Section 5-2 : Computing Definite Integrals

1. Evaluate $\int 4x^6 - 2x^3 + 7x - 4\,dx$.

Solution

There really isn't too much to do other than to evaluate the integral.

$$\int 4x^6 - 2x^3 + 7x - 4\,dx = \tfrac{4}{7}x^7 - \tfrac{2}{4}x^4 + \tfrac{7}{2}x^2 - 4x + c = \boxed{\tfrac{4}{7}x^7 - \tfrac{1}{2}x^4 + \tfrac{7}{2}x^2 - 4x + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

2. Evaluate $\int z^7 - 48z^{11} - 5z^{16}\,dz$.

Solution

There really isn't too much to do other than to evaluate the integral.

$$\int z^7 - 48z^{11} - 5z^{16}\,dz = \tfrac{1}{8}z^8 - \tfrac{48}{12}z^{12} - \tfrac{5}{17}z^{17} + c = \boxed{\tfrac{1}{8}z^8 - 4z^{12} - \tfrac{5}{17}z^{17} + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

3. Evaluate $\int 10t^{-3} + 12t^{-9} + 4t^3\,dt$.

Solution

There really isn't too much to do other than to evaluate the integral.

$$\int 10t^{-3} + 12t^{-9} + 4t^3\,dt = \tfrac{10}{-2}t^{-2} + \tfrac{12}{-8}t^{-8} + \tfrac{4}{4}t^4 + c = \boxed{-5t^{-2} - \tfrac{3}{2}t^{-8} + t^4 + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

4. Evaluate $\int w^{-2} + 10w^{-5} - 8\,dw$.

Solution

There really isn't too much to do other than to evaluate the integral.

$$\int w^{-2} + 10w^{-5} - 8\,dw = \tfrac{1}{-1}w^{-1} + \tfrac{10}{-4}w^{-4} - 8w + c = \boxed{-w^{-1} - \tfrac{5}{2}w^{-4} - 8w + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

5. Evaluate $\int 12\,dy$.

Solution

There really isn't too much to do other than to evaluate the integral.

$$\int 12\,dy = \boxed{12y + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

6. Evaluate $\int \sqrt[3]{w} + 10\,\sqrt[5]{w^3}\,dw$.

Hint : Don't forget to convert the roots to fractional exponents.

Step 1

We first need to convert the roots to fractional exponents.

$$\int \sqrt[3]{w} + 10\,\sqrt[5]{w^3}\,dw = \int w^{\frac{1}{3}} + 10\left(w^3\right)^{\frac{1}{5}}\,dw = \int w^{\frac{1}{3}} + 10w^{\frac{3}{5}}\,dw$$

Step 2

Once we've gotten the roots converted to fractional exponents there really isn't too much to do other than to evaluate the integral.

$$\int \sqrt[3]{w} + 10\,\sqrt[5]{w^3}\,dw = \int w^{\frac{1}{3}} + 10w^{\frac{3}{5}}\,dw = \tfrac{3}{4}w^{\frac{4}{3}} + 10\left(\tfrac{5}{8}\right)w^{\frac{8}{5}} + c = \boxed{\tfrac{3}{4}w^{\frac{4}{3}} + \tfrac{25}{4}w^{\frac{8}{5}} + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

7. Evaluate $\int \sqrt{x^7} - 7\sqrt[6]{x^5} + 17\sqrt[3]{x^{10}}\,dx$.

Hint : Don't forget to convert the roots to fractional exponents.

Step 1
We first need to convert the roots to fractional exponents.

$$\int \sqrt{x^7} - 7\sqrt[6]{x^5} + 17\sqrt[3]{x^{10}}\,dx = \int x^{\frac{7}{2}} - 7\left(x^5\right)^{\frac{1}{6}} + 17\left(x^{10}\right)^{\frac{1}{3}}\,dx = \int x^{\frac{7}{2}} - 7x^{\frac{5}{6}} + 17x^{\frac{10}{3}}\,dx$$

Step 2
Once we've gotten the roots converted to fractional exponents there really isn't too much to do other than to evaluate the integral.

$$\begin{aligned}\int \sqrt{x^7} - 7\sqrt[6]{x^5} + 17\sqrt[3]{x^{10}}\,dx &= \int x^{\frac{7}{2}} - 7x^{\frac{5}{6}} + 17x^{\frac{10}{3}}\,dx \\ &= \tfrac{2}{9}x^{\frac{9}{2}} - 7\left(\tfrac{6}{11}\right)x^{\frac{11}{6}} + 17\left(\tfrac{3}{13}\right)x^{\frac{13}{3}} + c = \boxed{\tfrac{2}{9}x^{\frac{9}{2}} - \tfrac{42}{11}x^{\frac{11}{6}} + \tfrac{51}{13}x^{\frac{13}{3}} + c}\end{aligned}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

8. Evaluate $\int \frac{4}{x^2} + 2 - \frac{1}{8x^3}\,dx$.

Hint : Don't forget to move the *x*'s in the denominator to the numerator with negative exponents.

Step 1
We first need to move the *x*'s in the denominator to the numerator with negative exponents.

$$\int \frac{4}{x^2} + 2 - \frac{1}{8x^3}\,dx = \int 4x^{-2} + 2 - \frac{1}{8}x^{-3}\,dx$$

Remember that the "8" in the denominator of the third term stays in the denominator and does not move up with the *x*.

Step 2

Once we've gotten the *x*'s out of the denominator there really isn't too much to do other than to evaluate the integral.

$$\int \frac{4}{x^2} + 2 - \frac{1}{8x^3}\,dx = \int 4x^{-2} + 2 - \frac{1}{8}x^{-3}\,dx$$

$$= 4\left(\frac{1}{-1}\right)x^{-1} + 2x - \frac{1}{8}\left(\frac{1}{-2}\right)x^{-2} + c = \boxed{-4x^{-1} + 2x + \frac{1}{16}x^{-2} + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

9. Evaluate $\displaystyle\int \frac{7}{3y^6} + \frac{1}{y^{10}} - \frac{2}{\sqrt[3]{y^4}}\,dy$.

Hint : Don't forget to convert the root to a fractional exponents and move the *y*'s in the denominator to the numerator with negative exponents.

Step 1

We first need to convert the root to a fractional exponent and move the *y*'s in the denominator to the numerator with negative exponents.

$$\int \frac{7}{3y^6} + \frac{1}{y^{10}} - \frac{2}{\sqrt[3]{y^4}}\,dy = \int \frac{7}{3y^6} + \frac{1}{y^{10}} - \frac{2}{y^{\frac{4}{3}}}\,dy = \int \frac{7}{3}y^{-6} + y^{-10} - 2y^{-\frac{4}{3}}\,dy$$

Remember that the "3" in the denominator of the first term stays in the denominator and does not move up with the *y*.

Step 2

Once we've gotten the root converted to a fractional exponent and the *y*'s out of the denominator there really isn't too much to do other than to evaluate the integral.

$$\int \frac{7}{3y^6} + \frac{1}{y^{10}} - \frac{2}{\sqrt[3]{y^4}}\,dy = \int \frac{7}{3}y^{-6} + y^{-10} - 2y^{-\frac{4}{3}}\,dy$$

$$= \frac{7}{3}\left(\frac{1}{-5}\right)y^{-5} + \left(\frac{1}{-9}\right)y^{-9} - 2\left(-\frac{3}{1}\right)y^{-\frac{1}{3}} + c$$

$$= \boxed{-\frac{7}{15}y^{-5} - \frac{1}{9}y^{-9} + 6y^{-\frac{1}{3}} + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

10. Evaluate $\int \left(t^2 - 1\right)\left(4 + 3t\right) dt$.

Hint : Remember that there is no "Product Rule" for integrals and so we'll need to eliminate the product before integrating.

Step 1
Since there is no "Product Rule" for integrals we'll need to multiply the terms out prior to integration.

$$\int \left(t^2 - 1\right)\left(4 + 3t\right) dt = \int 3t^3 + 4t^2 - 3t - 4\, dt$$

Step 2
At this point there really isn't too much to do other than to evaluate the integral.

$$\int \left(t^2 - 1\right)\left(4 + 3t\right) dt = \int 3t^3 + 4t^2 - 3t - 4\, dt = \boxed{\tfrac{3}{4}t^4 + \tfrac{4}{3}t^3 - \tfrac{3}{2}t^2 - 4t + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

11. Evaluate $\int \sqrt{z}\left(z^2 - \frac{1}{4z}\right) dz$.

Hint : Remember that there is no "Product Rule" for integrals and so we'll need to eliminate the product before integrating.

Step 1
Since there is no "Product Rule" for integrals we'll need to multiply the terms out prior to integration.

$$\int \sqrt{z}\left(z^2 - \frac{1}{4z}\right) dz = \int z^{\frac{5}{2}} - \frac{1}{4z^{\frac{1}{2}}}\, dz = \int z^{\frac{5}{2}} - \frac{1}{4} z^{-\frac{1}{2}}\, dz$$

Don't forget to convert the root to a fractional exponent and move the *z*'s out of the denominator.

Step 2
At this point there really isn't too much to do other than to evaluate the integral.

$$\int \sqrt{z}\left(z^2 - \frac{1}{4z}\right)dz = \int z^{\frac{5}{2}} - \frac{1}{4}z^{-\frac{1}{2}}\,dz = \boxed{\frac{2}{7}z^{\frac{7}{2}} - \frac{1}{2}z^{\frac{1}{2}} + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

12. Evaluate $\displaystyle\int \frac{z^8 - 6z^5 + 4z^3 - 2}{z^4}\,dz$.

Hint : Remember that there is no "Quotient Rule" for integrals and so we'll need to eliminate the quotient before integrating.

Step 1
Since there is no "Quotient Rule" for integrals we'll need to break up the integrand and simplify a little prior to integration.

$$\int \frac{z^8 - 6z^5 + 4z^3 - 2}{z^4}\,dz = \int \frac{z^8}{z^4} - \frac{6z^5}{z^4} + \frac{4z^3}{z^4} - \frac{2}{z^4}\,dz = \int z^4 - 6z + \frac{4}{z} - 2z^{-4}\,dz$$

Step 2
At this point there really isn't too much to do other than to evaluate the integral.

$$\int \frac{z^8 - 6z^5 + 4z^3 - 2}{z^4}\,dz = \int z^4 - 6z + \frac{4}{z} - 2z^{-4}\,dz = \boxed{\frac{1}{5}z^5 - 3z^2 + 4\ln|z| + \frac{2}{3}z^{-3} + c}$$

Don't forget to add on the "+c" since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

13. Evaluate $\displaystyle\int \frac{x^4 - \sqrt[3]{x}}{6\sqrt{x}}\,dx$.

Hint : Remember that there is no "Quotient Rule" for integrals and so we'll need to eliminate the quotient before integrating.

Step 1
Since there is no "Quotient Rule" for integrals we'll need to break up the integrand and simplify a little prior to integration.

$$\int \frac{x^4 - \sqrt[3]{x}}{6\sqrt{x}}\,dx = \int \frac{x^4}{6x^{\frac{1}{2}}} - \frac{x^{\frac{1}{3}}}{6x^{\frac{1}{2}}}\,dx = \int \frac{1}{6}x^{\frac{7}{2}} - \frac{1}{6}x^{-\frac{1}{6}}\,dx$$

Don’t forget to convert the roots to fractional exponents!

Step 2
At this point there really isn’t too much to do other than to evaluate the integral.

$$\int \frac{x^4 - \sqrt[3]{x}}{6\sqrt{x}}\,dx = \int \frac{1}{6}x^{\frac{7}{2}} - \frac{1}{6}x^{-\frac{1}{6}}\,dx = \boxed{\frac{1}{27}x^{\frac{9}{2}} - \frac{1}{5}x^{\frac{5}{6}} + c}$$

Don’t forget to add on the “+c” since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

14. Evaluate $\int \sin(x) + 10\csc^2(x)\,dx$.

Solution
There really isn’t too much to do other than to evaluate the integral.

$$\int \sin(x) + 10\csc^2(x)\,dx = \boxed{-\cos(x) - 10\cot(x) + c}$$

Don’t forget to add on the “+c” since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

15. Evaluate $\int 2\cos(w) - \sec(w)\tan(w)\,dw$.

Solution
There really isn’t too much to do other than to evaluate the integral.

$$\int 2\cos(w) - \sec(w)\tan(w)\,dw = \boxed{2\sin(w) - \sec(w) + c}$$

Don’t forget to add on the “+c” since we know that we are asking what function did we differentiate to get the integrand and the derivative of a constant is zero and so we do need to add that onto the answer.

---

16. Evaluate $\int 12+\csc(\theta)\left[\sin(\theta)+\csc(\theta)\right]d\theta$.

Hint : From previous problems in this set we should know how to deal with the product in the integrand.

Step 1

Before doing the integral we need to multiply out the product and don't forget the definition of cosecant in terms of sine.

$$\begin{aligned}\int 12+\csc(\theta)\left[\sin(\theta)+\csc(\theta)\right]d\theta &= \int 12+\csc(\theta)\sin(\theta)+\csc^2(\theta)\,d\theta \\ &= \int 13+\csc^2(\theta)\,d\theta\end{aligned}$$

Recall that,

$$\csc(\theta)=\frac{1}{\sin(\theta)}$$

and so,

$$\csc(\theta)\sin(\theta)=1$$

Doing this allows us to greatly simplify the integrand and, in fact, allows us to actually do the integral. Without this simplification we would not have been able to integrate the second term with the knowledge that we currently have.

Step 2

At this point there really isn't too much to do other than to evaluate the integral.

$$\int 12+\csc(\theta)\left[\sin(\theta)+\csc(\theta)\right]d\theta = \int 13+\csc^2(\theta)\,d\theta = \boxed{13\theta-\cot(\theta)+c}$$

Don't forget that with trig functions some terms can be greatly simplified just by recalling the definition of the trig functions and/or their relationship with the other trig functions.

---

17. Evaluate $\int 4\mathbf{e}^z+15-\frac{1}{6z}\,dz$.

Solution

There really isn't too much to do other than to evaluate the integral.

$$\int 4\mathbf{e}^z+15-\frac{1}{6z}\,dz = \int 4\mathbf{e}^z+15-\frac{1}{6}\frac{1}{z}\,dz = \boxed{4\mathbf{e}^z+15z-\frac{1}{6}\ln|z|+c}$$

Be careful with the "6" in the denominator of the third term. The "best" way of dealing with it in this case is to split up the third term as we've done above and then integrate.

Note that the "best" way to do a problem is always relative for many Calculus problems. There are other ways of dealing with this term (later section material) and so what one person finds the best another may not. For us, this seems to be an easy way to deal with the 6 and not overly complicate the integration process.

---

18. Evaluate $\int t^3 - \frac{\mathbf{e}^{-t} - 4}{\mathbf{e}^{-t}}\, dt$.

Hint : From previous problems in this set we should know how to deal with the quotient in the integrand.

Step 1
Before doing the integral we need to break up the quotient and do some simplification.

$$\int t^3 - \frac{\mathbf{e}^{-t} - 4}{\mathbf{e}^{-t}}\, dt = \int t^3 - \frac{\mathbf{e}^{-t}}{\mathbf{e}^{-t}} + \frac{4}{\mathbf{e}^{-t}}\, dt = \int t^3 - 1 + 4\mathbf{e}^{t}\, dt$$

Make sure that you correctly distribute the minus sign when breaking up the second term and don't forget to move the exponential in the denominator of the third term (after splitting up the integrand) to the numerator and changing the sign on the *t* to a "+" in the process.

Step 2
At this point there really isn't too much to do other than to evaluate the integral.

$$\int t^3 - \frac{\mathbf{e}^{-t} - 4}{\mathbf{e}^{-t}}\, dt = \int t^3 - 1 + 4\mathbf{e}^{t}\, dt = \boxed{\frac{1}{4}t^4 - t + 4\mathbf{e}^{t} + c}$$

---

19. Evaluate $\int \frac{6}{w^3} - \frac{2}{w}\, dw$.

Solution
There really isn't too much to do other than to evaluate the integral.

$$\int \frac{6}{w^3} - \frac{2}{w}\, dw = \int 6w^{-3} - \frac{2}{w}\, dw = \boxed{-3w^{-2} - 2\ln|w| + c}$$

---

20. Evaluate $\int \frac{1}{1+x^2} + \frac{12}{\sqrt{1-x^2}}\,dx$.

Solution

There really isn't too much to do other than to evaluate the integral.

$$\int \frac{1}{1+x^2} + \frac{12}{\sqrt{1-x^2}}\,dx = \boxed{\tan^{-1}(x) + 12\sin^{-1}(x) + c}$$

Note that because of the similarity of the derivative of inverse sine and inverse cosine an alternate answer is,

$$\int \frac{1}{1+x^2} + \frac{12}{\sqrt{1-x^2}}\,dx = \boxed{\tan^{-1}(x) - 12\cos^{-1}(x) + c}$$

---

21. Evaluate $\int 6\cos(z) + \frac{4}{\sqrt{1-z^2}}\,dz$.

Solution

There really isn't too much to do other than to evaluate the integral.

$$\int 6\cos(z) + \frac{4}{\sqrt{1-z^2}}\,dz = \boxed{6\sin(z) + 4\sin^{-1}(z) + c}$$

Note that because of the similarity of the derivative of inverse sine and inverse cosine an alternate answer is,

$$\int 6\cos(z) + \frac{4}{\sqrt{1-z^2}}\,dz = \boxed{6\sin(z) - 4\cos^{-1}(z) + c}$$

---

22. Determine $f(x)$ given that $f'(x) = 12x^2 - 4x$ and $f(-3) = 17$.

Hint : We know that integration is simply asking what function we differentiated to get the integrand and so we should be able to use this idea to arrive at a general formula for the function.

Step 1

Recall from the notes in this section that we saw,

$$f\left(x\right) = \int f'\left(x\right)\,dx$$

and so to arrive at a general formula for $f\left(x\right)$ all we need to do is integrate the derivative that we've been given in the problem statement.

$$f\left(x\right) = \int 12x^2 - 4x\,dx = 4x^3 - 2x^2 + c$$

Don't forget the "+c"!

Hint : To determine the value of the constant of integration, *c*, we have the value of the function at $x = -3$.

Step 2

Because we have the condition that $f\left(-3\right) = 17$ we can just plug $x = -3$ into our answer from the previous step, set the result equal to 17 and solve the resulting equation for *c*.

Doing this gives,

$$17 = f\left(-3\right) = -126 + c \qquad \Rightarrow \qquad c = 143$$

The function is then,

$$\boxed{f\left(x\right) = 4x^3 - 2x^2 + 143}$$

---

23. Determine $g\left(z\right)$ given that $g'\left(z\right) = 3z^3 + \dfrac{7}{2\sqrt{z}} - \mathbf{e}^z$ and $g\left(1\right) = 15 - \mathbf{e}$.

Hint : We know that integration is simply asking what function we differentiated to get the integrand and so we should be able to use this idea to arrive at a general formula for the function.

Step 1

Recall from the notes in this section that we saw,

$$g\left(z\right) = \int g'\left(z\right)\,dz$$

and so to arrive at a general formula for $g\left(z\right)$ all we need to do is integrate the derivative that we've been given in the problem statement.

$$g\left(z\right) = \int 3z^3 + \tfrac{7}{2}z^{-\frac{1}{2}} - \mathbf{e}^z\,dz = \tfrac{3}{4}z^4 + 7z^{\frac{1}{2}} - \mathbf{e}^z + c$$

Don't forget the "+c"!

Hint : To determine the value of the constant of integration, *c*, we have the value of the function at $z=1$.

Step 2
Because we have the condition that $g\left(1\right)=15-\mathbf{e}$ we can just plug $z=1$ into our answer from the previous step, set the result equal to 15 – **e** and solve the resulting equation for *c*.

Doing this gives,

$$15-\mathbf{e}=g\left(1\right)=\tfrac{31}{4}-\mathbf{e}+c \qquad \Rightarrow \qquad c=\tfrac{29}{4}$$

The function is then,

$$\boxed{g\left(z\right)=\tfrac{3}{4}z^{4}+7z^{\frac{1}{2}}-\mathbf{e}^{z}+\tfrac{29}{4}}$$

---

24. Determine $h\left(t\right)$ given that $h''\left(t\right)=24t^{2}-48t+2$, $h\left(1\right)=-9$ and $h\left(-2\right)=-4$.

Hint : We know how to find $h\left(t\right)$ from $h'\left(t\right)$ but we don't have that. We should however be able to determine the general formula for $h'\left(t\right)$ from $h''\left(t\right)$ which we are given.

Step 1
Because we know that the 2nd derivative is just the derivative of the 1st derivative we know that,

$$h'\left(t\right)=\int h''\left(t\right)\,dt$$

and so to arrive at a general formula for $h'\left(t\right)$ all we need to do is integrate the 2nd derivative that we've been given in the problem statement.

$$h'\left(t\right)=\int 24t^{2}-48t+2\;dt=8t^{3}-24t^{2}+2t+c$$

Don't forget the "+c"!

Hint : From the previous two problems you should be able to determine a general formula for $h\left(t\right)$. Just don't forget that *c* is just a constant!

Step 2
Now, just as we did in the previous two problems, all that we need to do is integrate the 1st derivative (which we found in the first step) to determine a general formula for $h\left(t\right)$.

$$h(t) = \int 8t^3 - 24t^2 + 2t + c \; dt = 2t^4 - 8t^3 + t^2 + ct + d$$

Don't forget that *c* is just a constant and so it will integrate just like we were integrating 2 or 4 or any other number. Also, the constant of integration from this step is liable to be different that the constant of integration from the first step and so we'll need to make sure to call it something different, *d* in this case.

Hint : To determine the value of the constants of integration, *c* and *d*, we have the value of the function at two values that should help with that.

Step 3
Now, we know the value of the function at two values of *z*. So let's plug both of these into the general formula for $h(t)$ that we found in the previous step to get,

$$\begin{aligned} -9 &= h(1) = -5 + c + d \\ -4 &= h(-2) = 100 - 2c + d \end{aligned}$$

Solving this system of equations (you do remember your Algebra class right?) for *c* and *d* gives,

$$c = \tfrac{100}{3} \qquad\qquad d = -\tfrac{112}{3}$$

The function is then,

$$\boxed{h(t) = 2t^4 - 8t^3 + t^2 + \tfrac{100}{3}t - \tfrac{112}{3}}$$