Problem 1

Matrix equation

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \right) = 96$$

Solving Matrix Equation through Determinants

It is difficult to show the circling process on this software, but I will try to use a spreadsheet to show you what is happening

The HIGHLIGHTED YELLOW is the original coefficient matrix

To find determinant of a 3x3 using Cramer's rule, to find the difference of the sum of products related to the three upper diagonals and sum of products related to the three lower diagonals

	A	B	C	D	E	F	G	H	I	J	K	L
=												
1												
2		-4	0	-6	-4	0						
3		-6	-4	-2	-6	-4						
4		-1	2	1	-1	2						
5												
6												

A1

Matrix equation

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix} \qquad \det\left(\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix}\right)=96$$

While the calculator does do this for you if you ask it to, we would like you to be able to verify it with paper and pencil methods

Step 1: copy the first two COLUMNS down outside of the coefficient matrix

(this is difficult to show with this software so I will use a spreadsheet to show this)

	A	B	C	D	E	F	G	H	I	J	K	L
=												
1												
2		-4	0	-6	-4	0						
3		-6	-4	-2	-6	-4						
4		-1	2	1	-1	2						
5												
6												
7												
8												
9												
10												

A1

Matrix equation

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

Step 2: find the sum of the products of the three upper diagonals

$(-4)(-4)(1)$ =product of first upper diagonal (this was highlighted in yellow) =16

$(0)(-2)(-1)$ =product of second upper diagonal (this was highlighted in blue) =0

$(-6)(-6)(2)$ =product of third upper diagonal (this was highlighted in grey) =72

OR $(-4)(-4)(1)+(0)(-2)(-1)+(-6)(-6)(2)=16+0+72=88$

So the sum of the products of the upper diagonals is 88

	A	B	C	D	E	F	G	H	I	J	K	L
=												
1												
2		-4	0	-6	-4	0						
3		-6	-4	-2	-6	-4						
4		-1	2	1	-1	2						
5												

A1

Matrix equation

$$\begin{bmatrix} ^-4 & 0 & ^-6 \\ ^-6 & ^-4 & ^-2 \\ ^-1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ^-12 \\ 6 \\ 9 \end{bmatrix}$$

Step 3: find the sum of the products of the three lower diagonals

$(^-1)(^-4)(^-6)$ =product of first lower diagonal (this was highlighted in yellow) $=^-24$

$(2)(^-2)(^-4)$ =product of second lower diagonal (this was highlighted in blue) $=16$

$(1)(^-6)(0)$ =product of third lower diagonal (this was highlighted in grey) $=0$

OR $(^-1)(^-4)(^-6)+(2)(^-2)(^-4)+(1)(^-6)(0)=^-24+16+0=^-8$

So the sum of the products of the lower diagonals is $^-8$

	A	B	C	D	E	F	G	H	I	J	K	L
=												
1												
2		-4	0	-6	-4	0						
3		-6	-4	-2	-6	-4						
4		-1	2	1	-1	2						
5												

A1

Matrix equation

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

Step 4: find the difference of the sums of the products in the upper and lower diagonals

the sum of the products of the upper diagonals is 88

the sum of the products of the lower diagonals is -8

the difference in these sums is $88 - -8 = 96$

So the determinant of $\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix}$ is 96 This will tell us if this equation has a solution

	A	B	C	D	E	F	G	H	I	J	K	L
=												
1												
2		-4	0	-6	-4	0						
3		-6	-4	-2	-6	-4						
4		-1	2	1	-1	2						
5												

A1

Matrix equation

$$\begin{bmatrix} ^-4 & 0 & ^-6 \\ ^-6 & ^-4 & ^-2 \\ ^-1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ^-12 \\ 6 \\ 9 \end{bmatrix}$$

So the determinant of $\begin{bmatrix} ^-4 & 0 & ^-6 \\ ^-6 & ^-4 & ^-2 \\ ^-1 & 2 & 1 \end{bmatrix}$ is 96

The determinant will tell us if this equation has a solution

when determinant of coefficient matrix is NONZERO then the related matrix has an inverse and the related matrix equation has a solution

WHY does 0 occur as a determinant?

Either there are parallel lines (2x2 matrices) or coinciding lines (same line)

Either there are parallel planes (3x3 matrices) or coinciding planes (same plane), or three planes that intersect with each other but not all together at the same place

BEYOND 3x3 hurts my head so GO TO COLLEGE and ask a REALLY smart mathematics professor

Now we know the denominator of three fractions that lead to the solution to a 3x3 matrix is 96 $x = \dfrac{\text{"det of replace X"}}{96}$ $y = \dfrac{\text{"det of replace Y"}}{96}$ $z = \dfrac{\text{"det of replace Z"}}{96}$

(this program moves negatives to numerator, be careful to look for it)

Matrix equation

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

What are the "replace matrices"?

replace X matrix will take the X column OUT and replace it with the constant matrix

Replace X matrix = $\begin{bmatrix} -12 & 0 & -6 \\ 6 & -4 & -2 \\ 9 & 2 & 1 \end{bmatrix}$

Replace Y matrix will take the Y column OUT and replace it with the constant matrix

Replace Y matrix = $\begin{bmatrix} -4 & -12 & -6 \\ -6 & 6 & -2 \\ -1 & 9 & 1 \end{bmatrix}$

Replace Z matrix will take the Z column OUT and replace it with the constant matrix

Replace Z matrix = $\begin{bmatrix} -4 & 0 & -12 \\ -6 & -4 & 6 \\ -1 & 2 & 9 \end{bmatrix}$

If the determinant of the coefficient matrix is NONZERO, then the determinants of the replace matrices become the numerators of the solution to the 3x3 matrix equation

Hopefully you understood the method from the coefficient matrix's determinant because you will need to repeat this process three more times to get the determinants of the replace matrices

$$\begin{bmatrix} ^-4 & 0 & ^-6 \\ ^-6 & ^-4 & ^-2 \\ ^-1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ^-12 \\ 6 \\ 9 \end{bmatrix}$$

	A	B	C	D	E	F
=						
1	-12	0	-6	-12	0	
2	6	-4	-2	6	-4	
3	9	2	1	9	2	
4						
5						
6						
7						
8						

A1 =b_11

	A	B	C	D	E	F
=						
1	-4	-12	-6	-4	-12	
2	-6	6	-2	-6	6	
3	-1	9	1	-1	9	
4						
5						
6						
7						
8						

A1 =a_11

	A	B	C	D	E	F
=						
1	-4	0	-12	-4	0	
2	-6	-4	6	-6	-4	
3	-1	2	9	-1	2	
4						
5						
6						
7						
8						

A1 =a_11

Replace x matrix process

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

The replace x matrix is $\begin{bmatrix} -12 & 0 & -6 \\ 6 & -4 & -2 \\ 9 & 2 & 1 \end{bmatrix}$

it has a determinant of -288

	A	B	C	D	E	F
=						
1	-12	0	-6	-12	0	
2	6	-4	-2	6	-4	
3	9	2	1	9	2	
4						
5						
6						
7						
8						

$(-12)(-4)(1)+(0)(-2)(9)+(-6)(6)(2)=48+0+-72=-24$

So the sum of the products of the upper diagonals of replace X matrix is -24

$(9)(-4)(-6)+(2)(-2)(-12)+(1)(6)(0)=216+48+12=264$

So the sum of the products of the lower diagonals of replace X matrix is 264

determinant of replace X matrix $-24-264=-288$

first coordinate in solution is $x=-288/96=-3$

Replace y matrix process

$$\begin{bmatrix} ^-4 & 0 & ^-6 \\ ^-6 & ^-4 & ^-2 \\ ^-1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ^-12 \\ 6 \\ 9 \end{bmatrix}$$

The replace y matrix is $\begin{bmatrix} ^-4 & ^-12 & ^-6 \\ ^-6 & 6 & ^-2 \\ ^-1 & 9 & 1 \end{bmatrix}$

it has a determinant of 96

	A	B	C	D	E	F
=						
1	-4	-12	-6	-4	-12	
2	-6	6	-2	-6	6	
3	-1	9	1	-1	9	
4						
5						
6						
7						
8						

$(^-4)(6)(1)+(^-12)(^-2)(^-1)+(^-6)(^-6)(9)=^-24+^-24+324=276$

So the sum of the products of the upper diagonals of replace Y matrix is 276

$(^-1)(6)(^-6)+(9)(^-2)(^-4)+(1)(^-6)(^-12)=36+72+72=180$

So the sum of the products of the lower diagonals of replace Y matrix is 180

determinant of replace Y matrix $276-180=96$

second coordinate in solution is $y=96/96=1$

Replace z matrix process

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

The replace z matrix is $\begin{bmatrix} -4 & 0 & -12 \\ -6 & -4 & 6 \\ -1 & 2 & 9 \end{bmatrix}$

it has a determinant of 384

	A	B	C	D	E	F
=						
1	-4	0	-12	-4	0	
2	-6	-4	6	-6	-4	
3	-1	2	9	-1	2	
4						
5						
6						
7						
8						

A1 =a_11

$(-4)(-4)(9)+(0)(6)(-1)+(-12)(-6)(2)=144+0+144=288$

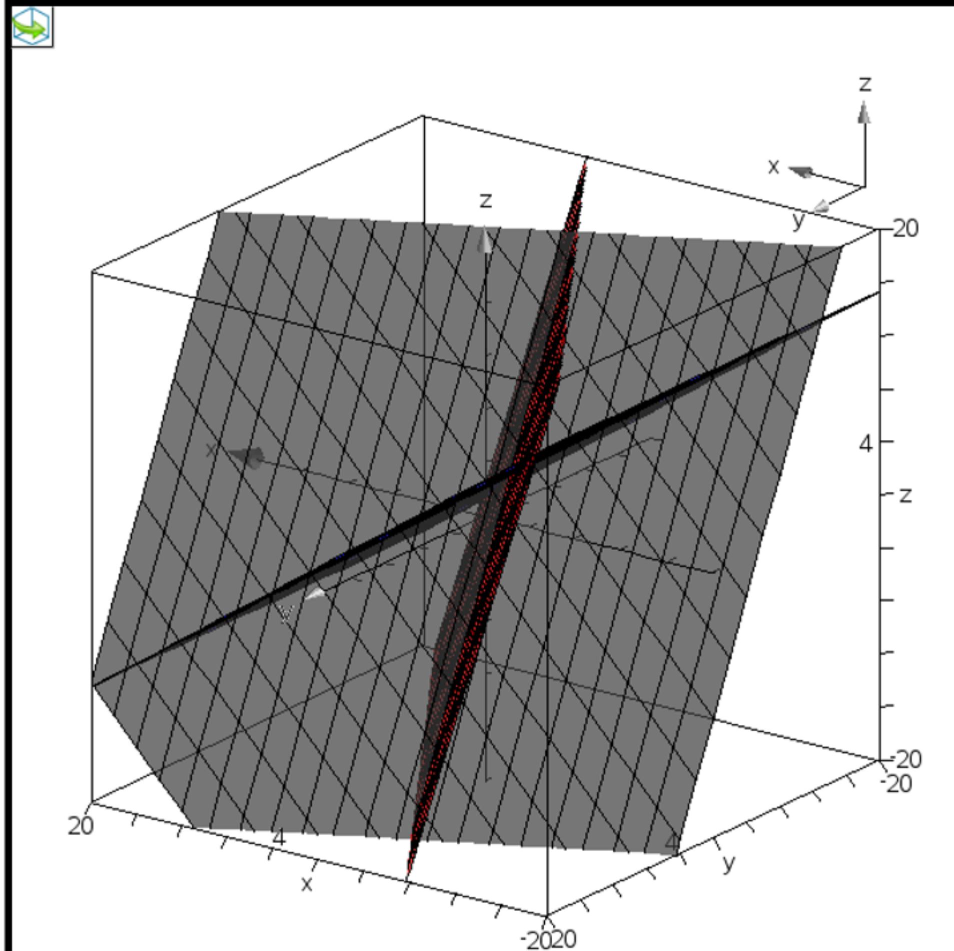
So the sum of the products of the upper diagonals of replace Z matrix is 288

$(-1)(-4)(-12)+(2)(6)(-4)+(9)(-6)(0)=-48+-48+0=-96$

So the sum of the products of the lower diagonals of replace Z matrix is -96

determinant of replace Z matrix $288 - -96 = 384$

last coordinate in solution is $z = 384/96 = 4$



The solution of a 3x3 Matrix is the place that three planes intersect

in this example we know this solution is

x = $^{-}3$ y = 1 z = 4

or ($^{-}3$,1 ,4)

or $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ^{-}3 \\ 1 \\ 4 \end{bmatrix}$

	A	B	C	D	E	F	G	H	I	J	K	L
1	a_11	-4	a_12	0	a_13	-6		b_11	-12	x_solve	-3	
2	a_21	-6	a_22	-4	a_23	-2		b_21	6	y_solve	1	
3	a_31	-1	a_32	2	a_33	1		b_31	9	z_solve	4	

K5

$^-4\cdot x-6\cdot z={}^-12$

$^-6\cdot x-4\cdot y-2\cdot z=6$

$^-x+2\cdot y+z=9$

Note: the GIANT −1 means applying the INVERSE to a matrix

Related matrix equation

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

Solving Matrix Equation through Inverses

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 0 & \frac{-1}{8} & \frac{-1}{4} \\ \frac{1}{12} & \frac{-5}{48} & \frac{7}{24} \\ \frac{-1}{6} & \frac{1}{12} & \frac{1}{6} \end{bmatrix}\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 & \frac{-1}{8} & \frac{-1}{4} \\ \frac{1}{12} & \frac{-5}{48} & \frac{7}{24} \\ \frac{-1}{6} & \frac{1}{12} & \frac{1}{6} \end{bmatrix}\begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$$

Matrix equation

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

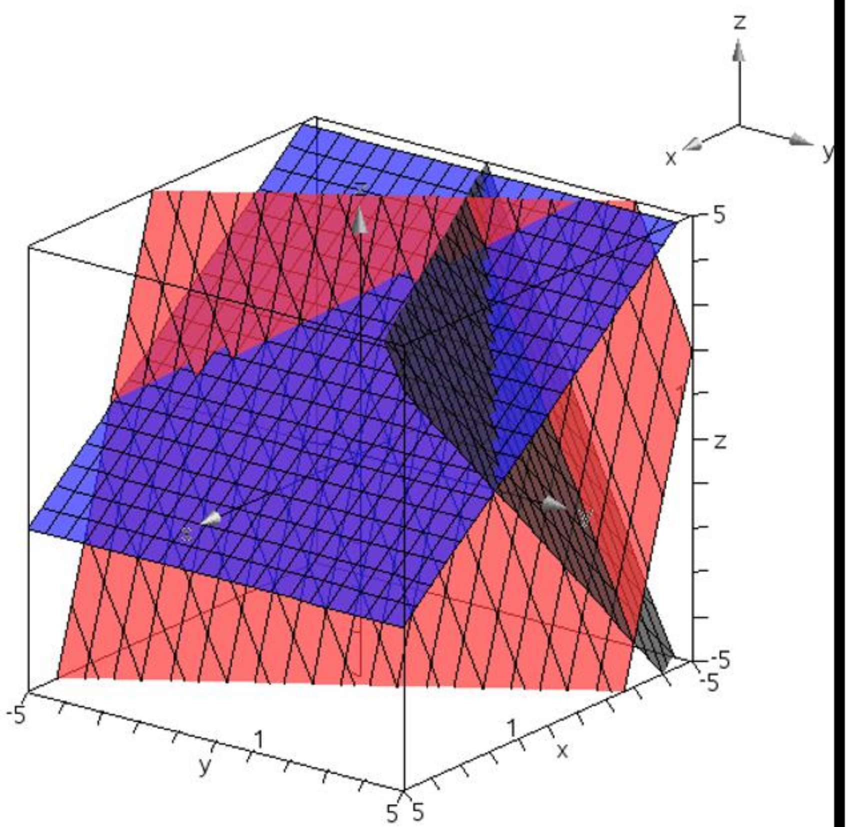
Solving Matrix Equation through Inverses

$$\begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \frac{-1}{8} & \frac{-1}{4} \\ \frac{1}{12} & \frac{-5}{48} & \frac{7}{24} \\ \frac{-1}{6} & \frac{1}{12} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} -4 & 0 & -6 \\ -6 & -4 & -2 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{8} & \frac{-1}{4} \\ \frac{1}{12} & \frac{-5}{48} & \frac{7}{24} \\ \frac{-1}{6} & \frac{1}{12} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} -12 \\ 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$$

Note: the GIANT −1 means applying the INVERSE to a matrix



The solution of a 3x3 Matrix is the place that three planes intersect

in this example we know this solution is

x = $^-3$ y = 1 z = 4

or ($^-3$,1 ,4)

or $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \mathbf{x\_solve} \\ \mathbf{y\_solve} \\ \mathbf{z\_solve} \end{bmatrix} \blacktriangleright \begin{bmatrix} {}^-3 \\ 1 \\ 4 \end{bmatrix}$

	A	B	C	D	E	F	G	H	I	J	K	L
=												
1	a_11	-4	a_12	0	a_13	-6		b_11	-12	x_solve	-3	
2	a_21	-6	a_22	-4	a_23	-2		b_21	6	y_solve	1	
3	a_31	-1	a_32	2	a_33	1		b_31	9	z_solve	4	
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K4