Section 4-8 : Optimization

1. Find two positive numbers whose sum of six times one of them and the second is 250 and whose product is a maximum.

2. Find two positive numbers whose sum of twice the first and seven times the second is 600 and whose product is a maximum.

3. Let x and y be two positive numbers whose sum is 175 and $(x+3)(y+4)$ is a maximum. Determine x and y.

4. Find two positive numbers such that the sum of the one and the square of the other is 200 and whose product is a maximum.

5. Find two positive numbers whose product is 400 and such that the sum of twice the first and three times the second is a minimum.

6. Find two positive numbers whose product is 250 and such that the sum of the first and four times the second is a minimum.

7. Let x and y be two positive numbers such that $y(x+2)=100$ and whose sum is a minimum. Determine x and y.

8. Find a positive number such that the sum of the number and its reciprocal is a minimum.

9. We are going to fence in a rectangular field and have 200 feet of material to construct the fence. Determine the dimensions of the field that will enclose the maximum area.

10. We are going to fence in a rectangular field. Starting at the bottom of the field and moving around the field in a counter clockwise manner the cost of material for each side is $6/ft, $9/ft, $12/ft and $14/ft respectively. If we have $1000 to buy fencing material determine the dimensions of the field that will maximize the enclosed area.

11. We are going to fence in a rectangular field that encloses 75 ft². Determine the dimensions of the field that will require the least amount of fencing material to be used.

12. We are going to fence in a rectangular field that encloses 200 m². If the cost of the material for of one pair of parallel sides is $3/m and cost of the material for the other pair of parallel sides is $8/m determine the dimensions of the field that will minimize the cost to build the fence around the field.

13. Show that a rectangle with a fixed area and minimum perimeter is a square.

14. Show that a rectangle with a fixed perimeter and a maximum area is a square.

15. We have 350 m² of material to build a box whose base width is four times the base length. Determine the dimensions of the box that will maximize the enclosed volume.

16. We have $1000 to buy the materials to build a box whose base length is seven times the base width and has no top. If the material for the sides cost $10/cm^2 and the material for the bottom cost $15/cm^2 determine the dimensions of the box that will maximize the enclosed volume.

17. We want to build a box whose base length is twice the base width and the box will enclose 80 ft^3. The cost of the material of the sides is $0.5/ft^2 and the cost of the top/bottom is $3/ft^2. Determine the dimensions of the box that will minimize the cost.

18. We want to build a box whose base is a square, has no top and will enclose 100 m^3. Determine the dimensions of the box that will minimize the amount of material needed to construct the box.

19. We want to construct a cylindrical can with a bottom but no top that will have a volume of 65 in^3. Determine the dimensions of the can that will minimize the amount of material needed to construct the can.

20. We want to construct a cylindrical can whose volume is 105 mm^3. The material for the wall of the can costs $3/mm^2, the material for the bottom of the can costs $7/mm^2 and the material for the top of the can costs $2/mm^2. Determine the dimensions of the can that will minimize the cost of the materials needed to construct the can.

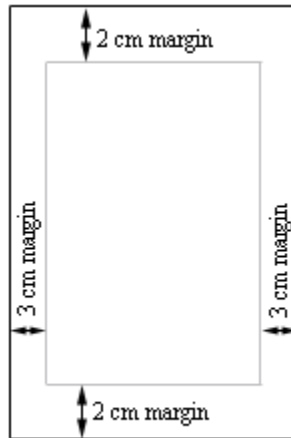21. We have a piece of cardboard that is 30 cm by 16 cm and we are going to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.
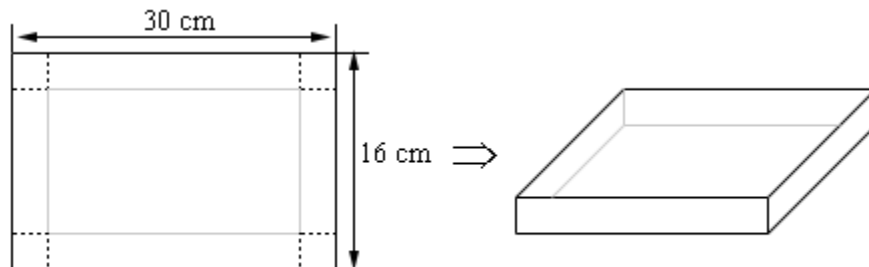
22. We have a piece of cardboard that is 5 in by 20 in and we are going to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.
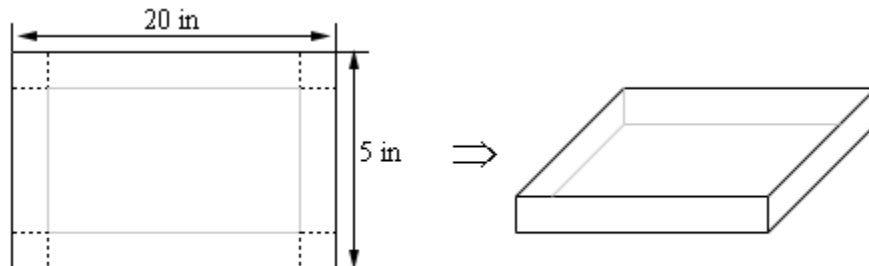
23. A printer needs to make a poster that will have a total of 500 cm^2 that will have 3 cm margins on the sides and 2 cm margins on the top and bottom. What dimensions of the poster will give the largest printed area?

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│        ↕ 2 cm margin        │
│ 3 ┌───────────────────┐ 3   │
│ c │                   │ c    │
│ m │                   │ m    │
│ m │                   │ m    │
│ a │                   │ a    │
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│        ↕ 2 cm margin        │
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24. A printer needs to make a poster that will have a total of 125 in^2 that will have ½ inch margin on the bottom, 1 inch margin on the right, 2 inch margin on the left and 4 inch margin on the top. What dimensions of the poster will give the largest printed area?

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┌─────────────────────────────┐
│            ↑                │
│   4 in margin               │
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│ 2 ┌───────────────┐ 1       │
│ i │               │ i        │
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│ i │               │ i        │
│ n └───────────────┘ n       │
│   1/2 in margin ↕           │
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