

Problem 1

$$a_2 = 120000, a_5 = 50625$$

This is a geometric sequence

Find a_n

$$a_n = a_1 r^{n-1}$$

$$a_2 = a_1 r^1 \quad \& \quad a_5 = a_1 r^4$$

$$120000 = a_1 r^1 \quad \& \quad 50625 = a_1 r^4$$

$$120000 = a_1 r^1 \text{ & } 50625 = a_1 r^4$$

$$\frac{50625}{120000} = \frac{a_1 r^4}{a_1 r^1} \rightarrow \frac{50625}{120000} = \frac{r^4}{r^1} \rightarrow \frac{50625}{120000} = r^3$$

$$\sqrt[3]{\frac{50625}{120000}} = r \rightarrow \sqrt[3]{\frac{27}{64}} = r \rightarrow r = \frac{3}{4}$$

We have r now!

$$120000 = a_1 r^1 \text{ with } r = \frac{3}{4}$$

$$120000 = \frac{3}{4} a_1$$

$$\frac{120000}{1} \cdot \frac{4}{3} = a_1$$

$$160000 = a_1$$

$$a_n = 160000 \left(\frac{3}{4}\right)^{n-1}$$

$$50625 = a_1 r^4 \text{ with } r = \frac{3}{4} \rightarrow 50625 = a_1 \cdot \left(\frac{3}{4}\right)^4$$

$$50625 = \frac{81}{256} a_1$$

$$50625 \cdot \frac{256}{81} = a_1$$

$$160000 = a_1$$

$$a_n = 160000 \left(\frac{3}{4}\right)^{n-1}$$

Problem 2

Problem 2

Given sequence

{1600, 1500, 1400, 1300, 1200, 1100}

Find S_{20}

Find a_n , a_{20} , S_n , & S_{20}

$$a_n = 1600 + (n-1)(-100)$$

$$a_{20} = 1600 + (20-1) \cdot -100 \rightarrow a_{20} = -300$$

$$S_n = \frac{n}{2}(1600 + a_n)$$

$$S_{20} = \frac{20}{2} \cdot (1600 + -300) \rightarrow S_{20} = 13000$$

$$S_{20} = \frac{20}{2} \cdot (3200 + (20-1) \cdot -100) \rightarrow S_{20} = 13000$$

This is an arithmetic sequence

$$a_n = a_1 + (n-1) \cdot d$$

$$d = a_n - a_{n-1}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

Problem 3

Problem 3

Given sequence

$$\{2400, -3600, 5400, -8100\}$$

Find a_{10}

$$r = \frac{-3600}{2400} \rightarrow r = \frac{-3}{2}$$

$$a_n = 2400 \left(\frac{-3}{2} \right)^{n-1}$$

$$a_{10} = 2400 \cdot \left(\frac{-3}{2} \right)^{10-1} \rightarrow a_{10} = \frac{-1476225}{16}$$

$$a_{10} = 2400 \cdot \left(\frac{-3}{2} \right)^{10-1} \rightarrow a_{10} = -92264.0625$$

This is a geometric sequence

$$a_n = a_1 r^{n-1}$$

$$r = \frac{a_n}{a_{n-1}}$$

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$S_\infty = \frac{a_1}{1-r} \text{ when } 0 < |r| < 1$$

Problem 4

Problem 4

$$256+248+240+232+224+\dots+184+176$$

Determine the number of terms in this series

$$d = 248 - 256 \rightarrow -8$$

$$a_n = 256 + (n-1)(-8)$$

$$176 = 256 - 8n + 8$$

$$176 = 264 - 8n$$

$$176 - 264 = -8n$$

$$-88 = -8n$$

$$\frac{-88}{-8} = \frac{-8n}{-8}$$

$$n = 11$$

$$a_n = 256 + (11-1) \cdot -8 \rightarrow a_n = 176$$

This is an arithmetic sequence

$$a_n = a_1 + (n-1) \cdot d$$

$$d = a_n - a_{n-1}$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

Problem 4

$$256+248+240+232+224+\dots+184+176$$

Determine the number of terms in this series

$$d = 248 - 256$$

$$a_n = 256 + (n-1) \cdot (-8)$$

$$176 = 256 + (n-1) \cdot (-8)$$

$$176 - 256 = (n-1) \cdot -8$$

$$-80 = (n-1)(-8)$$

$$\frac{-80}{-8} = \frac{(n-1)(-8)}{-8}$$

$$10 = n - 1$$

$$n = 11$$

$$a_n = 256 + (11-1) \cdot -8$$

This is an arithmetic sequence

$$a_n = a_1 + (n-1) \cdot d$$

$$d = a_n - a_{n-1}$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

Problem 5

Problem 5

Determine the number of terms in the sequence

$$250+255+260+265+\dots+a_n = 12150$$

$$d = 255 - 250 = 5$$

$$a_n = 250 + (n-1) \cdot 5$$

$$S_n = \frac{n}{2} (250 + a_n)$$

$$S_n = \frac{n}{2} \cdot (500 + (n-1) \cdot 5)$$

$$121500 = \frac{n}{2} (500 + (n-1) \cdot 5)$$

This is an arithmetic sequence

$$a_n = a_1 + (n-1) \cdot d$$

$$d = a_n - a_{n-1}$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

$$12150 = \frac{n}{2} \cdot (500 + (n-1) \cdot 5) \rightarrow 121500 = \frac{n}{2} (500 + 5n - 5)$$

$$12150 = \frac{n}{2} (495 + 5n) \rightarrow 121500 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2}$$

$$12150 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} \rightarrow 0 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} - 12150$$

$$0 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} - 12150$$

$$a = \frac{5}{2} \quad b = \frac{495}{2} \quad c = -12150 \quad \text{Use Quadratic Formula}$$

$$D = b^2 - 4 \cdot a \cdot c \text{ with } a = \frac{5}{2} \quad b = \frac{495}{2} \quad c = -12150$$

$$D = \left(\frac{495}{2}\right)^2 - 4 \cdot \frac{5}{2} \cdot -12150 = \frac{731025}{4}$$

$$n = \frac{-\frac{495}{2} + \sqrt{\frac{731025}{4}}}{2 \cdot \frac{5}{2}} = 36 \text{ or } n = \frac{-\frac{495}{2} - \sqrt{\frac{731025}{4}}}{2 \cdot \frac{5}{2}} = -135$$

since the number of terms in a sequence must be positive

$$n = 36 \quad S_{36} = \frac{36}{2} \cdot (500 + (36-1) \cdot 5) \rightarrow s_{36} = 12150$$

$$12150 = \frac{n}{2} \cdot (500 + (n-1) \cdot 5) \rightarrow 121500 = \frac{n}{2} (500 + 5n - 5)$$

$$12150 = \frac{n}{2} (495 + 5n) \rightarrow \text{expand} \left(121500 = \frac{n}{2} \cdot (495 + 5 \cdot n) \right)$$

$$12150 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} \rightarrow 0 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} - 12150$$

$$0 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} - 12150$$

$$a = 2.5 \quad b = 247.5 \quad c = -12150$$

Use Quadratic Formula

$$D = b^2 - 4 \cdot a \cdot c \text{ with } a = 2.5 \quad b = 247.5 \quad c = -12150$$

$$D = (247.5)^2 - 4 \cdot 2.5 \cdot -12150 = 182756.25$$

$$n = \frac{-247.5 + \sqrt{182756.25}}{2 \cdot 2.5} = 36 \text{ or } n = \frac{-247.5 - \sqrt{182756.25}}{2 \cdot 2.5} \rightarrow -135.$$

since the number of terms in a sequence must be positive

$$n = 36 \quad S_{36} = \frac{36}{2} \cdot (500 + (36-1) \cdot 5) \rightarrow s_{36} = 12150$$

$$12150 = \frac{n}{2} \cdot (500 + (n-1) \cdot 5) \rightarrow 121500 = \frac{n}{2} (500 + 5n - 5)$$

$$12150 = \frac{n}{2} (495 + 5n) \rightarrow \text{expand} \left(121500 = \frac{n}{2} \cdot (495 + 5 \cdot n) \right)$$

$$12150 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} \rightarrow 0 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} - 12150$$

$$0 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} - 12150 \rightarrow 0 = 5n^2 + 495n - 24300$$

$$a = 5 \quad b = 495 \quad c = -24300$$

Use Quadratic Formula

$$D = b^2 - 4 \cdot a \cdot c \text{ with } a=5 \quad b=495 \quad c=-24300$$

$$D = 495^2 - 4 \cdot 5 \cdot -24300 = 731025$$

$$n = \frac{-495 + \sqrt{731025}}{2 \cdot 5} = 36 \text{ or } n = \frac{-495 - \sqrt{731025}}{2 \cdot 5} \rightarrow -135$$

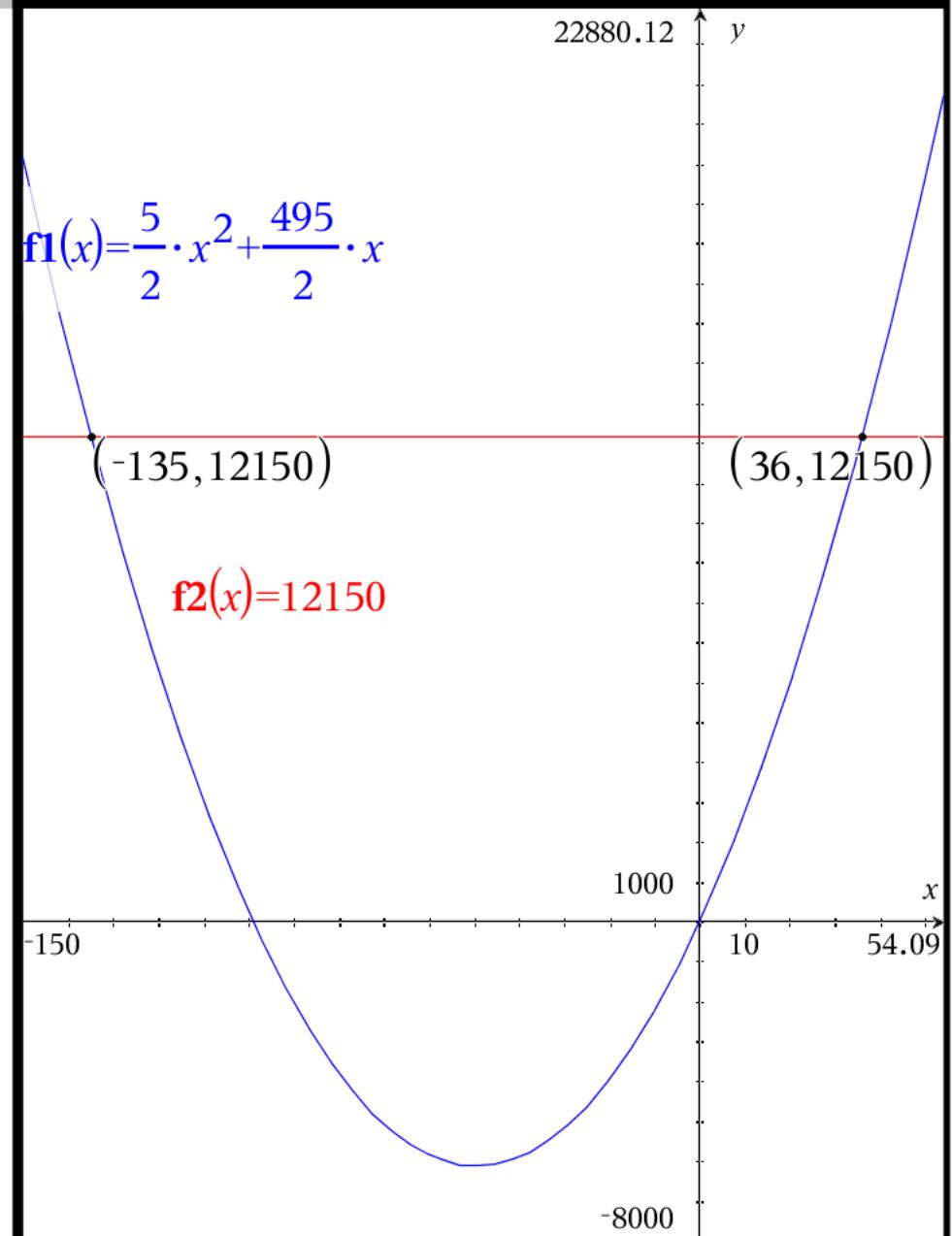
since the number of terms in a sequence must be positive

$$n = 36 \quad S_{36} = \frac{36}{2} \cdot (500 + (36-1) \cdot 5) \rightarrow s_{36} = 12150$$

You can also use a graph of a quadratic function to help determine n

$$12150 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2}$$

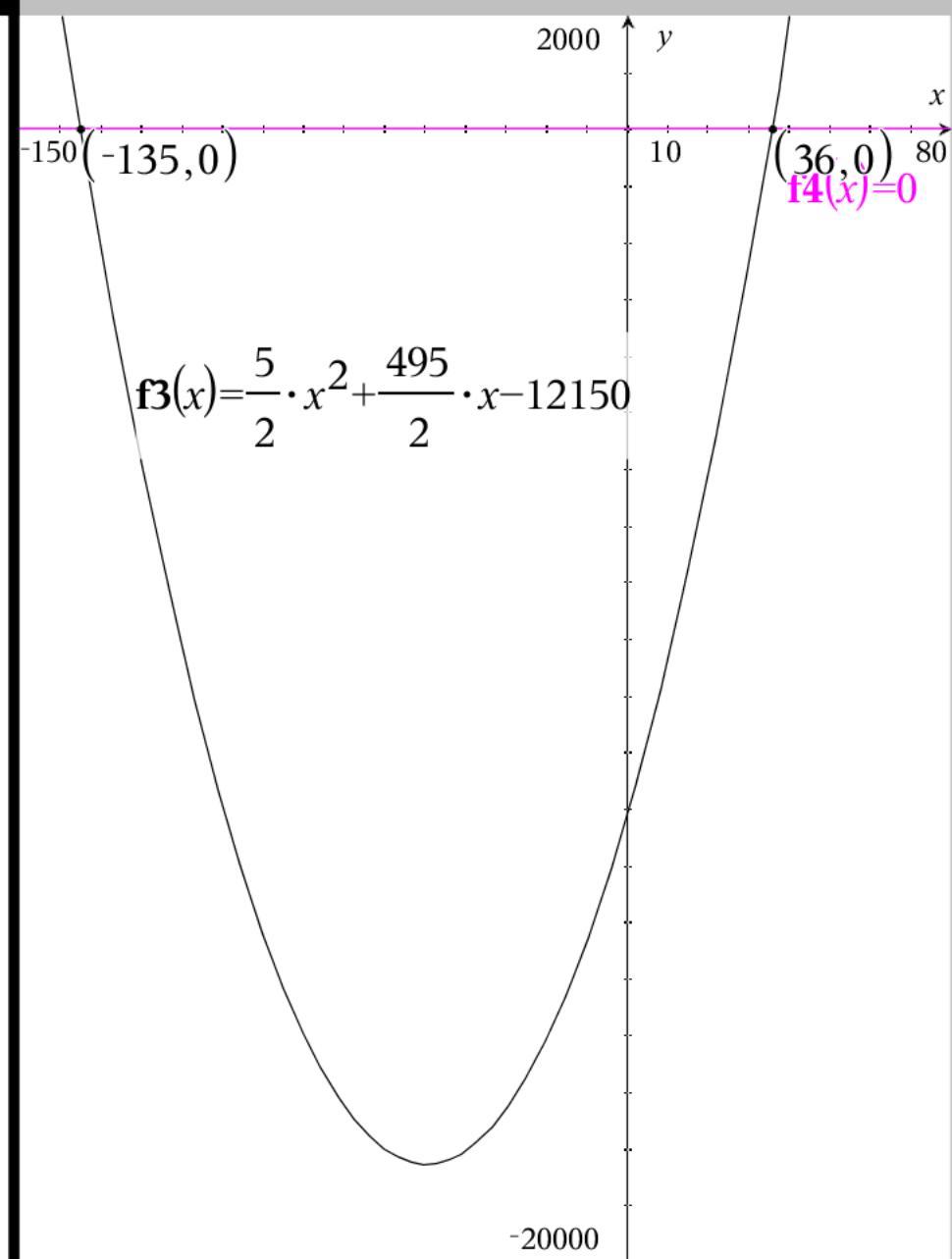
To receive credit for work, you would have to make a quick sketch showing the graph to the right



You can also use a graph of a quadratic function to help determine n

$$0 = \frac{5 \cdot n^2}{2} + \frac{495 \cdot n}{2} - 12150$$

To receive credit for work, you would have to make a quick sketch showing the graph to the right



Problem 6

Problem 6

Failure to SHOW WORK will result in no credit

Determine the following sum

$$\sum_{101}^{200} 10 + (n-1)(5)$$

Method 1: Find S_{200} & S_{100} and subtract

$$a_n = 10 + (n-1) \cdot 5$$

$$a_{100} = 10 + (100-1) \cdot 5 \rightarrow a_{100} = 505$$

$$a_{200} = 10 + (200-1) \cdot 5 \rightarrow a_{200} = 1005$$

Method 2: Find a_{101} and write new sequence

$$a_{100} = 10 + (101-1) \cdot 5 \rightarrow a_{100} = 510$$

This is an arithmetic sequence

$$a_n = a_1 + (n-1) \cdot d$$

$$d = a_n - a_{n-1}$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

Method 1 $S_n = \frac{n}{2}(10 + a_n)$ $a_{100} = 505$ & $a_{200} = 1005$

$$S_{200} = \frac{200}{2} \cdot (10 + 1005) \rightarrow S_{200} = 101500$$

$$S_{100} = \frac{100}{2} \cdot (10 + 505) \rightarrow S_{100} = 25750$$

$$S_{200} - S_{100} = 101500 - 25750 = 75750$$

$$\text{Method 2 } S_n = \frac{n}{2} (510 + b_n) \sum_{n=101}^{200} (10 + (n-1) \cdot 5) \rightarrow \sum_{n=1}^{100} (510 + (n-1) \cdot 5)$$

$$b_n = 510 + (n-1) \cdot 5 \quad b_{100} = 510 + (100-1) \cdot 5 \rightarrow b_{100} = 1005$$

$$S_{100} = \frac{100}{2} \cdot (510 + 1005) \rightarrow S_{100} = 75750$$

Problem 7

Problem 7

Failure to SHOW WORK will result in no credit

Determine the following sum

$$\sum_{n=1}^{\infty} 12 \left(\frac{7}{16}\right)^{n-1}$$

$$S_{\infty} = \frac{12}{1 - \frac{7}{16}} = \frac{12}{\frac{9}{16}}$$

$$= \frac{12}{1} \cdot \frac{16}{9} = \frac{64}{3}$$

$$= \frac{12}{1} \cdot \frac{16}{9} \approx 21.33$$

This is a geometric sequence

$$a_n = a_1 r^{n-1}$$

$$r = \frac{a_n}{a_{n-1}}$$

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$S_{\infty} = \frac{a_1}{1-r} \text{ when } 0 < |r| < 1$$

Problem 8

Problem 8

Failure to SHOW WORK will result in no credit

Determine the following sum

$$\sum_{n=8}^{19} 5(-2)^{n-1}$$

$$s_{19}=5 \cdot \frac{1-(-2)^{19}}{1-(-2)} = s_{19}=873815$$

$$s_7=5 \cdot \frac{1-(-2)^7}{1-(-2)} = s_7=215$$

$$S_{19}-S_7=873815-215 = 873600$$

This is a geometric sequence

$$a_n=a_1 r^{n-1}$$

$$r = \frac{a_n}{a_{n-1}}$$

$$S_n=a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$S_\infty=\frac{a_1}{1-r} \text{ when } 0 < |r| < 1$$