

Find the value of each limit. For a limit that does not exist, state why.

$$1. \lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\cos^2 \theta}{1 - \sin \theta} = \frac{1 - \sin^2 \theta}{1 - \sin \theta}$$

$$\frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 - \sin \theta)}$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}} 1 + \sin \theta = 1 + 1 = \boxed{2}$$

$$2. \lim_{x \rightarrow 0} \frac{x + \sin x}{x} = \frac{x}{x} + \frac{\sin x}{x}$$

$$\lim_{x \rightarrow 0} 1 + \frac{\sin x}{x} = 1 + 1 = \boxed{2}$$

$$3. \lim_{x \rightarrow 3} \begin{cases} 2x^2 - 3x, & x < 3 \\ 8 - \cos\left(\frac{\pi x}{3}\right), & x > 3 \end{cases}$$

$$\lim_{x \rightarrow 3^-} = 2(9) - 9 = 9$$

$$\lim_{x \rightarrow 3^+} = 8 - \cos\left(\frac{3\pi}{3}\right) = 9$$

$$\boxed{9}$$

$$4. \lim_{\theta \rightarrow 0} \frac{2 \sin 3\theta}{\theta}$$

$$\frac{3(2 \sin 3\theta)}{3\theta}$$

$$\lim_{\theta \rightarrow 0} 6 \left(\frac{\sin 3\theta}{3\theta}\right) = \boxed{6}$$

$$5. \lim_{x \rightarrow 0} \frac{\sin x}{2x^2 - x} = \frac{\sin x}{(2x - 1)x}$$

$$\lim_{x \rightarrow 0} \frac{1}{2x - 1} \cdot \frac{\sin x}{x}$$

$$= -1 \cdot 1 = \boxed{-1}$$

$$6. \lim_{x \rightarrow 0} \frac{5x + \sin 3x}{x}$$

$$\lim_{x \rightarrow 0} \frac{5x}{x} + \frac{\sin 3x}{x}$$

$$\lim_{x \rightarrow 0} 5 + \frac{3 \sin 3x}{3x}$$

$$= 5 + 3 = \boxed{8}$$

$$7. \lim_{x \rightarrow 0} \frac{\sin 2x}{6x}$$

$$\lim_{x \rightarrow 0} \frac{1}{3} \cdot \frac{\sin 2x}{2x}$$

$$= \frac{1}{3} \cdot 1 = \boxed{\frac{1}{3}}$$

$$8. \lim_{x \rightarrow 0} \frac{2 \sin 4x}{3x}$$

$$\lim_{x \rightarrow 0} \frac{2}{3} \cdot \frac{4 \sin 4x}{4x}$$

$$= \frac{2}{3} \cdot 4 = \boxed{\frac{8}{3}}$$

$$9. \lim_{\theta \rightarrow 0} \frac{\cos \theta \tan \theta}{3\theta}$$

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta \left( \frac{\sin \theta}{\cos \theta} \right)}{3\theta}$$

$$= \lim_{\theta \rightarrow 0} \frac{\sin \theta}{3\theta} = \boxed{\frac{1}{3}}$$

$$10. \lim_{\theta \rightarrow 0} \frac{3-3\cos \theta}{\theta}$$

$$\lim_{\theta \rightarrow 0} \frac{3(1-\cos \theta)}{\theta} = 3(0)$$

$$= \boxed{0}$$

$$11. \lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\cos \theta}{\cot \theta}$$

$$\frac{\cos \theta}{\frac{\cos \theta}{\sin \theta}} = \frac{\cos \theta \sin \theta}{\cos \theta}$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}} \sin \theta = \boxed{1}$$

$$12. \lim_{\theta \rightarrow 0} \frac{1-\tan \theta}{\sin \theta - \cos \theta}$$

$$\frac{1-0}{0-1} = \boxed{-1}$$