$
$= 0$
$(x - 2)$ is a factor of $P(x)$.

26. Let $P(x) = x^3 - 5x^2 - 2x + 24$
$P(-2) = (-2)^3 - 5(-2)^2 - 2(-2) + 24$
$= -8 - 20 + 4 + 24$
$= 0$
$(x + 2)$ is a factor of $P(x)$.

27.
$$\begin{array}{r} x^2 + 8x + 15 \\ x - 2\overline{)x^3 + 6x^2 - x - 30} \\ -(x^3 - 2x^2) \\ \hline 8x^2 - x \\ -(8x^2 - 16x) \\ \hline 15x - 30 \\ 15x - 30 \\ \hline 0 \end{array}$$

28.
$$\begin{array}{r} x - 1 \\ x^2 - x - 12\overline{)x^3 - 2x^2 - 11x + 12} \\ -(x^3 - x^2 - 12x) \\ \hline -x^2 + x + 12 \\ -(-x^2 + x + 12) \\ \hline 0 \end{array}$$

29.
$$\begin{array}{r|cccc} 3 & 1 & 3 & -34 & 48 \\ & & 3 & 18 & -48 \\ \hline & 1 & 6 & -16 & 0 \end{array}$$

The quotient is $x^2 + 6x - 16$.

30.
$$\begin{array}{r|cccc} -4 & 1 & 1 & -22 & -40 \\ & & -4 & 12 & 40 \\ \hline & 1 & -3 & -10 & 0 \end{array}$$

The quotient is $x^2 - 3x - 10$.

Optimized using
trial version
www.balesio.com

31. Substitute u for x^2 and solve for u.

$$x^4 - 8x^2 + 16 = 0$$
$$(x^2)^2 - 8x^2 + 16 = 0$$
$$u^2 - 8u + 16 = 0$$
$$(u - 4)^2 = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 4)^2 = 0$$
$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

32. Substitute u for x^2 and solve for u.

$$x^4 - 10x^2 + 24 = 0$$
$$(x^2)^2 - 10x^2 + 24 = 0$$
$$u^2 - 10u + 24 = 0$$
$$(u - 4)(u - 6) = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 4)(x^2 - 6) = 0$$
$$x^2 = 4 \quad \text{or} \quad x^2 = 6$$
$$x = \pm 2 \quad \text{or} \quad x = \pm\sqrt{6}$$

33. Substitute u for x^2 and solve for u.

$$x^4 - 10x^2 + 9 = 0$$
$$(x^2)^2 - 10x^2 + 9 = 0$$
$$u^2 - 10u + 9 = 0$$
$$(u - 1)(u - 9) = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 1)(x^2 - 9) = 0$$
$$x^2 = 1 \quad \text{or} \quad x^2 = 9$$
$$x = \pm 1 \quad \text{or} \quad x = \pm 3$$

34. Substitute u for x^2 and solve for u.

$$x^4 - 13x^2 + 12 = 0$$
$$(x^2)^2 - 13x^2 + 12 = 0$$
$$u^2 - 13u + 12 = 0$$
$$(u - 1)(u - 12) = 0$$

Replace u with x^2 and solve for x.

$$(x^2 - 1)(x^2 - 12) = 0$$
$$x^2 = 1 \quad \text{or} \quad x^2 = 12$$
$$x = \pm 1 \quad \text{or} \quad x = \pm\sqrt{12} = \pm 2\sqrt{3}$$

35.

$$x^3 - x^2 - 6x + 6 = 0$$
$$x^2(x - 1) - 6(x - 1) = 0$$
$$(x^2 - 6)(x - 1) = 0$$
$$x^2 = 6 \quad \text{or} \quad x = 1$$
$$x = \pm\sqrt{6} \quad \text{or} \quad x = 1$$

36.

$$x^5 - x^3 - 8x^2 + 8 = 0$$
$$x^3(x^2 - 1) - 8(x^2 - 1) = 0$$
$$(x^3 - 8)(x^2 - 1) = 0$$
$$x^3 = 8 \quad \text{or} \quad x^2 = 1$$
$$x = 2 \quad \text{or} \quad x = \pm 1$$

37.

$$x^4 + 2x^3 - x - 2 = 0$$
$$x^3(x + 2) - (x + 2) = 0$$
$$(x^3 - 1)(x + 2) = 0$$
$$x^3 = 1 \quad \text{or} \quad x = -2$$
$$x = 1 \quad \text{or} \quad x = -2$$

38.

$$x^5 - 9x^3 - x^2 + 9 = 0$$
$$x^3(x^2 - 9) - (x^2 - 9) = 0$$
$$(x^3 - 1)(x^2 - 9) = 0$$
$$x^3 = 1 \quad \text{or} \quad x^2 = 9$$
$$x = 1 \quad \text{or} \quad x = \pm 3$$

39.

-3	1	7	16	12
		-3	-12	-12
	1	4	4	0

$$x^3 + 7x^2 + 16x + 12 = 0$$
$$(x + 3)(x^2 + 4x + 4) = 0$$
$$(x + 3)(x + 2)^2 = 0$$

The remaining root is -2.

40.

-3	1	3	-16	-48
		-3	0	48
	1	0	-16	0

$$x^3 + 3x^2 - 16x - 48 = 0$$
$$(x + 3)(x^2 - 16) = 0$$
$$(x + 3)(x + 4)(x - 4) = 0$$

The remaning roots are -4 and 4.

41.

4	1	-11	38	-40
		4	-28	40
	1	-7	10	0

$$x^3 - 11x^2 + 38x - 40 = 0$$
$$(x - 4)(x^2 - 7x + 10) = 0$$
$$(x - 4)(x - 2)(x - 5) = 0$$

The remaining roots are 2 and 5.

42.

6	1	-6	-1	6
		6	0	-6
	1	0	-1	0

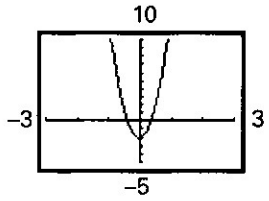
$$x^3 - 6x^2 - x + 6 = 0$$
$$(x - 6)(x^2 - 1) = 0$$
$$(x - 6)(x + 1)(x - 1) = 0$$

The remaining roots are -1 and 1.

Optimized using
trial version
www.balesio.com

43. $15x^2 + x - 2 = 0$

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2,$ and $q = \pm 1, \pm 3, \pm 5, \pm 15$. Use a graph to determine the roots from these possibilities.



From the graph, there are roots between -1 and 0 and between 0 and 1. Use synthetic division to test possible roots that are between these values.

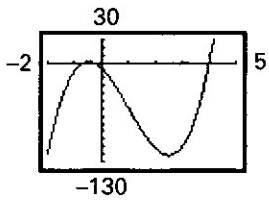
$-\frac{2}{5}$	15	1	-2
		-6	2
	15	-5	0

$\frac{1}{3}$	15	1	-2
		5	2
	15	6	0

The rational roots are $-\frac{2}{5}$ and $\frac{1}{3}$.

44. $9x^3 - 27x^2 - 34x - 8 = 0$

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2,$ $\pm 4, \pm 8,$ and $q = \pm 1, \pm 3, \pm 9$. Use a graph to determine the roots from these possibilities.



From the graph, there is a root at 4. There are also two roots between -1 and 0. Use synthetic division to test the possible roots that are between these values.

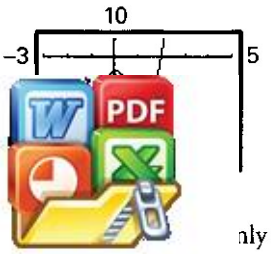
$-\frac{2}{3}$	9	-27	-34	-8
		-6	22	8
	9	-33	-12	0

$-\frac{1}{3}$	9	-27	-34	-8
		-3	10	8
	9	-30	-24	0

The rational roots are $-\frac{2}{3}$, $-\frac{1}{3}$, and 4.

45. $9x^3 - 18x^2 + 2x - 4 = 0$

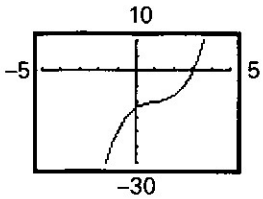
Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2, \pm 4$ and $q = \pm 1, \pm 3, \pm 9$. Use a graph to determine the roots from these possibilities.



nly root is at 2.

46. $x^3 - 3x^2 + 4x - 12 = 0$

Possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ and $q = \pm 1$. Use a graph to determine the roots from these possibilities.



From the graph, the only root is at 3.

Optimized using
trial version
www.balesio.com

47. $P(x) = a(x-2)(x-(-1))^2(x-3)$
$6 = a(0-2)(0+1)^2(0-3)$
$6 = a(-2)(1)(-3)$
$6 = 6a$
$a = 1$
$P(x) = (x-2)(x+1)^2(x-3)$
$= x^4 - 3x^3 - 3x^2 + 7x + 6$

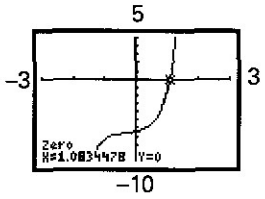
48. $P(x) = a(x-(-2))^3(x-1)$
$-16 = a(0+2)^3(0-1)$
$-16 = a(8)(-1)$
$-16 = -8a$
$a = 2$
$P(x) = 2(x+2)^3(x-1)$
$= 2x^4 + 10x^3 + 12x^2 - 8x - 16$

49. $P(x) = a(x-3)^2(x-(2+i))(x-(2-i))$
$-45 = a(0-3)^2(0-(2+i))(0-(2-i))$
$-45 = a(9)(-2-i)(-2+i)$
$-45 = a(9)(5)$
$-45 = 45a$
$a = -1$
$P(x) = -(x-3)^2(x-(2+i))(x-(2-i))$
$= -x^4 + 10x^3 - 38x^2 + 66x - 45$

50. $P(x) = a(x-5)(x-(-7))\left(x-\left(-\frac{2}{3}i\right)\right)\left(x-\frac{2}{3}i\right)$
$-140 = a(0-5)(0+7)\left(0+\frac{2}{3}i\right)\left(0-\frac{2}{3}i\right)$
$-140 = a(-5)(7)\left(\frac{4}{9}\right)$
$-140 = \frac{-140}{9}a$
$a = 9$
$P(x) = 9(x-5)(x+7)\left(x+\frac{2}{3}i\right)\left(x-\frac{2}{3}i\right)$
$= 9x^4 + 18x^3 - 311x^2 + 8x - 140$

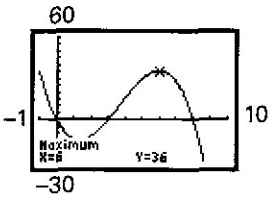
51. $500x^5 + 500x^4 + 500x^3 + 500x^2 + 500x = 3200$
$500x^5 + 500x^4 + 500x^3 + 500x^2 + 500x - 3200 = 0$
$500\left(x^5 + x^4 + x^3 + x^2 + x - \frac{32}{5}\right) = 0$
$x^5 + x^4 + x^3 + x^2 + x - \frac{32}{5} = 0$

Use a graph to approximate the zeros of the related function.



From the graph, there is a zero at approximately 1.083.
Thus, the effective yield is 8.3%.

52. $V = x(8-x)(x-3)$
$V = x[8(x-3) - x(x-3)]$
$V = x(8x - 24 - x^2 + 3x)$
$V = x(-x^2 + 11x - 24)$
$V = -x^3 + 11x^2 - 24x$



From the graph, the volume is a maximum of 36 cubic feet when $x = 6$ feet.
The crate has dimensions 6 feet × 2 feet × 3 feet.

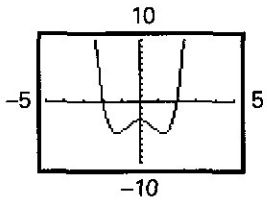
Optimized using
trial version
www.balesio.com

CHAPTER 7 ALTERNATIVE ASSESSMENT PAGE 471

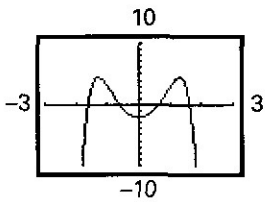
PERFORMANCE ASSESSMENT

1. If n is even and the leading coefficient is positive, the graph rises on both sides. If the leading coefficient is negative, the graph falls on both sides.
If n is odd and the leading coefficient is positive, the graph falls on the left and rises on the right. If the leading coefficient is negative, the graph rises on the left and falls on the right.
Examples: Answers may vary. Sample answer:

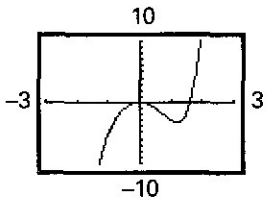
$y = x^4 - 3x^2 - 3$



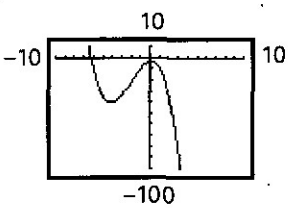
$y = -x^6 + x^4 + 5x^2 - 2$



$y = x^5 - 4x^2$

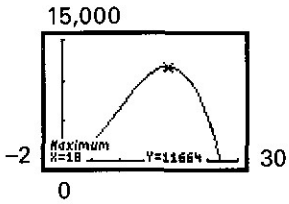


$y = -x^3 - 6x^2 + x - 4$



2. a. $V(x) = x(x)(108 - 4x)$
$V(x) = -4x^2 - 108x$

b.



The maximum volume is 11,664 cubic inches, when the length x is 18 inches.

c. $V(24) = 108(24)^2 - 4(24)^3$
$= 62{,}208 - 55{,}296$
$= 6912$ cubic inches

d. Answers may vary.
$V(x) = x(2x)(108 - 2(x + 2x))$
$V(x) = 2x^2(108 - 6x)$
$V(x) = 216x^2 - 12x^3$

3. $x - 2$ is not a factor of $x^3 + x^2 - 8x - 15$.
Method 1: Let $P(x) = x^3 + x^2 - 8x - 15$.
$P(2) = (2)^3 + (2)^2 - 8(2) - 15$
$= 8 + 4 - 16 - 15$
$= -19$

If $(x - 2)$ were a factor of $x^3 + x^2 - 8x - 15$, $P(2)$ would equal 0.

Method 2:

2	1	1	−8	−15
		2	6	−4
	1	3	−2	−19

Optimized using
trial version
www.balesio.com

$\frac{2}{15}$

15

$4)$

-19

If $(x - 2)$ were a factor of $x^3 + x^2 - 8x - 15$, the remainder of the synthetic and long divison methods would equal 0.

4. The function must also have $3 + 2i$ as a zero.

$P(x) = (x - (-5))(x - (3 - 2i))(x - (3 + 2i))$

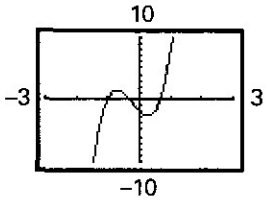
$P(x) = (x + 5)(x - 3 + 2i)(x - 3 - 2i)$

$P(x) = (x + 5)(x^2 - 6x + 13)$

$P(x) = x^3 - x^2 - 17x + 65$

5. $f(x) = 9x^3 + 6x^2 - 5x - 2$

Possible rational zeros are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2$ and $q = \pm 1, \pm 3, \pm 9$: $\pm\frac{1}{1}, \pm\frac{1}{3}, \pm\frac{1}{9}, \pm\frac{2}{1}, \pm\frac{2}{3}, \pm\frac{2}{9}$. Use a graph to determine the zeros from these possibilities.



From the graph, there is a zero at -1. There are also zeros between -1 and 0, and between 0 and 1. Use synthetic division to test the possible zeros that are between these values.

$-\frac{1}{3}$	9	6	-5	-2
		-3	-1	2
	9	3	-6	0

$\frac{2}{3}$	9	6	-5	-2
		6	8	2
	9	12	3	0

The actual zeros are -1, $-\frac{1}{3}$, and $\frac{2}{3}$.

PORTFOLIO PROJECTS

1. a. n must be an integer greater than or equal to 1. Fractional values mean parts of balls, which does not apply.

b. Tables may vary. Sample Table:

Number of balls on a side, n	Total number of balls in pyramid, $p(n)$
3	$\frac{3}{6}(3 + 1)(3 + 2) = \frac{60}{6} = 10$
4	$\frac{4}{6}(4 + 1)(4 + 2) = \frac{120}{6} = 20$
5	$\frac{5}{6}(5 + 1)(5 + 2) = \frac{210}{6} = 35$
6	$\frac{6}{6}(6 + 1)(6 + 2) = \frac{336}{6} = 56$
7	$\frac{7}{6}(7 + 1)(7 + 2) = \frac{504}{6} = 84$

Optimized using
trial version
www.balesio.com

c.

n	1	2	3	4	5	6	7
$p(n)$	1	4	10	20	35	56	84
First Differences		3	6	10	15	21	28
Second Differences			3	4	5	6	7

Each time the bottom layer of a pyramid has the side length increased by 1, the number of balls added is increased by the new number of balls on a side of the bottom layer. For example, 3 balls are added when the bottom layer goes from 1 ball to 2 balls per side, $6 = 3 + 3$ balls are added to go from 2 balls to 3 balls per side, and $10 = 6 + 4$ balls are added to go from 3 balls to 4 balls per side.

2. a. Answers will vary.

b. The formula has three variables: h, R, and r.
Determine a relationship between the variables and use substitution to rewrite the formula in 1 variable.

c. Answers will vary. Sample answer: If the height was increased, reducing R and r could maintain the same volume, and vice versa.

CHAPTERS 1–7 CUMULATIVE ASSESSMENT PAGES 472–473

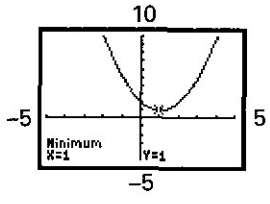
1. The degree of $x^4 + 2x^3 + x + 1$ is 4.
The degree of $3x^2 + 2x + 1$ is 2.
Choose A.

2. $\ln\left(\frac{1}{2}\right) < 0$
$\ln(2) > 0$
Choose B.

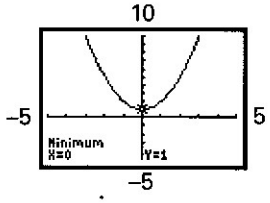
3. $e^{\ln 2} = 2$
Choose C.

4.
$$x^3 - 4x = 0$$
$$x(x^2 - 4) = 0$$
$$x(x-2)(x+2) = 0$$
$x = 0$ or $x = 2$ or $x = -2$
$$x^3 - 5x^2 + 6x = 0$$
$$x(x^2 - 5x + 6) = 0$$
$$x(x-6)(x+1) = 0$$
$x = 0$ or $x = 6$ or $x = -1$
Choose C.

$f(x) = (x-1)^2 + 1$



5. $f(x) = x^2 + 1$



Choose C.

6. a. $-5 \geq -1$ False
$(1, -5)$ is not a solution for the system

b. $5 \geq 0$ True
$5 \geq 3(0) + 2$
$5 \geq 2$ True
$(0, 5)$ is a solution of the system.
Choose b.

8.
$$(y - y_1) = m(x - x_1)$$
$$y - (-3) = -\frac{1}{2}(x - 1)$$
$$2(y+3) = -x + 1$$
$$2y = -x - 5$$
Choose b.

Optimized using trial version www.balesio.com

9. $\begin{cases} x + 2y = 4 \\ 2x + y = 5 \end{cases}$

Solve the first equation for x and substitute into the second equation.

$x + 2y = 4$

$x = -2y + 4$

$2(-2y + 4) + y = 5$

$-4y + 8 + y = 5$

$-3y = -3$

$y = 1$

Substitute $y = 1$ into the first equation.

$x + 2(1) = 4$

$x = 2$

Solution: (2, 1)

Choose b.

10. $\{(2, 1), (4, -1), (6, 2)\}$ is the only set that represents a function, since each element of the domain is paired with exactly one element of the range.

Choose b.

11. Rewrite each equation in standard form

$a(x) = 3x^2 + 5x$

$b(x) = 7x + 5x - 3x^2 = -3x^2 + 12x$

$c(x) = 3 + 5x + \frac{1}{3}x^2 = \frac{1}{3}x^2 + 5x + 3$

$d(x) = \frac{1}{3}x^2$

$b(x) = -3x^2 + 12x$ is the only one of the quadratics with a negative leading coefficient, so it is the only equation in which the vertex gives a maximum value.

Choose b.

12. A consistent system of linear equations has at least one solution. (It may have exactly one solution or infinitely many solutions.)

Choose c.

13. $x - 5y = 15$

$-5y = -x + 15$

$y = \frac{1}{5}x - 3$

The y-intercept of the graph is -3.

Choose d.

14. $4x + 2 < 2x + 1$

$2x + 2 < 1$

$2x < -1$

$x < -\frac{1}{2}$

Choose d.

15. $|x| \leq 5$

$x \leq 5$ and $x \geq -5$

$-5 \leq x \leq 5$

Choose a.

16. $x^2 - 5x + 6 = (x - 2)(x - 3)$

Choose a.

17. $(x + 1)(x + 2)(x - 4) = (x^2 + 3x + 2)(x - 4)$

$= x^3 - x^2 - 10x - 8$

18. $\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 + 6 & 2 + 9 \\ 4 + 6 & 4 + 9 \end{bmatrix} = \begin{bmatrix} 8 & 11 \\ 10 & 13 \end{bmatrix}$

19. $(f + g)(x) = f(x) + g(x)$

$= (3x + 5) + (x^2 + x + 5)$

$= x^2 + 4x + 10$

20. $\ln x = 5$

$e^{\ln x} = e^5$

$x = e^5 \approx 148.41$

21. $f(x) = 2x^2 - 8x + 9$

$= 2(x^2 - 4x) + 9$

$- 4x + 4) + 9 - 8$

$- 2)^2 + 1$

22. $(2x^3 + 3x^2 + 1) + (5x^2 - 2x + 2)$

$= 2x^3 + 3x^2 + 5x^2 - 2x + 1 + 2$

$= 2x^3 + 8x^2 - 2x + 3$

$- x) - (x^3 + 2x^2 - 1) = 5x^3 - x^3 + 4x^2 - 2x^2 - x + 1$

$= 4x^3 + 2x^2 - x + 1$

Optimized using
trial version
www.balesio.com

24. $f(x) = 3x + 2$

$y = 3x + 2$

$x = 3y + 2$

$x - 2 = 3y$

$y = \frac{1}{3}x - \frac{2}{3}$

$f^{-1}(x) = \frac{1}{3}x - \frac{2}{3}$

25. $3x^2 + 5x - 2 = 0$

According to the Rational Root Theorem, the possible rational roots are all the quotients $\frac{p}{q}$ that can be formed from $p = \pm 1, \pm 2$ and $q = \pm 1, \pm 3$. Testing the possibilities, you can determine that -2 and $\frac{1}{3}$ are the rational roots:

$3(-2)^2 + 5(-2) - 2 = 0$

$12 - 10 - 2 = 0$

$0 = 0$

and

$3\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) - 2 = 0$

$\frac{1}{3} + \frac{5}{3} - \frac{6}{3} = 0$

$0 = 0$

26. $(4 - 2i)(4 + 2i) = 4(4 + 2i) - 2i(4 + 2i)$

$= 16 + 8i - 8i - 4i^2$

$= 16 + 4$

$= 20$

27. $-3y^2 - 5y = -y(3y + 5)$

28. $x^2 - 5x - 36 = (x + 4)(x - 9)$

29. $24x^2 + 5x - 36 = (8x - 9)(3x + 4)$

30. $36x^2 - 46x - 12 = 2(18x^2 - 23x - 6)$

$= 2(9x + 2)(2x - 3)$

31. $\lceil 3.1 \rceil + [2.5]$

$= 4 + 2$

$= 6$

32. $\log_5 25 = 2$

33. $\ln x + \ln (2 - x) = 0$

$\ln (2x - x^2) = 0$

$2x - x^2 = e^0$

$-x^2 + 2x - 1 = 0$

$-(x - 1)^2 = 0$

$x = 1$

34. $5[3 - (3 - 2)]^2$

$= 5(3 - 1)^2$

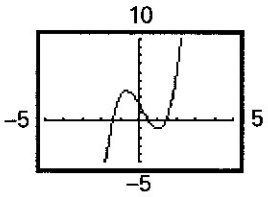
$= 5(4)$

$= 20$

35. $f(x) = 2x^3 - x^2 - 4x + 2$

Possible rational zeros are $\pm 1, \pm 2, \pm\frac{1}{2}$.

Use the graph of $f(x)$ to determine the roots.



From the graph, the only possible rational root is $\frac{1}{2}$.

36. $\frac{x + 2}{2} = \frac{2x}{3}$

$3(x + 2) = 2(2x)$

$3x + 6 = 4x$

$x = 6$

37. $|-2.5| - |3.2| = 2.5 - 3.2$

$= -0.7$

38. $-3x + 4 = 5 - x$

$-1 = 2x$

$-\frac{1}{2} = x$

39. $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

$A(5) = 1000\left(1 + \frac{0.08}{1}\right)^{(1)(5)}$

$\approx \$1469.33$

Optimized using
trial version
www.balesio.com

$.00012t$

$.00012t$

$012t$

$.00012t)$

$012t$

$\frac{0)}{012} \approx 4257$

roximately 4257 years old.

41. $A = Pe^{rt}$

$= 1000e^{(0.08)(5)}$

$\approx \$1491.82$