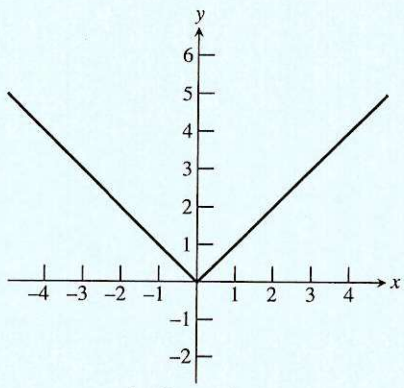
EXAMPLE 10 **Analyzing a Basic Function**

Give a complete analysis of the properties of the function $f(x) = |x|$.

FUNCTION SPOTLIGHT — The Absolute Value Function



$f(x) = |x|$

- Domain: $(-\infty, \infty)$
- Range: $[0, \infty)$
- Continuous on $(-\infty, \infty)$
- Decreasing on $(-\infty, 0]$; increasing on $[0, \infty)$
- Concave up on no interval; concave down on no interval
- No points of inflection
- Symmetric with respect to the y-axis (an even function)
- Bounded below but not above
- Absolute minimum of 0 at $x = 0$
- No horizontal asymptotes
- No vertical asymptotes
- End behavior: $\lim_{x\to-\infty} |x| = \infty$ and $\lim_{x\to\infty} |x| = \infty$

Now try Exercise 63.

GROUP ACTIVITY Modeling with Function Composition

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1. Your favorite store is having a 20% off clothing sale and will give you \$10 off purchases of \$100. (Remember that if you live in an area that has a sales tax on clothes, you must include that in your calculations.) You decide to buy a pair of pants that cost \$39.99.
   - **(a)** In what order would you apply the discounts and sales tax?
   - **(b)** Write a function for each step of the process. Use the notation $f(x)$ and $g(x)$.
   - **(c)** State the composition of functions that aligns with the order you chose in part 1a.
   - **(d)** Solve the composition of functions. Label your answer and round according to the context of the situation.
2. You really want that \$10 off \$100 discount, so you decide to buy more clothes. Research the cost of an item or items that would bring your subtotal to \$100 for your clothes.
   - **(a)** What items did you buy, and what was your subtotal?
   - **(b)** Write a function for each step of the process. Use the notation $f(x)$ and $g(x)$.
   - **(c)** State the composition of functions that relates to your problem.
   - **(d)** Solve the composition of functions. Label your answer and round according to the context of the situation.

SECTION 1.4 Exercises

Exercise numbers with a red underline indicate problems that the authors have designed to be solved *without a calculator*.

Quick Review

In Exercises 1–10, find the domain of the function and express it in interval notation.

1. $f(x) = \dfrac{x-2}{x+3}$
2. $g(x) = \ln(x-1)$
3. $f(t) = \sqrt{5-t}$
4. $g(x) = \dfrac{3}{\sqrt{2x-1}}$
5. $f(x) = \sqrt{\ln(x)}$
6. $h(x) = \sqrt{1-x^2}$
7. $f(t) = \dfrac{t+5}{t^2+1}$
8. $g(t) = \ln(|t|)$
9. $f(x) = \dfrac{1}{\sqrt{1-x^2}}$
10. $g(x) = 2$

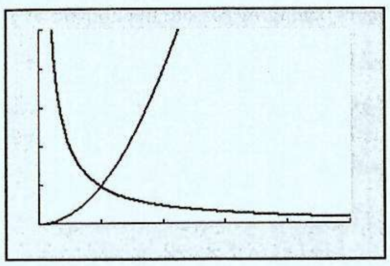
Section Exercises

In Exercises 11–14, find formulas for the functions $f + g$, $f - g$, and fg. Give the domain of each.

11. $f(x) = 2x - 1$; $g(x) = x^2$
12. $f(x) = (x - 1)^2$; $g(x) = 3 - x$
13. $f(x) = \sqrt{x}$; $g(x) = \sin x$
14. $f(x) = \sqrt{x + 5}$; $g(x) = |x + 3|$

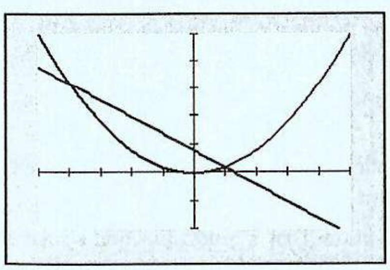
In Exercises 15–18, find formulas for f/g and g/f. Give the domain of each.

15. $f(x) = \sqrt{x + 3}$; $g(x) = x^2$
16. $f(x) = \sqrt{x - 2}$; $g(x) = \sqrt{x + 4}$
17. $f(x) = x^2$; $g(x) = \sqrt{1 - x^2}$
18. $f(x) = x^3$; $g(x) = \sqrt[3]{1 - x^3}$
19. $f(x) = x^2$ and $g(x) = 1/x$ are shown below in the viewing window $[0, 5]$ by $[0, 5]$. Sketch the graph of the sum $(f + g)(x)$ by adding the y-coordinates directly from the graphs. Then graph the sum on your calculator and see how close you came.



[0, 5] by [0, 5]

20. The graphs of $f(x) = x^2$ and $g(x) = 4 - 3x$ are shown in the viewing window $[-5, 5]$ by $[-10, 25]$. Sketch the graph of the difference $(f - g)(x)$ by subtracting the y-coordinates directly from the graphs. Then graph the difference on your calculator and see how close you came.



[–5, 5] by [–10, 25]

In Exercises 21–24, find $(f \circ g)(3)$ and $(g \circ f)(-2)$.

21. $f(x) = 2x - 3$; $g(x) = x + 1$
22. $f(x) = x^2 - 1$; $g(x) = 2x - 3$
23. $f(x) = x^2 + 4$; $g(x) = \sqrt{x + 1}$
24. $f(x) = \dfrac{x}{x + 1}$; $g(x) = 9 - x^2$

In Exercises 25–32, find $f(g(x))$ and $g(f(x))$. State the domain of each.

25. $f(x) = 3x + 2$; $g(x) = x - 1$
26. $f(x) = x^2 - 1$; $g(x) = \dfrac{1}{x - 1}$
27. $f(x) = x^2 - 2$; $g(x) = \sqrt{x + 1}$
28. $f(x) = \dfrac{1}{x - 1}$; $g(x) = \sqrt{x}$
29. $f(x) = x^2$; $g(x) = \sqrt{1 - x^2}$
30. $f(x) = x^3$; $g(x) = \sqrt[3]{1 - x^3}$
31. $f(x) = \dfrac{1}{2x}$; $g(x) = \dfrac{1}{3x}$
32. $f(x) = \dfrac{1}{x + 1}$; $g(x) = \dfrac{1}{x - 1}$

In Exercises 33–40, find $f(x)$ and $g(x)$ so that the function can be described as $y = f(g(x))$. (There may be more than one possible decomposition.)

33. $y = \sqrt{x^2 - 5x}$
34. $y = (x^3 + 1)^2$
35. $y = |3x - 2|$
36. $y = \dfrac{1}{x^3 - 5x + 3}$
37. $y = (x - 3)^5 + 2$
38. $y = e^{\sin x}$
39. $y = \cos(\sqrt{x})$
40. $y = (\tan x)^2 + 1$

41. **Weather Balloons** A high-altitude spherical weather balloon expands as it rises due to the drop in atmospheric pressure. Suppose that the radius r increases at the rate of 0.03 in./sec and that $r = 48$ in. at time $t = 0$. Determine an equation that models the volume V of the balloon at time t and find the volume when $t = 300$ sec.

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42. **A Snowball's Chance** Jake stores a small cache of 4-inch-diameter snowballs in the basement freezer, unaware that the freezer's self-defrosting feature will cause each snowball to lose about 1 cubic inch of volume every 40 days. He remembers them a year later (call it 360 days) and goes to retrieve them. What is their diameter then?
43. **Satellite Photography** A satellite camera takes a rectangle-shaped picture. The smallest region that can be photographed is a 5-km by 7-km rectangle. As the camera zooms out, the length l and width w of the rectangle increase at a rate of 2 km/sec. How long does it take for the area A to be at least 5 times its original size?
44. **Computer Imaging** New Age Special Effects, Inc., prepares computer software based on specifications prepared by film directors. To simulate an approaching vehicle, they begin with a computer image of a 5-cm by 7-cm by 3-cm box. The program increases each dimension at a rate of 2 cm/sec. How long does it take for the volume V of the box to be at least 5 times its initial size?
45. Which of the ordered pairs $(1, 1)$, $(4, -2)$, and $(3, -1)$ is or are in the relation given by $3x + 4y = 5$?
46. Which of the ordered pairs $(5, 1)$, $(3, 4)$, and $(0, -5)$ is or are in the relation given by $x^2 + y^2 = 25$?

In Exercises 47–54, find two functions defined implicitly by the given relation.

47. $x^2 + y^2 = 25$
48. $x + y^2 = 25$
49. $x^2 - y^2 = 25$
50. $3x^2 - y^2 = 25$
51. $x + |y| = 1$
52. $x - |y| = 1$
53. $y^2 = x^2$
54. $y^2 = x$

In Exercises 55–62, sketch the graph of the piecewise-defined function. (Try doing it without a calculator.) In each case, give any points of discontinuity.

55. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$

56. $g(x) = \begin{cases} x^3 & \text{if } x \le 0 \\ e^x & \text{if } x > 0 \end{cases}$

57. $h(x) = \begin{cases} |x| & \text{if } x < 0 \\ \sin x & \text{if } x \ge 0 \end{cases}$

58. $w(x) = \begin{cases} 1/x & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \ge 0 \end{cases}$

59. $f(x) = \begin{cases} \cos x & \text{if } x \le 0 \\ e^x & \text{if } x > 0 \end{cases}$

60. $g(x) = \begin{cases} |x| & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

61. $f(x) = \begin{cases} -3 - x & \text{if } x \le 0 \\ 1 & \text{if } 0 < x < 1 \\ x^2 & \text{if } x \ge 1 \end{cases}$

62. $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ |x| & \text{if } -1 \le x < 1 \\ \text{int}(x) & \text{if } x \ge 1 \end{cases}$

63. Three on a Match Match each function f with a function g and a domain D so that $(f \circ g)(x) = x^2$ with domain D.

f	g	D
e^x	$\sqrt{2 - x}$	$x \ne 0$
$(x^2 + 2)^2$	$x + 1$	$x \ne 1$
$(x^2 - 2)^2$	$2 \ln x$	$(0, \infty]$
$\dfrac{1}{(x - 1)^2}$	$\dfrac{1}{x - 1}$	$[2, \infty)$
$x^2 - 2x + 1$	$\sqrt{x - 2}$	$(-\infty, 2]$
$\left(\dfrac{x + 1}{x}\right)^2$	$\dfrac{x + 1}{x}$	$(-\infty, \infty)$

64. Be a g Whiz Let $f(x) = x^2 + 1$. Find a function g so that

(a) $(fg)(x) = x^4 - 1$ **(b)** $(f + g)(x) = 3x^2$

(c) $(f/g)(x) = 1$ **(d)** $f(g(x)) = 9x^4 + 1$

(e) $g(f(x)) = 9x^4 + 1$

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A graphing calculator is permitted for solving the following problems.

65. Multiple Choice Suppose f and g are functions with domain all real numbers. Which of the following statements is *not* necessarily true?

(A) $(f + g)(x) = (g + f)(x)$

(B) $(fg)(x) = (gf)(x)$

(C) $f(g(x)) = g(f(x))$

(D) $(f - g)(x) = -(g - f)(x)$

66. Multiple Choice If $f(x) = x - 7$ and $g(x) = \sqrt{4 - x}$, what is the domain of the function f/g?

(A) $(-\infty, 4)$ (B) $(-\infty, 4]$

(C) $(4, \infty)$ (D) $[4, \infty)$

67. Multiple Choice If $f(x) = x^2 + 1$, then $(f \circ f)(x) =$

(A) $2x^2 + 2$ (B) $2x^2 + 1$

(C) $x^4 + 1$ (D) $x^4 + 2x^2 + 2$

68. Multiple Choice Which of the following relations defines the function $y = |x|$ implicitly?

(A) $y = x$ (B) $y^2 = x^2$

(C) $x^2 + y^2 = 1$ (D) $x = |y|$

69. Answer and Explain Is the domain of the quotient function $\dfrac{f}{g}$ the set of all numbers belonging to both the domain of f and the domain of g? Justify your answer.

70. Answer and Explain Suppose the function f is increasing on $(-\infty, \infty)$, the function g is decreasing on $(-\infty, \infty)$, and $f(2) = g(2)$. Define the function h piecewise by

$$h(x) = \begin{cases} f(x) & \text{if } x \le 2 \\ g(x) & \text{if } x > 2. \end{cases}$$

Does the function h have an absolute maximum at $x = 2$? Justify your answer.

Extending the Ideas

71. Identifying Identities An *identity* for a function operation is a function that combines with a given function f to return the same function f. Find the identity functions for the following operations:

(a) Function addition. That is, find a function g such that $(f + g)(x) = (g + f)(x) = f(x)$.

(b) Function multiplication. That is, find a function g such that $(fg)(x) = (gf)(x) = f(x)$.

(c) Function composition. That is, find a function g such that $(f \circ g)(x) = (g \circ f)(x) = f(x)$.

72. Is Function Composition Associative? You already know that function composition is not commutative; that is, $(f \circ g)(x) \ne (g \circ f)(x)$. But is function composition associative? That is, does $(f \circ (g \circ h))(x) = ((f \circ g) \circ h)(x)$? Explain your answer.

73. Revisiting Example 6 Solve $x^2y + y^2 = 5$ for y using the quadratic formula and graph the pair of implicit functions.