Algebra

Practice Problems

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Table of Contents

Preface

First, here's a little bit of history on how this material was created (there's a reason for this, I promise). A long time ago (2002 or so) when I decided I wanted to put some mathematics stuff on the web I wanted a format for the source documents that could produce both a pdf version as well as a web version of the material. After some investigation I decided to use MS Word and MathType as the easiest/quickest method for doing that. The result was a pretty ugly HTML (*i.e* web page code) and had the drawback of the mathematics were images which made editing the mathematics painful. However, it was the quickest way or dealing with this stuff.

Fast forward a few years (don't recall how many at this point) and the web had matured enough that it was now much easier to write mathematics in LaTeX (https://en.wikipedia.org/wiki/LaTeX) and have it display on the web (LaTeX was my first choice for writing the source documents). So, I found a tool that could convert the MS Word mathematics in the source documents to LaTeX. It wasn't perfect and I had to write some custom rules to help with the conversion but it was able to do it without "messing" with the mathematics and so I didn't need to worry about any math errors being introduced in the conversion process. The only problem with the tool is that all it could do was convert the mathematics and not the rest of the source document into LaTeX. That meant I just converted the math into LaTeX for the website but didn't convert the source documents.

Now, here's the reason for this history lesson. Fast forward even more years and I decided that I really needed to convert the source documents into LaTeX as that would just make my life easier and I'd be able to enable working links in the pdf as well as a simple way of producing an index for the material. The only issue is that the original tool I'd use to convert the MS Word mathematics had become, shall we say, unreliable and so that was no longer an option and it still has the problem on not converting anything else into proper LaTeX code.

So, the best option that I had available to me is to take the web pages, which already had the mathematics in proper LaTeX format, and convert the rest of the HTML into LaTeX code. I wrote a set of tools to do his and, for the most part, did a pretty decent job. The only problem is that the tools weren't perfect. So, if you run into some "odd" stuff here (things like `<sup>`, `<span>`, `</span>`, `<div>`, *etc*.) please let me know the section with the code that I missed. I did my best to find all the "orphaned" HTML code but I'm certain I missed some on occasion as I did find my eyes glazing over every once in a while as I went over the converted document.

Now, with that out of the way, here are a set of practice problems for the Calculus notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a couple of places on the site.

1. If you'd like a pdf document containing the solutions go to http://tutorial.math.lamar.edu and navigate to the class you want a pdf file for the solutions. In the download tab you will find a link to the pdf's containing the solutions.

2. If you'd like to view the solutions on the web go to the problem set web page, click the solution link for any problem and it will take you to the solution for that problem.

Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Outline

Here is a listing (and brief description) of the material in this set of notes.

Review - In this chapter we give a brief review of selected topics from Algebra and Trig that are vital to surviving a Calculus course. Included are Functions, Trig Functions, Solving Trig Equations, Exponential/Logarithm Functions and Solving Exponential/Logarithm Equations.

Preliminaries - The purpose of this chapter is to review several topics that will arise time and again throughout this material. Many of the topics here are so important to an Algebra class that if you don't have a good working grasp of them you will find it very difficult to successfully complete the course. Also, it is assumed that you've seen the topics in this chapter somewhere prior to this class and so this chapter should be mostly a review for you. However, since most of these topics are so important to an Algebra class we will make sure that you do understand them by doing a quick review of them here.

Exponents and polynomials are integral parts of any Algebra class. If you do not remember the basic exponent rules and how to work with polynomials you will find it very difficult, if not impossible, to pass an Algebra class. This is especially true with factoring polynomials. There are more than a few sections in an Algebra course where the ability to factor is absolutely essential to being able to do the work in those sections. In fact, in many of these sections factoring will be the first step taken.

It is important that you leave this chapter with a good understanding of this material! If you don't understand this material you will find it difficult to get through the remaining chapters. Here is a brief listing of the material covered in this chapter.

> Integer Exponents - In this section we will start looking at exponents. We will give the basic properties of exponents and illustrate some of the common mistakes students make in working with exponents. Examples in this section we will be restricted to integer exponents. Rational exponents will be discussed in the next section.
>
> Rational Exponents - In this section we will define what we mean by a rational exponent and extend the properties from the previous section to rational exponents. We will also discuss how to evaluate numbers raised to a rational exponent.
>
> Radicals - In this section we will define radical notation and relate radicals to rational exponents. We will also give the properties of radicals and some of the common mistakes students often make with radicals. We will also define simplified radical form and show how to rationalize the denominator.

Polynomials - In this section we will introduce the basics of polynomials a topic that will appear throughout this course. We will define the degree of a polynomial and discuss how to add, subtract and multiply polynomials.

Factoring Polynomials - In this section we look at factoring polynomials a topic that will appear in pretty much every chapter in this course and so is vital that you understand it. We will discuss factoring out the greatest common factor, factoring by grouping, factoring quadratics and factoring polynomials with degree greater than 2.

Rational Expressions - In this section we will define rational expressions. We will discuss how to reduce a rational expression lowest terms and how to add, subtract, multiply and divide rational expressions.

Complex Numbers - In this section we give a very quick primer on complex numbers including standard form, adding, subtracting, multiplying and dividing them.

Solving Equations and Inequalities In this chapter we will look at one of the standard topics in any Algebra class. The ability to solve equations and/or inequalities is very important and is used time and again both in this class and in later classes. We will cover a wide variety of solving topics in this chapter that should cover most of the basic equations/inequalities/techniques that are involved in solving.

Solutions and Solution Sets - In this section we introduce some of the basic notation and ideas involved in solving equations and inequalities. We define solutions for equations and inequalities and solution sets.

Linear Equations - In this section we give a process for solving linear equations, including equations with rational expressions, and we illustrate the process with several examples. In addition, we discuss a subtlety involved in solving equations that students often overlook.

Applications of Linear Equations - In this section we discuss a process for solving applications in general although we will focus only on linear equations here. We will work applications in pricing, distance/rate problems, work rate problems and mixing problems.

Equations With More Than One Variable - In this section we will look at solving equations with more than one variable in them. These equations will have multiple variables in them and we will be asked to solve the equation for one of the variables. This is something that we will be asked to do on a fairly regular basis.

Quadratic Equations, Part I - In this section we will start looking at solving quadratic equations. Specifically, we will concentrate on solving quadratic equations by factoring and the square root property in this section.

Quadratic Equations, Part II - In this section we will continue solving quadratic equations. We will use completing the square to solve quadratic equations in this section and use that to derive the quadratic formula. The quadratic formula is a quick way that will allow us to quickly solve any quadratic equation.

Quadratic Equations : A Summary - In this section we will summarize the topics from the last two sections. We will give a procedure for determining which method to use in solving quadratic equations and we will define the discriminant which will allow us to quickly determine what kind of solutions we will get from solving a quadratic equation.

Applications of Quadratic Equations - In this section we will revisit some of the applications we saw in the linear application section, only this time they will involve solving a quadratic equation. Included are examples in distance/rate problems and work rate problems.

Equations Reducible to Quadratic Form - Not all equations are in what we generally consider quadratic equations. However, some equations, with a proper substitution can be turned into a quadratic equation. These types of equations are called quadratic in form. In this section we will solve this type of equation.

Equations with Radicals - In this section we will discuss how to solve equations with square roots in them. As we will see we will need to be very careful with the potential solutions we get as the process used in solving these equations can lead to values that are not, in fact, solutions to the equation.

Linear Inequalities - In this section we will start solving inequalities. We will concentrate on solving linear inequalities in this section (both single and double inequalities). We will also introduce interval notation.

Polynomial Inequalities - In this section we will continue solving inequalities. However, in this section we move away from linear inequalities and move on to solving inequalities that involve polynomials of degree at least 2.

Rational Inequalities - We continue solving inequalities in this section. We now will solve inequalities that involve rational expressions, although as we'll see the process here is pretty much identical to the process used when solving inequalities with polynomials.

Absolute Value Equations - In this section we will give a geometric as well as a mathematical definition of absolute value. We will then proceed to solve equations that involve an absolute value. We will also work an example that involved two absolute values.

Absolute Value Inequalities - In this final section of the Solving chapter we will solve inequalities that involve absolute value. As we will see the process for solving inequalities with a $<$ (i.e. a less than) is very different from solving an inequality with a $>$ (i.e. greater than).

Graphing and Functions In this chapter we will be introducing two topics that are very important in an algebra class. We will start off the chapter with a brief discussion of graphing. This is not really the main topic of this chapter, but we need the basics down before moving into the second topic of this chapter. The next chapter will contain the remainder of the graphing discussion.

The second topic that we'll be looking at is that of functions. This is probably one of the more important ideas that will come out of an Algebra class. When first studying the concept of functions many students don't really understand the importance or usefulness of functions and function notation. The importance and/or usefulness of functions and function notation will only become apparent in later chapters and later classes. In fact, there are some topics that can only be done easily with function and function notation.

Graphing - In this section we will introduce the Cartesian (or Rectangular) coordinate system. We will define/introduce ordered pairs, coordinates, quadrants, and x and y-intercepts. We will illustrate these concepts with a quick example.

Lines - In this section we will discuss graphing lines. We will introduce the concept of slope and discuss how to find it from two points on the line. In addition, we will introduce the standard form of the line as well as the point-slope form and slope-intercept form of the line. We will finish off the section with a discussion on parallel and perpendicular lines.

Circles - In this section we discuss graphing circles. We introduce the standard form of the circle and show how to use completing the square to put an equation of a circle into standard form.

The Definition of a Function - In this section we will formally define relations and functions. We also give a "working definition" of a function to help understand just what a function is. We introduce function notation and work several examples illustrating how it works. We also define the domain and range of a function. In addition, we introduce piecewise functions in this section.

Graphing Functions - In this section we discuss graphing functions including several examples of graphing piecewise functions.

Combining functions - In this section we will discuss how to add, subtract, multiply and divide functions. In addition, we introduce the concept of function composition.

Inverse Functions - In this section we define one-to-one and inverse functions. We also discuss a process we can use to find an inverse function and verify that the function we get from this process is, in fact, an inverse function.

Common Graphs We started the process of graphing in the previous chapter. However, since the main focus of that chapter was functions we didn't graph all that many equations or functions. In this chapter we will now look at graphing a wide variety of equations and functions.

Lines, Circles and Piecewise Functions - This section is here only to acknowledge that we've already talked about graphing these in a previous chapter.

Parabolas - In this section we will be graphing parabolas. We introduce the vertex and axis of symmetry for a parabola and give a process for graphing parabolas. We also illustrate how to use completing the square to put the parabola into the form $f(x) = a(x - h)^2 + k$.

Ellipses - In this section we will graph ellipses. We introduce the standard form of an ellipse and how to use it to quickly graph an ellipse.

Hyperbolas - In this section we will graph hyperbolas. We introduce the standard form of a hyperbola and how to use it to quickly graph a hyperbola.

Miscellaneous Functions - In this section we will graph a couple of common functions that don't really take all that much work to do but will be needed in later sections. We'll be looking at the constant function, square root, absolute value and a simple cubic function.

Transformations - In this section we will be looking at vertical and horizontal shifts of graphs as well as reflections of graphs about the x and y-axis. Collectively these are often called transformations and if we understand them they can often be used to allow us to quickly graph some fairly complicated functions.

Symmetry - In this section we introduce the idea of symmetry. We discuss symmetry about the x-axis, y-axis and the origin and we give methods for determining what, if any symmetry, a graph will have without having to actually graph the function.

Rational Functions - In this section we will discuss a process for graphing rational functions. We will also introduce the ideas of vertical and horizontal asymptotes as well as how to determine if the graph of a rational function will have them.

Polynomial Functions In this chapter we are going to take a more in depth look at polynomials. We've already solved and graphed second degree polynomials (*i.e.* quadratic equations/functions) and we now want to extend things out to more general polynomials. We will take a look at finding solutions to higher degree polynomials and how to get a rough sketch for a higher degree polynomial.

We will also be looking at Partial Fractions in this chapter. It doesn't really have anything to do with graphing polynomials but needed to be put somewhere and this chapter seemed like as good a place as any.

Dividing Polynomials - In this section we'll review some of the basics of dividing polynomials. We will define the remainder and divisor used in the division process and introduce the idea of synthetic division. We will also give the Division Algorithm.

Zeroes/Roots of Polynomials - In this section we'll define the zero or root of a polynomial and whether or not it is a simple root or has multiplicity k. We will also give the Fundamental Theorem of Algebra and The Factor Theorem as well as a couple of other useful Facts.

Graphing Polynomials - In this section we will give a process that will allow us to get a rough sketch of the graph of some polynomials. We discuss how to determine the behavior of the graph at x-intercepts and the leading coefficient test to determine the behavior of the graph as we allow x to increase and decrease without bound.

Finding Zeroes of Polynomials - As we saw in the previous section in order to sketch the graph of a polynomial we need to know what it's zeroes are. However, if we are not able to factor the polynomial we are unable to do that process. So, in this section we'll look at a process using the Rational Root Theorem that will allow us to find some of the zeroes of a polynomial and in special cases all of the zeroes.

Partial Fractions - In this section we will take a look at the process of partial fractions and finding the partial fraction decomposition of a rational expression. What we will be asking here is what "smaller" rational expressions did we add and/or subtract to get the given rational expression. This is a process that has a lot of uses in some later math classes. It can show up in Calculus and Differential Equations for example.

Exponential Functions - In this section we will introduce exponential functions. We will give some of the basic properties and graphs of exponential functions. We will also discuss what many people consider to be the exponential function, $f(x) = \mathbf{e}^x$.

Logarithm Functions - In this section we will introduce logarithm functions. We give the basic properties and graphs of logarithm functions. In addition, we discuss how to evaluate some basic logarithms including the use of the change of base formula. We will also discuss the common logarithm, $\log(x)$, and the natural logarithm, $\ln(x)$.

Solving Exponential Equations - In this section we will discuss a couple of methods for solving equations that contain exponentials.

Solving Logarithm Equations - In this section we will discuss a couple of methods for solving equations that contain logarithms. Also, as we'll see, with one of the methods we will need to be careful of the results of the method as it is always possible that the method gives values that are, in fact, not solutions to the equation.

Applications - In this section we will look at a couple of applications of exponential functions and an application of logarithms. We look at compound interest, exponential growth and decay and earthquake intensity.

Systems of Equations This is a fairly short chapter devoted to solving systems of equations. A system of equations is a set of equations each containing one or more variable.

We will focus exclusively on systems of two equations with two unknowns and three equations with three unknowns although the methods looked at here can be easily extended to more equations. Also, with the exception of the last section we will be dealing only with systems of linear equations.

Linear Systems with Two Variables - In this section we will solve systems of two equations and two variables. We will use the method of substitution and method of elimination to solve the systems in this section. We will also introduce the concepts of inconsistent systems of equations and dependent systems of equations.

Linear Systems with Three Variables - In this section we will work a couple of quick examples illustrating how to use the method of substitution and method of elimination introduced in the previous section as they apply to systems of three equations.

Augmented Matrices - In this section we will look at another method for solving systems. We will introduce the concept of an augmented matrix. This will allow us to use the method of Gauss-Jordan elimination to solve systems of equations. We will use the method with systems of two equations and systems of three equations.

More on the Augmented Matrix - In this section we will revisit the cases of inconsistent and dependent solutions to systems and how to identify them using the augmented matrix method.

Nonlinear Systems - In this section we will take a quick look at solving nonlinear systems of equations. A nonlinear system of equations is a system in which at least one of the equations is not linear, i.e. has degree of two or more. Note as well that the discussion here does not cover all the possible solution methods for nonlinear systems. Solving nonlinear systems is often a much more involved process than solving linear systems.

1 Preliminaries

This chapter consists of some material that many students in a College Algebra course should have seen somewhere prior to the course. However, these topics are so important to a College Algebra course that we need to make sure they are covered so those that haven't seen them prior to the course can get caught up and to allows those that have seen them a chance for a refresher. Most of these topics will arise time and again as we cover the rest of the material for the course and if you don't have a good grasp of them you will find it difficult to successfully complete the course.

Exponents and polynomials are integral parts of any College Algebra class. If you do not remember the basic exponent rules and how to work with polynomials you will find it very difficult, if not impossible, to pass a College Algebra class. This is especially true with factoring polynomials. There are more than a few sections in a College Algebra course where the ability to factor is absolutely essential to being able to do the work in those sections. In fact, in many of these sections factoring will be the first step taken.

So, once again, it is important that you leave this chapter with a good understanding of this material! If you don't understand this material you will find it difficult to get through the remaining chapters.

The following sections are the practice problems (without solutions) for this material.

If you are looking for the solutions to these problems you can go to the Practice Problems on the website and download a pdf with the solutions from there.

1.1 Integer Exponents

For problems 1 - 4 evaluate the given expression and write the answer as a single number with no exponents.

1. $-6^2 + 4 \cdot 3^2$
2. $\dfrac{(-2)^4}{(3^2+2^2)^2}$
3. $\dfrac{4^0 \cdot 2^{-2}}{3^{-1} \cdot 4^{-2}}$
4. $2^{-1} + 4^{-1}$

For problems 5 - 9 simplify the given expression and write the answer with only positive exponents.

5. $\left(2w^4v^{-5}\right)^{-2}$
6. $\dfrac{2x^4y^{-1}}{x^{-6}y^3}$
7. $\dfrac{m^{-2}n^{-10}}{m^{-7}n^{-3}}$
8. $\dfrac{\left(2p^2\right)^{-3}q^4}{(6q)^{-1}p^{-7}}$
9. $\left(\dfrac{z^2y^{-1}x^{-3}}{x^{-8}z^6y^4}\right)^{-4}$

1.2 Rational Exponents

For problems 1 - 6 evaluate the given expression and write the answer as a single number with no exponents.

1. $36^{\frac{1}{2}}$
2. $(-125)^{\frac{1}{3}}$
3. $-16^{\frac{3}{2}}$
4. $27^{-\frac{5}{3}}$
5. $\left(\frac{9}{4}\right)^{\frac{1}{2}}$
6. $\left(\frac{8}{343}\right)^{-\frac{2}{3}}$

For problems 7 - 10 simplify the given expression and write the answer with only positive exponents.

7. $\left(a^3\,b^{-\frac{1}{4}}\right)^{\frac{2}{3}}$
8. $x^{\frac{1}{4}}\,x^{-\frac{1}{5}}$
9. $\left(\frac{q^3\,p^{-\frac{1}{2}}}{q^{-\frac{1}{3}}\,p}\right)^{\frac{3}{7}}$
10. $\left(\frac{m^{\frac{1}{2}}\,n^{-\frac{1}{3}}}{n^{\frac{2}{3}}\,m^{-\frac{7}{4}}}\right)^{-\frac{1}{6}}$

1.3 Radicals

For problems 1 - 4 write the expression in exponential form.

1. $\sqrt[7]{y}$
2. $\sqrt[3]{x^2}$
3. $\sqrt[6]{ab}$
4. $\sqrt{w^2v^3}$

For problems 5 - 7 evaluate the radical.

5. $\sqrt[4]{81}$
6. $\sqrt[3]{-512}$
7. $\sqrt[3]{1000}$

For problems 8 - 12 simplify each of the following. Assume that x, y and z are all positive.

8. $\sqrt[3]{x^8}$
9. $\sqrt{8y^3}$
10. $\sqrt[4]{x^7y^{20}z^{11}}$
11. $\sqrt[3]{54x^6y^7z^2}$
12. $\sqrt[4]{4x^3y}\ \sqrt[4]{8x^2y^3z^5}$

For problems 13 - 15 multiply each of the following. Assume that x is positive.

13. $\sqrt{x}\,(4-3\sqrt{x})$
14. $(2\sqrt{x}+1)\,(3-4\sqrt{x})$
15. $\left(\sqrt[3]{x}+2\ \sqrt[3]{x^2}\right)\left(4-\sqrt[3]{x^2}\right)$

For problems 16 - 19 rationalize the denominator. Assume that x and y are both positive.

16. $\dfrac{6}{\sqrt{x}}$

17. $\dfrac{9}{\sqrt[3]{2x}}$

18. $\dfrac{4}{\sqrt{x}+2\sqrt{y}}$

19. $\dfrac{10}{3-5\sqrt{x}}$

1.4 Polynomials

For problems 1 - 10 perform the indicated operation and identify the degree of the result.

1. Add $4x^3 - 2x^2 + 1$ to $7x^2 + 12x$
2. Subtract $4z^6 - 3z^2 + 2z$ from $-10z^6 + 7z^2 - 8$
3. Subtract $-3x^2 + 7x + 8$ from $x^4 + 7x^3 - 12x - 1$
4. $12y\left(3y^4 - 7y^2 + 1\right)$
5. $(3x + 1)\left(2 - 9x^2\right)$
6. $\left(w^2 + 2\right)\left(3w^2 + w\right)$
7. $\left(4x^6 - 3x\right)\left(4x^6 + 3x\right)$
8. $3\left(10 - 4y^3\right)^2$
9. $\left(x^2 + x - 2\right)\left(3x^2 - 8x - 7\right)$
10. Subtract $3\left(x^2 + 1\right)^2$ from $6x^3 - 9x^2 - 13x - 4$

1.5 Factoring Polynomials

For problems 1 - 4 factor out the greatest common factor from each polynomial.

1. $6x^7 + 3x^4 - 9x^3$
2. $a^3b^8 - 7a^{10}b^4 + 2a^5b^2$
3. $2x\left(x^2+1\right)^3 - 16\left(x^2+1\right)^5$
4. $x^2\left(2-6x\right) + 4x\left(4-12x\right)$

For problems 5 & 6 factor each of the following by grouping.

5. $7x + 7x^3 + x^4 + x^6$
6. $18x + 33 - 6x^4 - 11x^3$

For problems 7 - 15 factor each of the following.

7. $x^2 - 2x - 8$
8. $z^2 - 10z + 21$
9. $y^2 + 16y + 60$
10. $5x^2 + 14x - 3$
11. $6t^2 - 19t - 7$
12. $4z^2 + 19z + 12$
13. $x^2 + 14x + 49$
14. $4w^2 - 25$
15. $81x^2 - 36x + 4$

For problems 16 - 18 factor each of the following.

16. $x^6 + 3x^3 - 4$
17. $3z^5 - 17z^4 - 28z^3$
18. $2x^{14} - 512x^6$

1.6 Rational Expressions

For problems 1 - 3 reduce each of the following to lowest terms.

1. $\dfrac{x^2-6x-7}{x^2-10x+21}$

2. $\dfrac{x^2+6x+9}{x^2-9}$

3. $\dfrac{2x^2-x-28}{20-x-x^2}$

For problems 4 - 7 perform the indicated operation and reduce the answer to lowest terms.

4. $\dfrac{x^2+5x-24}{x^2+6x+8}\cdot\dfrac{x^2+4x+4}{x^2-3x}$

5. $\dfrac{x^2-49}{2x^2-3x-5}\div\dfrac{x^2-x-42}{x^2+7x+6}$

6. $\dfrac{x^2-2x-8}{2x^2-8x-24}\div\dfrac{x^2-9x+20}{x^2-11x+30}$

7. $\dfrac{\dfrac{3}{x+1}}{\dfrac{x+4}{x^2+11x+10}}$

For problems 8 - 12 perform the indicated operations.

8. $\dfrac{3}{x-4}+\dfrac{x}{2x+7}$

9. $\dfrac{2}{3x^2}-\dfrac{1}{9x^4}+\dfrac{2}{x+4}$

10. $\dfrac{x}{x^2+12x+36}-\dfrac{x-8}{x+6}$

11. $\dfrac{1}{x^2-13x+42}+\dfrac{x+1}{x-6}-\dfrac{x^2}{x-7}$

12. $\dfrac{x+10}{(3x+8)^3}+\dfrac{x}{(3x+8)^2}$

1.7 Complex Numbers

Perform the indicated operation and write your answer in standard form.

1. $(4-5i)(12+11i)$
2. $(-3-i)-(6-7i)$
3. $(1+4i)-(-16+9i)$
4. $8i(10+2i)$
5. $(-3-9i)(1+10i)$
6. $(2+7i)(8+3i)$
7. $\dfrac{7-i}{2+10i}$
8. $\dfrac{1+5i}{-3i}$
9. $\dfrac{6+7i}{8-i}$

2 Solving Equations and Inequalities

This chapter can, in many ways, be considered the heart of a College Algebra class. The ability to solve equations and inequalities is very important and will be used time and again both in this class and in later classes.

The majority of the chapter will be spent discussing how to solve linear and quadratic equations. We will also look at a few applications of linear and quadratic equations. As we will eventually see the ability to solve quadratic equations will arise in other topics in this section. Some equations can, for example, be reduced to a quadratic equation and when solving equations involving roots we will often end up solving a quadratic equation in the solution process.

In addition we'll solve inequalities involving linear equations, polynomial and rational expressions. In these sections we'll again see that the ability to solve linear and quadratic equations key to being able to successfully solve inequalities.

We'll the close out the section by solving equations and inequalities that involve absolute value equations.

The following sections are the practice problems (without solutions) for this material.

If you are looking for the solutions to these problems you can go to the Practice Problems on the website and download a pdf with the solutions from there.

2.1 Solutions and Solution Sets

For each of the following determine if the given number is a solution to the given equation or inequality.

1. Is $x = 6$ a solution to $2x - 5 = 3(1 - x) + 22$?
2. Is $t = 7$ a solution to $t^2 + 3t - 10 = 4 + 8t$?
3. Is $t = -3$ a solution to $t^2 + 3t - 10 = 4 + 8t$?
4. Is $w = -2$ a solution to $\dfrac{w^2 + 8w + 12}{w + 2} = 0$?
5. Is $z = 4$ a solution to $6z - z^2 \geq z^2 + 3$?
6. Is $y = 0$ a solution to $2(y + 7) - 1 < 4(y + 1) + 3(4y + 10)$?
7. Is $x = 1$ a solution to $(x + 1)^2 > 3x + 1$?

2.2 Linear Equations

Solve each of the following equations and check your answer.

1. $4x - 7(2 - x) = 3x + 2$

2. $2(w + 3) - 10 = 6(32 - 3w)$

3. $\dfrac{4 - 2z}{3} = \dfrac{3}{4} - \dfrac{5z}{6}$

4. $\dfrac{4t}{t^2 - 25} = \dfrac{1}{5 - t}$

5. $\dfrac{3y + 4}{y - 1} = 2 + \dfrac{7}{y - 1}$

6. $\dfrac{5x}{3x - 3} + \dfrac{6}{x + 2} = \dfrac{5}{3}$

2.3 Applications of Linear Equations

1. A widget is being sold in a store for \$135.40 and has been marked up 7%. How much did the store pay for the widget?

2. A store is having a 30% off sale and one item is now being sold for \$9.95. What was the original price of the item?

3. Two planes start out 2800 km apart and move towards each other meeting after 3.5 hours. One plane flies at 75 km/hour slower than the other plane. What was the speed of each plane?

4. Mike starts out 35 feet in front of Kim and they both start moving towards the right at the same time. Mike moves at 2 ft/sec while Kim moves at 3.4 ft/sec. How long will it take for Kim to catch up with Mike?

5. A pump can empty a pool in 7 hours and a different pump can empty the same pool in 12 hours. How long does it take for both pumps working together to empty the pool?

6. John can paint a house in 28 hours. John and Dave can paint the house in 17 hours working together. How long would it take Dave to paint the house by himself?

7. How much of a 20% acid solution should we add to 20 gallons of a 42% acid solution to get a 35% acid solution?

8. We need 100 liters of a 25% saline solution and we only have a 14% solution and a 60% solution. How much of each should we mix together to get the 100 liters of the 25% solution?

9. We want to fence in a field whose length is twice the width and we have 80 feet of fencing material. If we use all the fencing material what would the dimensions of the field be?

2.4 Equations With More Than One Variable

1. Solve $E = 3v\left(4 - \frac{2}{r}\right)$ for r.
2. Solve $Q = \frac{6h}{7s} + 4\left(1 - h\right)$ for s.
3. Solve $Q = \frac{6h}{7s} + 4\left(1 - h\right)$ for h.
4. Solve $A - \frac{1-2t}{4p} = \frac{4+3t}{5p}$ for t.
5. Solve $y = \frac{10}{3-7x}$ for x.
6. Solve $y = \frac{3+x}{12-9x}$ for x.

2.5 Quadratic Equations - Part I

For problems 1 - 7 solve the quadratic equation by factoring.

1. $u^2 - 5u - 14 = 0$

2. $x^2 + 15x = -50$

3. $y^2 = 11y - 28$

4. $19x = 7 - 6x^2$

5. $6w^2 - w = 5$

6. $z^2 - 16z + 61 = 2z - 20$

7. $12x^2 = 25x$

For problems 8 & 9 use factoring to solve the equation.

8. $x^4 - 2x^3 - 3x^2 = 0$

9. $t^5 = 9t^3$

For problems 10 - 12 use factoring to solve the equation.

10. $\dfrac{w^2 - 10}{w + 2} + w - 4 = w - 3$

11. $\dfrac{4z}{z + 1} + \dfrac{5}{z} = \dfrac{6z + 5}{z^2 + z}$

12. $x + 1 = \dfrac{2x - 7}{x + 5} - \dfrac{5x + 8}{x + 5}$

For problems 13 - 16 use the Square Root Property to solve the equation.

13. $9u^2 - 16 = 0$

14. $x^2 + 15 = 0$

15. $(z - 2)^2 - 36 = 0$

16. $(6t + 1)^2 + 3 = 0$

2.6 Quadratic Equations - Part II

For problems 1 - 3 complete the square.

1. $x^2 + 8x$
2. $u^2 - 11u$
3. $2z^2 - 12z$

For problems 4 - 8 solve the quadratic equation by completing the square.

4. $t^2 - 10t + 34 = 0$
5. $v^2 + 8v - 9 = 0$
6. $x^2 + 9x + 16 = 0$
7. $4u^2 - 8u + 5 = 0$
8. $2x^2 + 5x + 3 = 0$

For problems 9 - 13 use the quadratic formula to solve the quadratic equation.

9. $x^2 - 6x + 4 = 0$
10. $9w^2 - 6w = 101$
11. $8u^2 + 5u + 70 = 5 - 7u$
12. $169 - 20t + 4t^2 = 0$
13. $2z^2 + z - 72 = z^2 - 2z + 58$

2.7 Quadratic Equations : A Summary

For problems 1 - 4 use the discriminant to determine the type of roots for the equation. Do not find any roots.

1. $169x^2 - 182x + 49 = 0$
2. $x^2 + 28x + 61 = 0$
3. $49x^2 - 126x + 102 = 0$
4. $9x^2 + 151 = 0$

2.8 Applications of Quadratic Equations

1. The width of a rectangle is 1 m less than twice the length. If the area of the rectangle is 100 m2 what are the dimensions of the rectangle?

2. Two cars start out at the same spot. One car starts to drive north at 40 mph and 3 hours later the second car starts driving to the east at 60 mph. How long after the first car starts driving does it take for the two cars to be 500 miles apart?

3. Two people can paint a house in 14 hours. Working individually one of the people takes 2 hours more than it takes the other person to paint the house. How long would it take each person working individually to paint the house?

2.9 Equations Reducible to Quadratic in Form

Solve each of the following equations.

1. $x^6 - 9x^3 + 8 = 0$
2. $x^{-4} - 7x^{-2} - 18 = 0$
3. $4x^{\frac{2}{3}} + 21x^{\frac{1}{3}} + 27 = 0$
4. $x^8 - 6x^4 + 7 = 0$
5. $\dfrac{2}{x^2} + \dfrac{17}{x} + 21 = 0$
6. $\dfrac{1}{x} - \dfrac{11}{\sqrt{x}} + 18 = 0$

2.10 Equations with Radicals

Solve each of the following equations.

1. $2x = \sqrt{x+3}$
2. $\sqrt{33-2x} = x+1$
3. $7 = \sqrt{39+3x} - x$
4. $x = 1 + \sqrt{2x-2}$
5. $1 + \sqrt{1-x} = \sqrt{2x+4}$

2.11 Linear Inequalities

For problems 1 - 6 solve each of the following inequalities. Give the solution in both inequality and interval notations.

1. $4(z+2)-1>5-7(4-z)$
2. $\frac{1}{2}(3+4t)\le 6\left(\frac{1}{3}-\frac{1}{2}t\right)-\frac{1}{4}(2+10t)$
3. $-1<4x+2<10$
4. $8\le 3-5z<12$
5. $0\le 10w-15\le 23$
6. $2<\frac{1}{6}-\frac{1}{2}x\le 4$
7. If $0\le x<3$ determine a and b for the inequality : $a\le 4x+1<b$

2.12 Polynomial Inequalities

Solve each of the following inequalities.

1. $u^2 + 4u \geq 21$
2. $x^2 + 8x + 12 < 0$
3. $4t^2 \leq 15 - 17t$
4. $z^2 + 34 > 12z$
5. $y^2 - 2y + 1 \leq 0$
6. $t^4 + t^3 - 12t^2 < 0$

2.13 Rational Inequalities

Solve each of the following inequalities.

1. $\dfrac{4-x}{x+3} > 0$

2. $\dfrac{2z-5}{z-7} \le 0$

3. $\dfrac{w^2+5w-6}{w-3} \ge 0$

4. $\dfrac{3x+8}{x-1} < -2$

5. $u \le \dfrac{4}{u-3}$

6. $\dfrac{t^3-6t^2}{t-2} > 0$

2.14 Absolute Value Equations

For problems 1 - 5 solve each of the equation.

1. $|4p - 7| = 3$
2. $|2 - 4x| = 1$
3. $6u = |1 + 3u|$
4. $|2x - 3| = 4 - x$
5. $\left|\frac{1}{2}z + 4\right| = |4z - 6|$

For problems 6 & 7 find all the real valued solutions to the equation.

6. $\left|x^2 + 2x\right| = 15$
7. $\left|x^2 + 4\right| = 1$

2.15 Absolute Value Inequalities

Solve each of the following inequalities.

1. $|4t + 9| < 3$
2. $|6 - 5x| \leq 10$
3. $|12x + 1| \leq -9$
4. $|2w - 1| < 1$
5. $|2z - 7| > 1$
6. $|10 - 3w| \geq 4$
7. $|4 - 3z| > 7$

3 Graphing and Functions

In this chapter we will be introducing two topics that are very important in an algebra class. We will start off the chapter with a brief discussion of graphing including graphing lines and circles. This is not really the main topic of this chapter, but we need the basics down before moving into the second topic of this chapter. The next chapter will contain the remainder of the graphing discussion.

The second topic that we'll be looking at is that of functions. This is probably one of the more important ideas that will come out of an Algebra class. When first studying the concept of functions many students don't really understand the importance or usefulness of functions and function notation. The importance and/or usefulness of functions and function notation will only become apparent in later chapters and later classes. In fact, there are some topics that can only be done easily with function and function notation.

We'll discuss the formal definition of a function, how to graph functions and the various ways to combine functions. We'll also introduce the idea of an inverse function.

The following sections are the practice problems (without solutions) for this material.

If you are looking for the solutions to these problems you can go to the Practice Problems on the website and download a pdf with the solutions from there.

3.1 Graphing

For problems 1 - 3 construct a table of at least 4 ordered pairs of points on the graph of the equation and use the ordered pairs from the table to sketch the graph of the equation.

1. $y = 3x + 4$
2. $y = 1 - x^2$
3. $y = 2 + \sqrt{x}$

For problems 4 - 9 determine the x-intercepts and y-intercepts for the equation. Do not sketch the graph.

4. $3x - 7y = 10$
5. $y = 6 - x^2$
6. $y = x^2 + 6x - 7$
7. $y = x^2 + 10$
8. $y = x^2 + 6x + 58$
9. $y = (x + 3)^2 - 8$

3.2 Lines

For problems 1 & 2 determine the slope of the line containing the two points and sketch the graph of the line.

1. $(-2, 4)\,,\ \ (1, 10)$
2. $(8, 2)\,,\ \ (14, -7)$

For problems 3 - 5 write down the equation of the line that passes through the two points. Give your answer in point-slope form and slope-intercept form.

3. $(-2, 4)\,,\ \ (1, 10)$
4. $(8, 2)\,,\ \ (14, -7)$
5. $(-4, 8)\,,\ \ (-1, -20)$

For problems 6 & 7 determine the slope of the line and sketch the graph of the line.

6. $4y + x = 8$
7. $5x - 2y = 6$

For problems 8 & 9 determine if the two given lines are parallel, perpendicular or neither.

8. $y = \frac{3}{7}x + 1$ and $3y + 7x = -10$
9. $8x - y = 2$ and the line containing the two points $(1, 3)$ and $(2, -4)$.
10. Find the equation of the line through $(-7, 2)$ and is parallel to the line $3x - 14y = 4$.
11. Find the equation of the line through $(-7, 2)$ and is perpendicular to the line $3x - 14y = 4$.

3.3 Circles

1. Write the equation of the circle with radius 3 and center $(6, 0)$.
2. Write the equation of the circle with radius $\sqrt{7}$ and center $(-1, -9)$.

For problems 3 - 5 determine the radius and center of the circle and sketch the graph of the circle.

3. $(x-9)^2 + (y+4)^2 = 25$
4. $x^2 + (y-5)^2 = 4$
5. $(x+1)^2 + (y+3)^2 = 6$

For problems 6 - 8 determine the radius and center of the circle. If the equation is not the equation of a circle clearly explain why not.

6. $x^2 + y^2 + 14x - 8y + 56 = 0$
7. $9x^2 + 9y^2 - 6x - 36y - 107 = 0$
8. $x^2 + y^2 + 8x + 20 = 0$

3.4 The Definition of a Function

For problems 1 - 3 determine if the given relation is a function.

1. $\{(2,4),(3,-7),(6,10)\}$

2. $\{(-1,8),(4,-7),(-1,6),(0,0)\}$

3. $\{(2,1),(9,10),(-4,10),(-8,1)\}$

For problems 4 - 6 determine if the given equation is a function.

4. $y = 14 - \frac{1}{3}x$

5. $y = \sqrt{3x^2+1}$

6. $y^4 - x^2 = 16$

7. Given $f(x) = 3 - 2x^2$ determine each of the following.

   (a) $f(0)$ **(b)** $f(2)$ **(c)** $f(-4)$ **(d)** $f(3t)$ **(e)** $f(x+2)$

8. Given $g(w) = \frac{4}{w+1}$ determine each of the following.

   (a) $g(-6)$ **(b)** $g(-2)$ **(c)** $g(0)$ **(d)** $g(t-1)$ **(e)** $g(4w+3)$

9. Given $h(t) = t^2 + 6$ determine each of the following.

   (a) $h(0)$ **(b)** $h(-2)$ **(c)** $h(2)$ **(d)** $h(\sqrt{x})$ **(e)** $h(3-t)$

10. Given $h(z) = \begin{cases} 3z & \text{if } z < 2 \\ 1+z^2 & \text{if } z \geq 2 \end{cases}$ determine each of the following.

    (a) $h(0)$ **(b)** $h(2)$ **(c)** $h(7)$

11. Given $f(x) = \begin{cases} 6 & \text{if } x \geq 9 \\ x+9 & \text{if } 2 < x < 9 \\ x^2 & \text{if } x \leq 2 \end{cases}$ determine each of the following.

    (a) $f(-4)$ **(b)** $f(2)$ **(c)** $f(6)$ **(d)** $f(9)$ **(e)** $f(12)$

For problems 12 & 13 compute the difference quotient for the given function. The difference quotient for the function $f(x)$ is defined to be,

$$\frac{f(x+h)-f(x)}{h}$$

12. $f(x) = 4 - 9x$

13. $f(x) = 2x^2 - x$

For problems 14 - 18 determine the domain of the function.

14. $A(x) = 6x + 14$

15. $f(x) = \dfrac{1}{x^2 - 25}$

16. $g(t) = \dfrac{8t - 24}{t^2 - 7t - 18}$

17. $g(w) = \sqrt{9w + 7}$

18. $f(x) = \dfrac{1}{\sqrt{x^2 - 8x + 15}}$

3.5 Graphing Functions

For problems 1 - 5 construct a table of at least 4 ordered pairs of points on the graph of the function and use the ordered pairs from the table to sketch the graph of the function.

1. $f\left(x\right) = x^2 - 2$

2. $f\left(x\right) = \sqrt{x+1}$

3. $f\left(x\right) = 9$

4. $f\left(x\right) = \begin{cases} 10 - 2x & \text{if } x < 2 \\ x^2 + 2 & \text{if } x \geq 2 \end{cases}$

5. $f\left(x\right) = \begin{cases} 5 + x & \text{if } x \geq 1 \\ 2 & \text{if } -2 \leq x < 1 \\ 1 - x^2 & \text{if } x < -2 \end{cases}$

3.6 Combining Functions

1. Given $f(x) = 6x + 2$ and $g(x) = 10 - 7x$ compute each of the following.

   (a) $(f - g)(2)$ **(b)** $(g - f)(2)$ **(c)** $f\,g$ **(d)** $\left(\frac{f}{g}\right)(x)$

2. Given $P(t) = 4t^2 + 3t - 1$ and $A(t) = 2 - t^2$ compute each of the following.

   (a) $(P + A)(t)$ **(b)** $A - P$ **(c)** $(PA)(t)$ **(d)** $\left(\frac{P}{A}\right)(-2)$

3. Given $h(z) = 7z + 6$ and $f(z) = 4 - z$ compute each of the following.

   (a) $(f\,h)(z)$ **(b)** $(f \circ h)(z)$ **(c)** $(h \circ f)(z)$ **(d)** $(h \circ h)(z)$

4. Given $f(x) = 6x^2$ and $g(x) = x^2 + 3x - 1$ compute each of the following.

   (a) $(g\,f)(x)$ **(b)** $(f \circ g)(x)$ **(c)** $(g \circ f)(x)$ **(d)** $(f \circ f)(x)$

5. Given $R(t) = \sqrt{t} - 2$ and $A(t) = (t + 2)^2$, $t \geq 0$ compute each of the following.

   (a) $(R \circ A)(t)$ **(b)** $(A \circ R)(t)$

3.7 Inverse Functions

1. Given $h(x) = 5 - 9x$ find $h^{-1}(x)$.
2. Given $g(x) = \frac{1}{2}x + 7$ find $g^{-1}(x)$.
3. Given $f(x) = (x-2)^3 + 1$ find $f^{-1}(x)$.
4. Given $A(x) = \sqrt[5]{2x + 11}$ find $A^{-1}(x)$.
5. Given $f(x) = \dfrac{4x}{5 - x}$ find $f^{-1}(x)$.
6. Given $h(x) = \dfrac{1 + 2x}{7 + x}$ find $h^{-1}(x)$.

4 Common Graphs

In this chapter we will continue the process of investigating graphing that we started in the last chapter. In the last chapter we discussed how to graph functions in general, lines, circles and piecewise functions. In this chapter we will take a look at parabolas, ellipses, hyperbolas, rational functions as well as a few other functions that will arise occasionally. In addition we will take a look at various transformations that we can make to functions and how these transformations impact the graph of the function. We will also introduce the concept of symmetry of a the graph of a function and how to determine if the function has symmetry without graphing the function.

The following sections are the practice problems (without solutions) for this material.

If you are looking for the solutions to these problems you can go to the Practice Problems on the website and download a pdf with the solutions from there.

4.1 Lines, Circles and Piecewise Functions

We looked at these topics in the previous chapter. Problems for these topics can be found in the following sections.

Here are the appropriate sections to see for these.

Lines : Graphing and Functions - Lines

Circles : Graphing and Functions - Circles

Piecewise Functions : Graphing and Functions - Graphing Functions

4.2 Parabolas

For problems 1 - 7 sketch the graph of the following parabolas. The graph should contain the vertex, the y intercept, x-intercepts (if any) and at least one point on either side of the vertex.

1. $f(x) = (x+4)^2 - 3$
2. $f(x) = 5(x-1)^2 - 20$
3. $f(x) = 3x^2 + 7$
4. $f(x) = x^2 + 12x + 11$
5. $f(x) = 2x^2 - 12x + 26$
6. $f(x) = 4x^2 - 4x + 1$
7. $f(x) = -3x^2 + 6x + 3$

For problems 8 - 10 convert the following equations into the form $y = a(x-h)^2 + k$.

8. $f(x) = x^2 - 24x + 157$
9. $f(x) = 6x^2 + 12x + 3$
10. $f(x) = -x^2 - 8x - 18$

4.3 Ellipses

For problems 1 - 3 sketch the ellipse.

1. $\dfrac{(x+3)^2}{9} + \dfrac{(y-5)^2}{3} = 1$

2. $x^2 + \dfrac{(y-1)^2}{4} = 1$

3. $4(x+2)^2 + \dfrac{(y+4)^2}{4} = 1$

For problems 4 & 5 complete the square on the x and y portions of the equation and write the equation into the standard form of the equation of the ellipse.

4. $x^2 + 8x + 3y^2 - 6y + 7 = 0$

5. $9x^2 + 126x + 4y^2 - 32y + 469 = 0$

4.4 Hyperbolas

For problems 1 - 3 sketch the hyperbola.

1. $\dfrac{y^2}{16} - \dfrac{(x-2)^2}{9} = 1$
2. $\dfrac{(x+3)^2}{4} - \dfrac{(y-1)^2}{9} = 1$
3. $3(x-1)^2 - \dfrac{(y+1)^2}{2} = 1$

For problems 4 & 5 complete the square on the x and y portions of the equation and write the equation into the standard form of the equation of the hyperbola.

4. $4x^2 - 32x - y^2 - 4y + 24 = 0$
5. $25y^2 + 250y - 16x^2 - 32x + 209 = 0$

4.5 Miscellaneous Functions

The sole purpose of this section was to get you familiar with the basic shape of some miscellaneous functions for the next section. As such there are no problems for this section. You will see quite a few problems utilizing these functions in the Transformations section.

4.6 Transformations

Use transformations to sketch the graph of the following functions.

1. $f(x) = \sqrt{x} + 4$
2. $f(x) = x^3 - 2$
3. $f(x) = |x + 2|$
4. $f(x) = (x - 5)^2$
5. $f(x) = -x^3$
6. $f(x) = \sqrt{x + 4} - 3$
7. $f(x) = |x - 7| + 2$

4.7 Symmetry

Determine the symmetry of each of the following equations.

1. $x = 4y^6 - y^2$
2. $\dfrac{y^2}{4} = 1 + \dfrac{x^2}{9}$
3. $x^2 = 7y - x^3 + 2$
4. $y = 4x^2 + x^6 - x^8$
5. $y = 7x + 4x^5$

4.8 Rational Functions

Sketch the graph of each of the following functions. Clearly identify all intercepts and asymptotes.

1. $f(x) = \dfrac{-4}{x-2}$
2. $f(x) = \dfrac{6-2x}{1-x}$
3. $f(x) = \dfrac{8}{x^2+x-6}$
4. $f(x) = \dfrac{4x^2-36}{x^2-2x-8}$

5 Polynomial Functions

In this chapter we are going to take a more in depth look at polynomials. We've already solved and graphed second degree polynomials (i.e. quadratic equations/functions) and we now want to extend things out to more general polynomials. We will investigate dividing polynomials and determining where a polynomial equal to zero. After we know how to determine where a polynomial is zero we will take a look at how to get a rough sketch for a higher degree polynomial.

We will also be looking at Partial Fractions in this chapter. It doesn't really have anything to do with graphing polynomials but needed to be put somewhere and this chapter seemed like as good a place as any.

The following sections are the practice problems (without solutions) for this material.

If you are looking for the solutions to these problems you can go to the Practice Problems on the website and download a pdf with the solutions from there.

5.1 Dividing Polynomials

For problems 1 - 3 use long division to perform the indicated division.

1. Divide $3x^4 - 5x^2 + 3$ by $x + 2$
2. Divide $x^3 + 2x^2 - 3x + 4$ by $x - 7$
3. Divide $2x^5 + x^4 - 6x + 9$ by $x^2 - 3x + 1$

For problems 4 - 6 use synthetic division to perform the indicated division.

4. Divide $x^3 + x^2 + x + 1$ by $x + 9$
5. Divide $7x^3 - 1$ by $x + 2$
6. Divide $5x^4 + x^2 - 8x + 2$ by $x - 4$

5.2 Zeroes/Roots of Polynomials

For problems 1 - 3 list all of the zeros of the polynomial and give their multiplicities.

1. $f(x) = 2x^2 + 13x - 7$

2. $g(x) = x^6 - 3x^5 - 6x^4 + 10x^3 + 21x^2 + 9x = x(x-3)^2(x+1)^3$

3. $A(x) = x^8 + 2x^7 - 29x^6 - 76x^5 + 199x^4 + 722x^3 + 261x^2 - 648x - 432$
   $= (x+1)^2(x-4)^2(x-1)(x+3)^3$

For problems 4 - 6 $x = r$ is a root of the given polynomial. Find the other two roots and write the polynomial in fully factored form.

4. $P(x) = x^3 - 6x^2 - 16x$; $r = -2$

5. $P(x) = x^3 - 7x^2 - 6x + 72$; $r = 4$

6. $P(x) = 3x^3 + 16x^2 - 33x + 14$; $r = -7$

5.3 Graphing Polynomials

Sketch the graph of each of the following polynomials.

1. $f(x) = x^3 - 2x^2 - 24x$
2. $g(x) = -x^3 + 3x - 2 = -(x-1)^2(x+2)$
3. $h(x) = x^4 + x^3 - 12x^2 + 4x + 16 = (x-2)^2(x+1)(x+4)$
4. $P(x) = x^5 - 12x^3 - 16x^2 = x^2(x+2)^2(x-4)$

5.4 Finding Zeroes of Polynomials

Find all the zeroes of the following polynomials.

1. $f(x) = 2x^3 - 13x^2 + 3x + 18$
2. $P(x) = x^4 - 3x^3 - 5x^2 + 3x + 4$
3. $A(x) = 2x^4 - 7x^3 - 2x^2 + 28x - 24$
4. $g(x) = 8x^5 + 36x^4 + 46x^3 + 7x^2 - 12x - 4$

5.5 Partial Fractions

Determine the partial fraction decomposition of each of the following expressions.

1. $\dfrac{17x-53}{x^2-2x-15}$

2. $\dfrac{34-12x}{3x^2-10x-8}$

3. $\dfrac{125+4x-9x^2}{(x-1)(x+3)(x+4)}$

4. $\dfrac{10x+35}{(x+4)^2}$

5. $\dfrac{6x+5}{(2x-1)^2}$

6. $\dfrac{7x^2-17x+38}{(x+6)(x-1)^2}$

7. $\dfrac{4x^2-22x+7}{(2x+3)(x-2)^2}$

8. $\dfrac{3x^2+7x+28}{x(x^2+x+7)}$

9. $\dfrac{4x^3+16x+7}{(x^2+4)^2}$

6 Exponential and Logarithm Functions

In this chapter we are going to look at exponential and logarithm functions. Both of these functions are very important and need to be understood by anyone who is going on to later math courses. These functions also have applications in science, engineering, and business to name a few areas. In fact, these functions can show up in just about any field that uses even a small degree of mathematics.

Many students find both of these functions, especially logarithm functions, difficult to deal with. This is probably because they are so different from any of the other functions that they've looked at to this point and logarithms use a notation that will be new to almost everyone in an algebra class. However, you will find that once you get past the notation and start to understand some of their properties they really aren't too bad. So, we'll make sure to go over the notation and various properties of both exponential and logarithm functions in the hope that you will agree that once you understand those they aren't too bad.

In addition, we'll take a look at solving equations with exponentials and solving equations with logarithms. We'll also take a quick look a couple of applications involving exponentials and logarithms.

The following sections are the practice problems (without solutions) for this material.

If you are looking for the solutions to these problems you can go to the Practice Problems on the website and download a pdf with the solutions from there.

6.1 Exponential Functions

1. Given the function $f(x) = 4^x$ evaluate each of the following.

   (a) $f(-2)$ **(b)** $f\left(-\frac{1}{2}\right)$ **(c)** $f(0)$ **(d)** $f(1)$ **(e)** $f\left(\frac{3}{2}\right)$

2. Given the function $f(x) = \left(\frac{1}{5}\right)^x$ evaluate each of the following.

   (a) $f(-3)$ **(b)** $f(-1)$ **(c)** $f(0)$ **(d)** $f(2)$ **(e)** $f(3)$

3. Sketch each of the following.

   (a) $f(x) = 6^x$ **(b)** $g(x) = 6^x - 9$ **(c)** $g(x) = 6^{x+1}$

4. Sketch the graph of $f(x) = \mathbf{e}^{-x}$.

5. Sketch the graph of $f(x) = \mathbf{e}^{x-3} + 6$.

6.2 Logarithm Functions

For problems 1 - 3 write the expression in logarithmic form.

1. $7^5 = 16807$
2. $16^{\frac{3}{4}} = 8$
3. $\left(\frac{1}{3}\right)^{-2} = 9$

For problems 4 - 6 write the expression in exponential form.

4. $\log_2 32 = 5$
5. $\log_{\frac{1}{5}} \frac{1}{625} = 4$
6. $\log_9 \frac{1}{81} = -2$

For problems 7 - 12 determine the exact value of each of the following without using a calculator.

7. $\log_3 81$
8. $\log_5 125$
9. $\log_2 \frac{1}{8}$
10. $\log_{\frac{1}{4}} 16$
11. $\ln \mathbf{e}^4$
12. $\log \frac{1}{100}$

For problems 13 - 15 write each of the following in terms of simpler logarithms

13. $\log\left(3x^4y^{-7}\right)$
14. $\ln\left(x\sqrt{y^2+z^2}\right)$
15. $\log_4\left(\frac{x-4}{y^2\sqrt[5]{z}}\right)$

For problems 16 - 18 combine each of the following into a single logarithm with a coefficient of one.

16. $2\log_4 x + 5\log_4 y - \frac{1}{2}\log_4 z$
17. $3\ln(t+5) - 4\ln(t) - 2\ln(s-1)$
18. $\dfrac{1}{3}\log a - 6\log b + 2$

For problems 19 & 20 use the change of base formula and a calculator to find the value of each of the following.

19. $\log_{12} 35$
20. $\log_{\frac{2}{3}} 53$

For problems 21 - 23 sketch each of the given functions.

21. $g(x) = -\ln(x)$
22. $g(x) = \ln(x+5)$
23. $g(x) = \ln(x) - 4$

6.3 Solving Exponential Equations

Solve each of the following equations.

1. $6^{2x} = 6^{1-3x}$
2. $5^{1-x} = 25$
3. $8^{x^2} = 8^{3x+10}$
4. $7^{4-x} = 7^{4x}$
5. $2^{3x} = 10$
6. $7^{1-x} = 4^{3x+1}$
7. $9 = 10^{4+6x}$
8. $\mathbf{e}^{7+2x} - 3 = 0$
9. $\mathbf{e}^{4-7x} + 11 = 20$

6.4 Solving Logarithm Equations

Solve each of the following equations.

1. $\log_4\left(x^2-2x\right)=\log_4(5x-12)$
2. $\log(6x)-\log(4-x)=\log(3)$
3. $\ln(x)+\ln(x+3)=\ln(20-5x)$
4. $\log_3\left(25-x^2\right)=2$
5. $\log_2(x+1)-\log_2(2-x)=3$
6. $\log_4(-x)+\log_4(6-x)=2$
7. $\log(x)=2-\log(x-21)$
8. $\ln(x-1)=1+\ln(3x+2)$
9. $2\log(x)-\log(7x-1)=0$

6.5 Applications

10. We have \$10,000 to invest for 44 months. How much money will we have if we put the money into an account that has an annual interest rate of 5.5% and interest is compounded

    (a) quarterly **(b)** monthly **(c)** continuously

11. We are starting with \$5000 and we're going to put it into an account that earns an annual interest rate of 12%. How long should we leave the money in the account in order to double our money if interest is compounded

    (a) quarterly **(b)** monthly **(c)** continuously

12. A population of bacteria initially has 250 present and in 5 days there will be 1600 bacteria present.

    (a) Determine the exponential growth equation for this population.

    (b) How long will it take for the population to grow from its initial population of 250 to a population of 2000?

13. We initially have 100 grams of a radioactive element and in 1250 years there will be 80 grams left.

    (a) Determine the exponential decay equation for this element.

    (b) How long will it take for half of the element to decay?

    (c) How long will it take until there is only 1 gram of the element left?

7 Systems of Equations

This is a fairly short chapter devoted to solving systems of equations. A system of equations is a set of equations each containing one or more variable.

We will focus exclusively on systems of two equations with two unknowns and three equations with three unknowns although the methods looked at here can be easily extended to more equations. We'll look at a couple of methods of solving systems including the idea of an augmented matrix to solve systems.

In addition, with the exception of the last section we will be dealing only with systems of linear equations.

The following sections are the practice problems (without solutions) for this material.

If you are looking for the solutions to these problems you can go to the Practice Problems on the website and download a pdf with the solutions from there.

7.1 Linear Systems with Two Variables

For problems 1 - 3 use the Method of Substitution to find the solution to the given system or to determine if the system is inconsistent or dependent.

1. $$\begin{aligned} x - 7y &= -11 \\ 5x + 2y &= -18 \end{aligned}$$

2. $$\begin{aligned} 7x - 8y &= -12 \\ -4x + 2y &= 3 \end{aligned}$$

3. $$\begin{aligned} 3x + 9y &= -6 \\ -4x - 12y &= 8 \end{aligned}$$

For problems 4 - 6 use the Method of Elimination to find the solution to the given system or to determine if the system is inconsistent or dependent.

4. $$\begin{aligned} 6x - 5y &= 8 \\ -12x + 2y &= 0 \end{aligned}$$

5. $$\begin{aligned} -2x + 10y &= 2 \\ 5x - 25y &= 3 \end{aligned}$$

6. $$\begin{aligned} 2x + 3y &= 20 \\ 7x + 2y &= 53 \end{aligned}$$

7.2 Linear Systems with Three Variables

Find the solution to each of the following systems of equations.

1. $$\begin{aligned} 2x + 5y + 2z &= -38 \\ 3x - 2y + 4z &= 17 \\ -6x + y - 7z &= -12 \end{aligned}$$

2. $$\begin{aligned} 3x - 9z &= 33 \\ 7x - 4y - z &= -15 \\ 4x + 6y + 5z &= -6 \end{aligned}$$

7.3 Augmented Matrices

1. For the following augmented matrix perform the indicated elementary row operations.

$$\left[\begin{array}{ccc|c} 4 & -1 & 3 & 5 \\ 0 & 2 & 5 & 9 \\ -6 & 1 & -3 & 10 \end{array}\right]$$

(a) $8R_1$ **(b)** $R_2 \leftrightarrow R_3$ **(c)** $R_2 + 3R_1 \to R_2$

2. For the following augmented matrix perform the indicated elementary row operations.

$$\left[\begin{array}{ccc|c} 1 & -6 & 2 & 0 \\ 2 & -8 & 10 & 4 \\ 3 & -4 & -1 & 2 \end{array}\right]$$

(a) $\frac{1}{2}R_2$ **(b)** $R_1 \leftrightarrow R_3$ **(c)** $R_1 - 6R_3 \to R_1$

3. For the following augmented matrix perform the indicated elementary row operations.

$$\left[\begin{array}{ccc|c} 10 & -1 & -5 & 1 \\ 4 & 0 & 7 & -1 \\ 0 & 7 & -2 & 3 \end{array}\right]$$

(a) $-9R_3$ **(b)** $R_1 \leftrightarrow R_2$ **(c)** $R_3 - R_1 \to R_3$

Note : Problems using augmented matrices to solve systems of equations are in the next section.

7.4 More on the Augmented Matrix

For each of the following systems of equations convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system or to determine if the system is inconsistent or dependent.

1. $$\begin{aligned} x - 7y &= -11 \\ 5x + 2y &= -18 \end{aligned}$$

2. $$\begin{aligned} 7x - 8y &= -12 \\ -4x + 2y &= 3 \end{aligned}$$

3. $$\begin{aligned} 3x + 9y &= -6 \\ -4x - 12y &= 8 \end{aligned}$$

4. $$\begin{aligned} 6x - 5y &= 8 \\ -12x + 2y &= 0 \end{aligned}$$

5. $$\begin{aligned} 5x - 25y &= 3 \\ -2x + 10y &= 2 \end{aligned}$$

6. $$\begin{aligned} 2x + 3y &= 20 \\ 7x + 2y &= 53 \end{aligned}$$

7. $$\begin{aligned} 2x + 5y + 2z &= -38 \\ 3x - 2y + 4z &= 17 \\ -6x + y - 7z &= -12 \end{aligned}$$

8. $$\begin{aligned} 3x - 9z &= 33 \\ 7x - 4y - z &= -15 \\ 4x + 6y + 5z &= -6 \end{aligned}$$

7.5 Nonlinear Systems

Find the solution to each of the following system of equations.

1. $$\begin{aligned} y &= x^2 + 6x - 8 \\ y &= 4x + 7 \end{aligned}$$

2. $$\begin{aligned} y &= 1 - 3x \\ \tfrac{x^2}{4} + y^2 &= 1 \end{aligned}$$

3. $$\begin{aligned} xy &= 4 \\ \tfrac{x^2}{4} + \tfrac{y^2}{25} &= 1 \end{aligned}$$

4. $$\begin{aligned} y &= 1 - 2x^2 \\ x^2 - \tfrac{y^2}{9} &= 1 \end{aligned}$$